



**Spin-Singlet Resonance State
in Proton-Embedded Metals:
Discovery of novel high- T_K system
leading to high- T_c superconductivity**

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1. Introduction

Heavy fermion superconductors

Concept of metallic screening

2. Proton Embedded in Metals as Modeled by the Electron Gas

Transition from a positive H^+ ion to a negative closed-shell H^- ion

3. Emergence of a Spin-Singlet Resonance State

Appearance of Kondo resonance between H^+ and H^- states with $T_K \sim 2,000K$ at $r_s=4$.

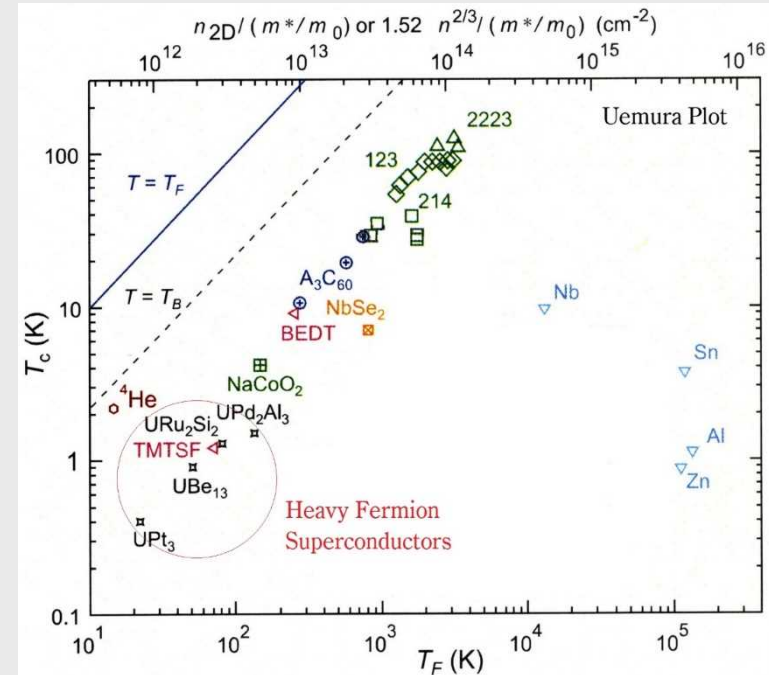
4. Summary



Heavy Fermion Superconductors

- ◎ T_c (and also T_F) is low, but T_c / T_F is large.
- ◎ Some of them are considered as Kondo-lattice systems and strong correlation exists between T_c and the Kondo temperature T_K .

Material	T_c (K)	T_K (K)
CeCu ₂ Si ₂	0.7	15-20
CePd ₂ Si ₂	0.43	10
CeNi ₂ Ge ₂	0.2	30
CeCu ₂ Ge ₂	0.6	4
CeCu ₂ Si ₂	0.6	10
URu ₂ Si ₂	1.5	60



→ Spin-fluctuation mechanism of superconductivity in the Kondo lattice;

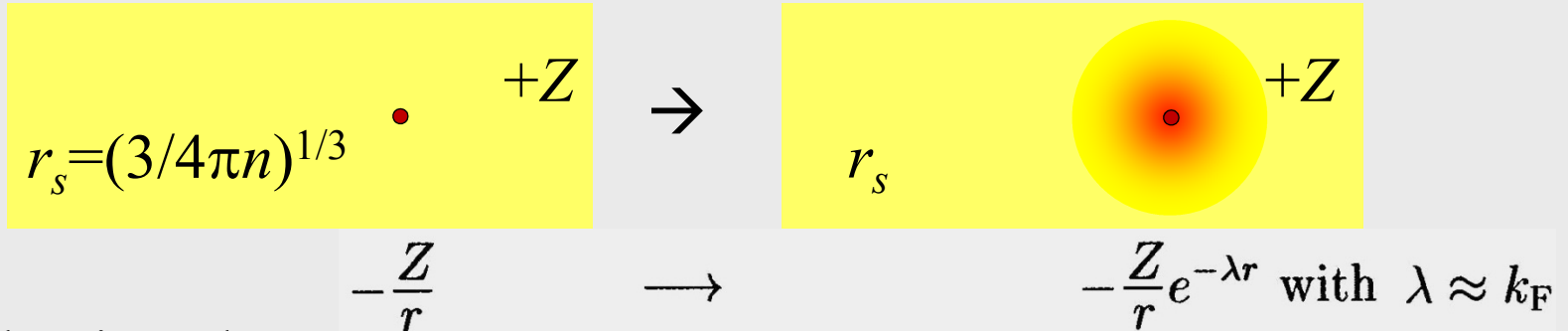
$$T_c / T_K \text{ is } 0.04\text{-}0.15.$$

- (1) Look for a new Kondo system with $T_K > 1,000\text{K}$.
 - (2) Construct a Kondo lattice to obtain a high- T_c superconductor!
- ← Make all energy scales hundred times up!



Charged Point Impurity in a Metal

- ◎ Put an impurity of charge $+Z$ in a metal with its density n or the r_s parameter, defined by $r_s = (3/4\pi n)^{1/3}$



- ◎ Conduction electrons respond to screen the charge, as described by the linear-response theory (Debye-Hückel or Thomas-Fermi)
 \rightarrow **Charge resonance** of metallic electrons and their **Friedel oscillation**

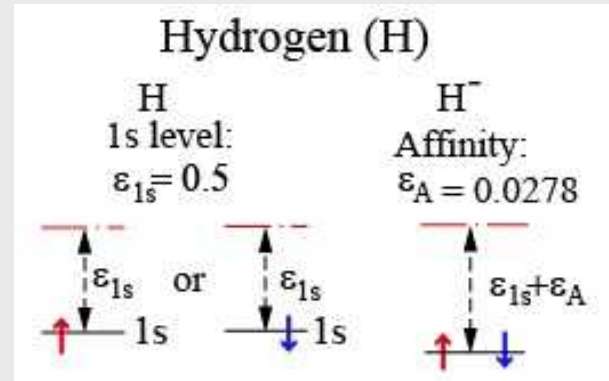
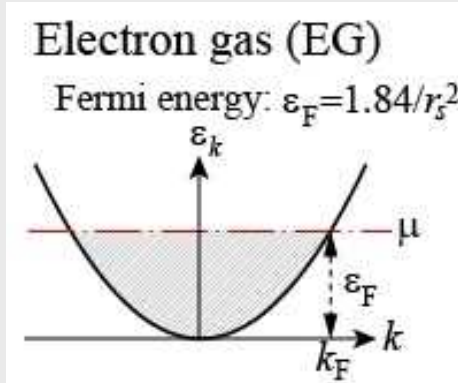
◎ Basic Question:

If Z is small enough, this is always a valid concept, but if Z becomes large, the effects of **nonlinear response** become important. Then, what about the effect of **spin resonance**?
This spin effect is enhanced for an odd integer Z .

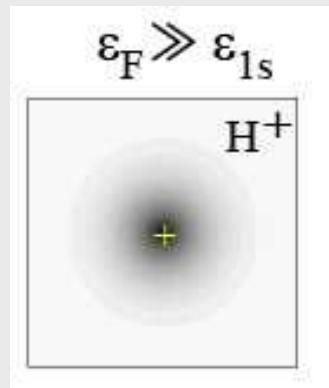


Proton-Embedded Electron Gas

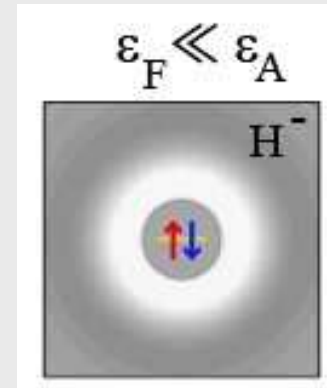
- Case of a proton ($Z=1$) embedded in the electron gas
- Compare the energy scales of the electron gas and the H atom:



(i) $r_s < 2$:



(ii) $r_s > 10$



“Charge resonance state”
screening bare $Z=1$ charge

“H⁻ ion confinement state”
screening composite $Z=-1$ charge

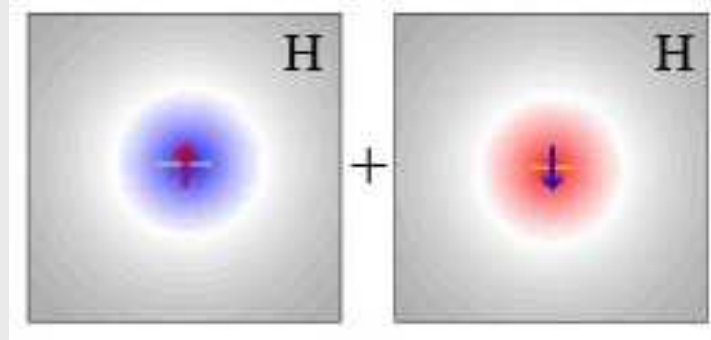
→ What is the state in between?



Kondo Spin-Singlet Resonance State

◎ Case of $\varepsilon_A < \varepsilon_F < \varepsilon_{1s}$:

- Naively, we may imagine a single-electron localized state with either up or down spin. → **Spin polarized!**
- Because of **Kondo physics**, this cannot be a true ground state. Rather, a **spin-singlet resonance** state is expected in which the conduction electrons at the Fermi level hybridize with the electron in the 1s level with changing spin orientation.



- Here the spin-screening length ξ_K is long, i.e., $\xi_K \gg k_F^{-1}$.
cf. The Thomas-Fermi screening length λ^{-1} in the **charge resonance** state is short, i.e., $\lambda^{-1} \sim k_F^{-1}$.



Purpose of Research

- Usually, Kondo physics is discussed in **the impurity Anderson model**,

$$H_A = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^+ c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma} (V c_{\mathbf{k}\sigma}^+ c_{d\sigma} + V^* c_{d\sigma}^+ c_{\mathbf{k}\sigma}) + \sum_{\sigma} E_d c_{d\sigma}^+ c_{d\sigma} + U c_{d\uparrow}^+ c_{d\uparrow} c_{d\downarrow}^+ c_{d\downarrow}$$

but in our many-electron system with a proton at the origin of coordinates,

$$H = - \sum_i \frac{\nabla_i^2}{2} + \frac{1}{2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} - \sum_i \frac{1}{|\mathbf{r}_i|}$$

- (i) no predetermined **localized d state** exists;
- (ii) the **long-range Coulomb interaction** works even among conduction electrons.

→ Must confirm **the existence of the Kondo spin-singlet state in the actual proton embedded electron gas.**

- Because the long-range Coulomb interaction is the origin for the charge resonance state, **competition between charge and spin resonances** is also an important issue to investigate.



Strategy for Confirmation

⊙ Difficulties of the problem:

- (i) The techniques developed for H_A cannot be applied to our H .
- (ii) The Kondo-singlet wave function Ψ_0 can only be constructed in the infinite system (or the total electron number $N \rightarrow \infty$), so that the Diffusion Monte Carlo (DMC) cannot be used as it is.
- (iii) The DFT approach such as the LSDA calculation cannot treat such a highly correlated many-body wave function.

⊙ Our basic strategy:

We focus on the ground-state electron density $n(\mathbf{r})$ rather than Ψ_0 .

cf. In principle, it is proved that DFT can provide the exact $n(\mathbf{r})$ even for strongly-correlated systems.

- Check $n(\mathbf{r})$ in LSDA at finite N in comparison with DMC.
- Obtain $n(\mathbf{r})$ in either LSDA at $N \rightarrow \infty$ or extrapolation in DMC.
- Check the obtained $n(\mathbf{r})$ at $N \rightarrow \infty$ reproducing the behavior characteristic to the Kondo spin-singlet resonance state, or “the anomalous (shortened-period) Friedel oscillation”.



LSDA Calculation

M. Shimomoto & K. Yoshizawa (ISSP)

Kohn-Sham scheme for single-atom embedding

$$\left[-\frac{\Delta}{2} + U_{\sigma}^{\text{KS}}(\mathbf{r})\right] \phi_{i\sigma}(\mathbf{r}) = \epsilon_{i\sigma} \phi_{i\sigma}(\mathbf{r})$$

$$U_{\sigma}^{\text{KS}}(\mathbf{r}) = -\frac{Z}{r} + \int d\mathbf{r}' \frac{n(\mathbf{r}') - n_0}{|\mathbf{r} - \mathbf{r}'|} + U_{xc}(\mathbf{r}\sigma; [n_{\sigma}(\mathbf{r})]) - U_{xc}(n_0)$$

n_0 : the uniform density of the host electron gas

$$U_{xc}(\mathbf{r}\sigma; [n_{\sigma}(\mathbf{r})]) = \frac{\delta E_{xc}^{\text{LDA}}[n_{\uparrow}, n_{\downarrow}]}{\delta n_{\sigma}(\mathbf{r})}, \quad U_{xc}(n_0) = \frac{\delta E_{xc}^{\text{LDA}}[n_0]}{\delta n_0} \quad \text{and} \quad n_{\sigma}(\mathbf{r}) = \sum_{i(\epsilon_{i\sigma} \leq \mu)} |\phi_{i\sigma}(\mathbf{r})|^2$$

Boundary condition for an infinite system: \rightarrow Phase shift: $\delta_{l\sigma}(p)$

Write $\phi_{i\sigma}(\mathbf{r}) = R_{pl\sigma}(r) Y_{lm}(\theta, \varphi)$,

then at $r \rightarrow \infty$, $R_{pl\sigma} \rightarrow \cos[\delta_{l\sigma}(p)] j_l(pr) - \sin[\delta_{l\sigma}(p)] n_l(pr)$

Change in the total energy associated with this embedding: δE **Immersion Energy**

$$\delta E = \sum_{i\sigma \in \{\text{Bound states}\}} \epsilon_{i\sigma} + \frac{1}{\pi} \sum_{l\sigma} (2l+1) \int_0^{k_F} dp \frac{p^2}{2} \delta'_{l\sigma}(p) + \int d\mathbf{r} \left\{ \frac{Z}{r} (n_0 - n(\mathbf{r})) - \sum_{\sigma} U_{xc}(\mathbf{r}\sigma; [n_{\sigma}(\mathbf{r})]) n_{\sigma}(\mathbf{r}) \right\} + \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \frac{(n(\mathbf{r}) - n_0)(n(\mathbf{r}') - n_0)}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}^{\text{LDA}}[n_{\uparrow}, n_{\downarrow}] - E_{xc}^{\text{LDA}}[n_0]$$



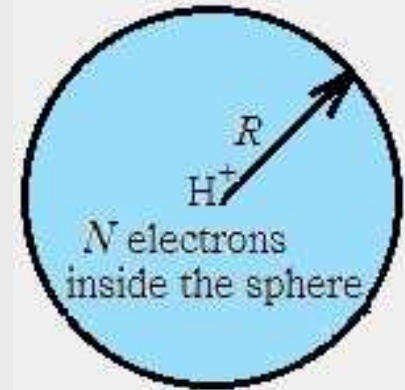
VMC + Diffusion Monte Carlo

Ryo Maezono (JAIST)

N -electron systems confined in a sphere of R (in which the embedded atom is situated at the center of the sphere) with the fixed boundary condition at the surface.

Slater-Jastrow trial function for VMC

$$\Psi(\vec{x}_1, \dots, \vec{x}_N) = e^{J(\vec{X})} \cdot \begin{vmatrix} \psi_1(\vec{x}_1) & \cdots & \psi_N(\vec{x}_1) \\ \vdots & \ddots & \vdots \\ \psi_1(\vec{x}_N) & \cdots & \psi_N(\vec{x}_N) \end{vmatrix}$$



Jastrow factor

$$J(\vec{X}) = \underbrace{\sum_{i>j} u(r_{ij})}_{\text{two-electron correlation}} + \underbrace{\sum_{i,I} \chi_I(r_{iI})}_{\text{electron-proton correlation}} + \underbrace{\sum_{I,j>j} f_I(r_{iI}, r_{jI}, r_{ij})}_{\text{two-electron-proton correlation}}$$

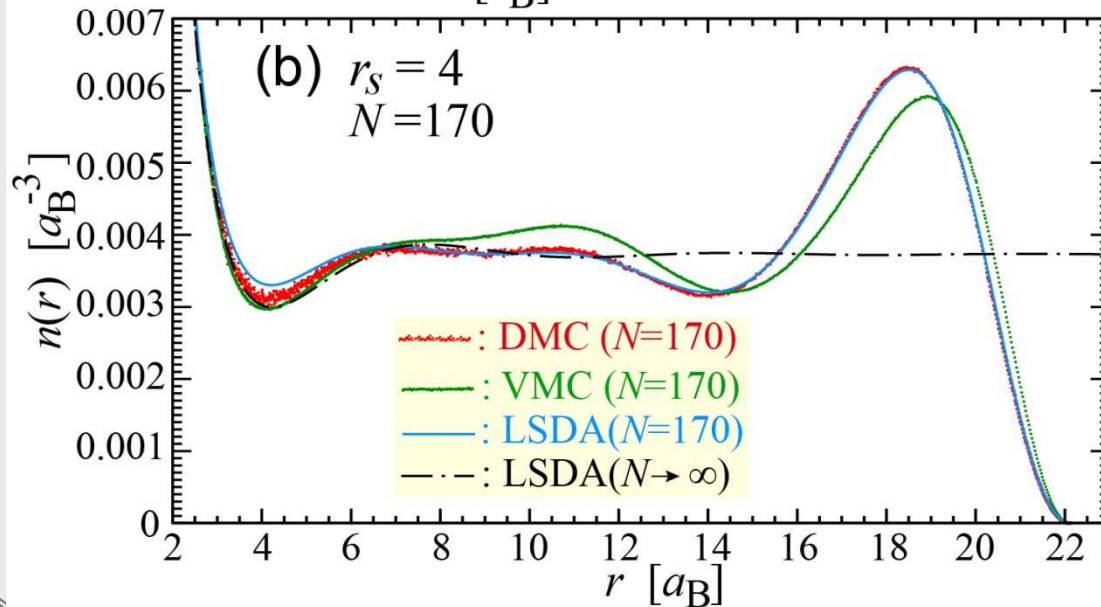
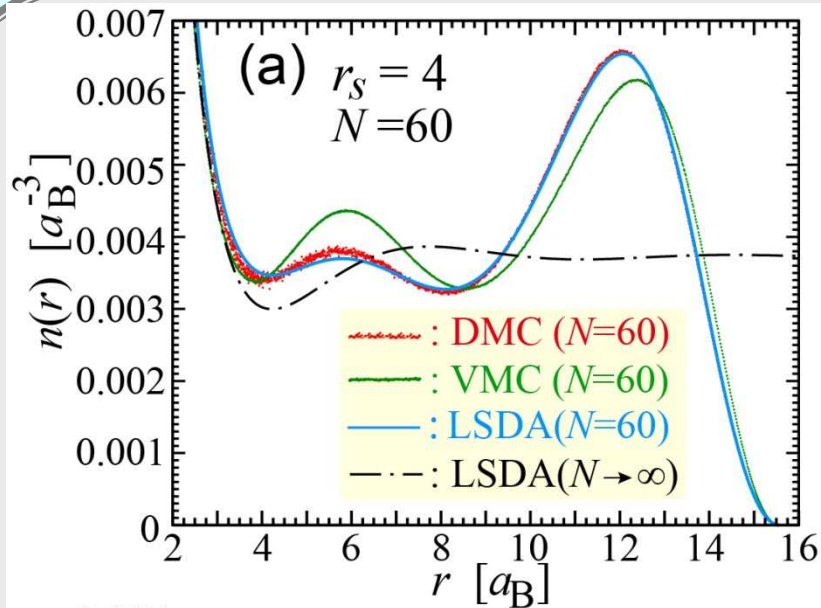
Fixed-node DMC: **Casino code** developed by Needs' group at Cambridge.

Mostly at $N=58$ or 60 , but for checking size effects, N is increased up to 132 and 170 .

→ N -independent quantities are supposed to be independent of the position of fixed nodes, because the node-positions vary with R or N for a given value of r_s .



Comparison between LSDA & DMC



○ No stable spin-polarized state is obtained in either LSDA and DMC.

Thus LSDA gives the same results as the simple LDA.

○ Rather strong N dependence is seen.

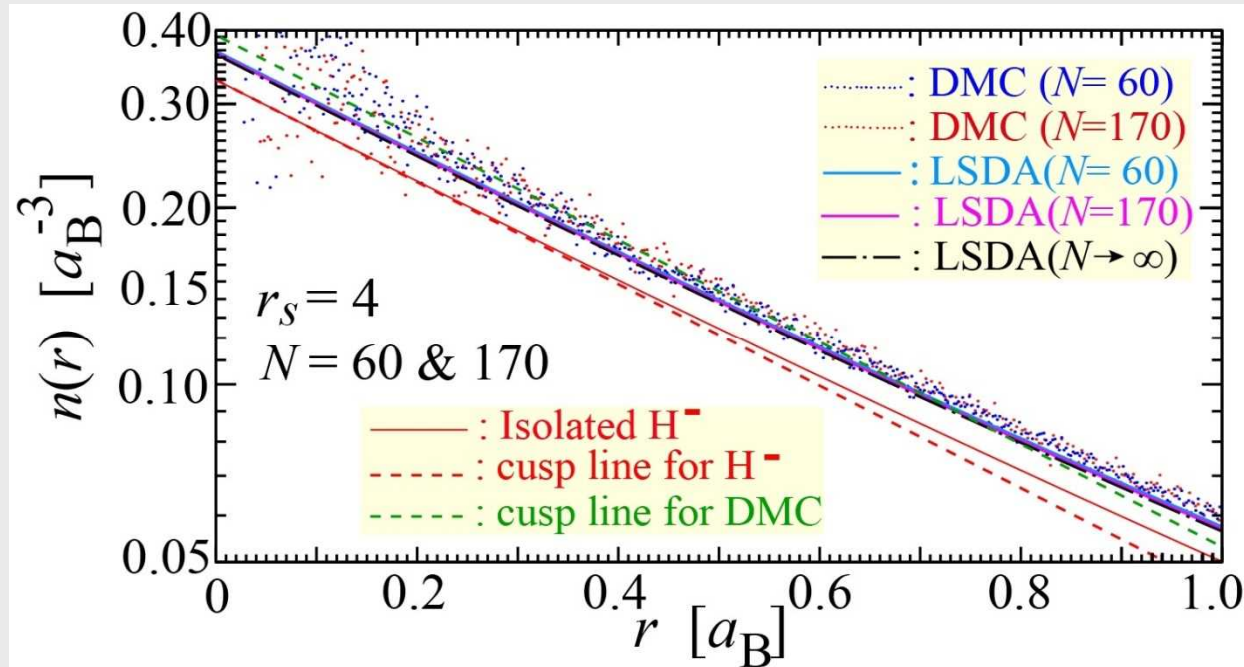
cf. This should be, as long as R is smaller than ξ_K .

○ At each N , DMC and LSDA agree surprisingly well in the whole range of r , though VMC deviates much from both.



Cusp Theorem and $n(0)$

- For $r \sim 0$, **the cusp theorem** around the proton ($Z=1$) predicts:
 $n(r) \sim n(0) \exp(-2Zr)$

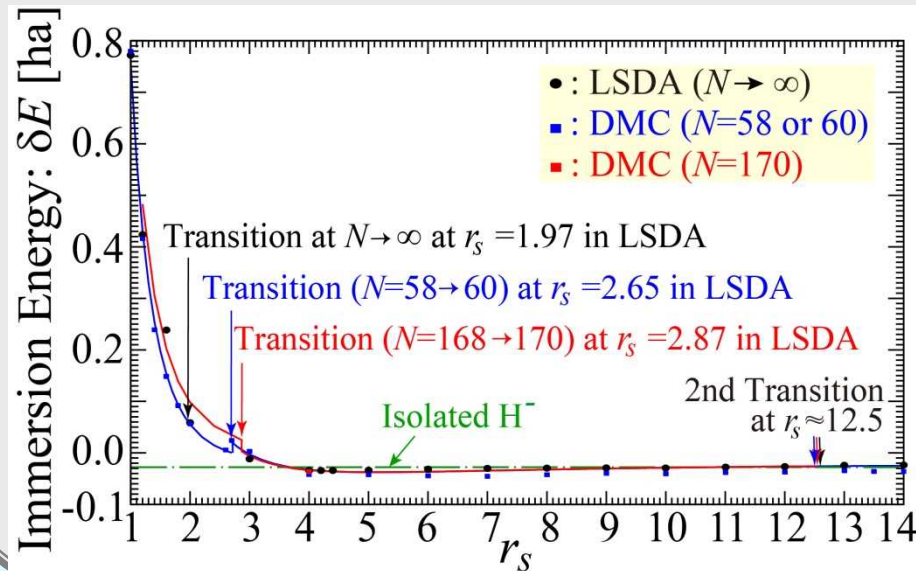
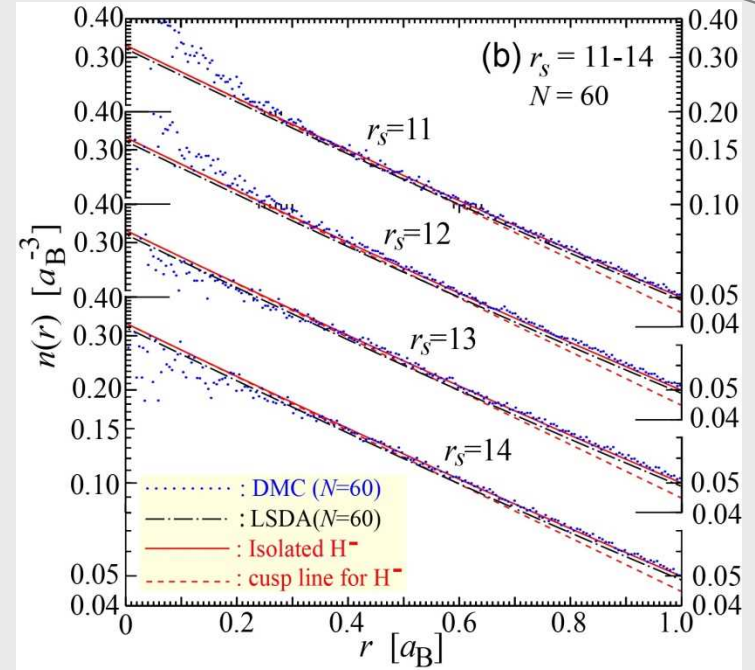
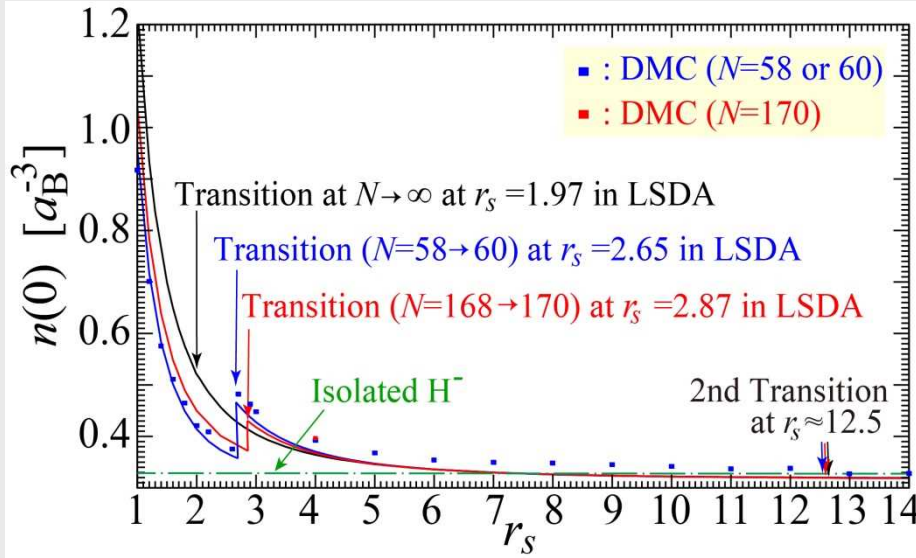


- From the cusp line in the semi-log plot, we can determine $n(0)$ accurately. For $r_s=4$, $n(0)=0.395$ in atomic units in DMC, while it is 0.369 in LSDA. Note that it is 0.329 for an isolated H^- ion.
- In the vicinity of $r=0$, $N=60$ is already large enough to obtain the convergent results for $n(r)$ in the limit of $N \rightarrow \infty$.

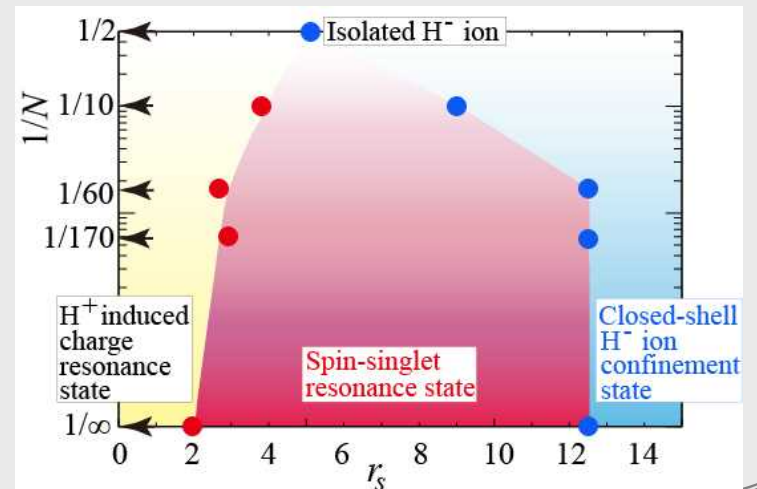


Two Sequential Transitions

○ Plot as a function of r_s



○ Plot of the transitions in (r_s, N) space





Friedel Oscillation

○ For metals **without** the Kondo resonance state:

$$\lim_{r \rightarrow \infty} [n(\mathbf{r}) - n_0] \approx \frac{1}{8\pi^2 r^3} \sum_{l\sigma} (2l+1)(-1)^l \left[\cos\left(2k_F r - \frac{3\pi}{2} + 2\delta_{l\sigma}(k_F)\right) - \cos\left(2k_F r - \frac{3\pi}{2}\right) \right]$$

Consider the s-wave part. ↓ Affleck, Borda & Saleur, PRB77, 180404(R) (2008)

○ For metals **with** the Kondo resonance state:

Put $F(r/\xi_K)$ here.

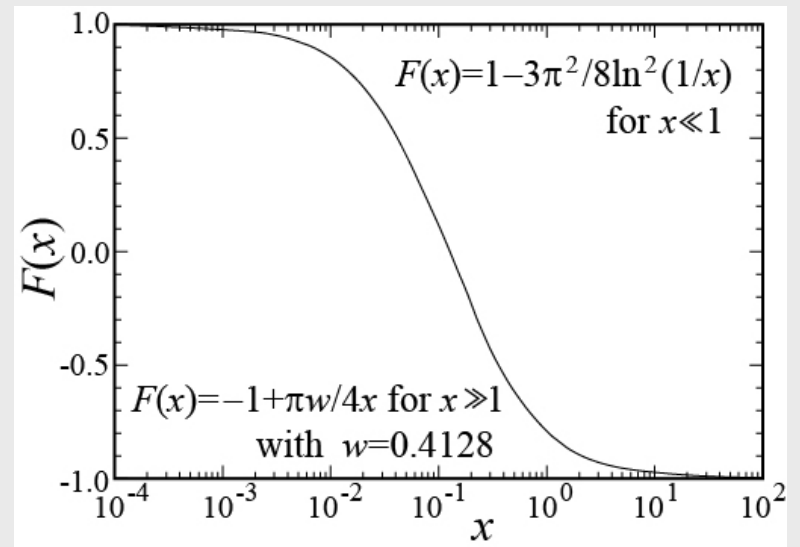
$$n(\mathbf{r}) \xrightarrow{r \gg k_F^{-1}} n_0 + \frac{A(r)}{4\pi^2 r^3} \cos[2k_F r + \delta(r)]$$

$$A(r) = \sqrt{1 - 2F(r/\xi_K) \cos(2\delta_0) + F(r/\xi_K)^2}, \quad \delta(r) = \tan^{-1} \left[\frac{1 - F(r/\xi_K) \cos(2\delta_0)}{F(r/\xi_K) \sin(2\delta_0)} \right]$$

s-wave phase shift: $\delta_0 [\equiv \delta_{0\uparrow}(k_F) = \delta_{0\downarrow}(k_F)]$

In accordance with the behavior of the universal function $F(x)$, $\delta(r) = \delta_0$ for $r \ll \xi_K$, changing into $\delta(r) = \delta_0 + \pi/2$ for $r \gg \xi_K$

cf. Without the Kondo resonance state, $F \equiv 1 \rightarrow \delta(r) \equiv \delta_0$.





Anomalous Friedel Oscillation

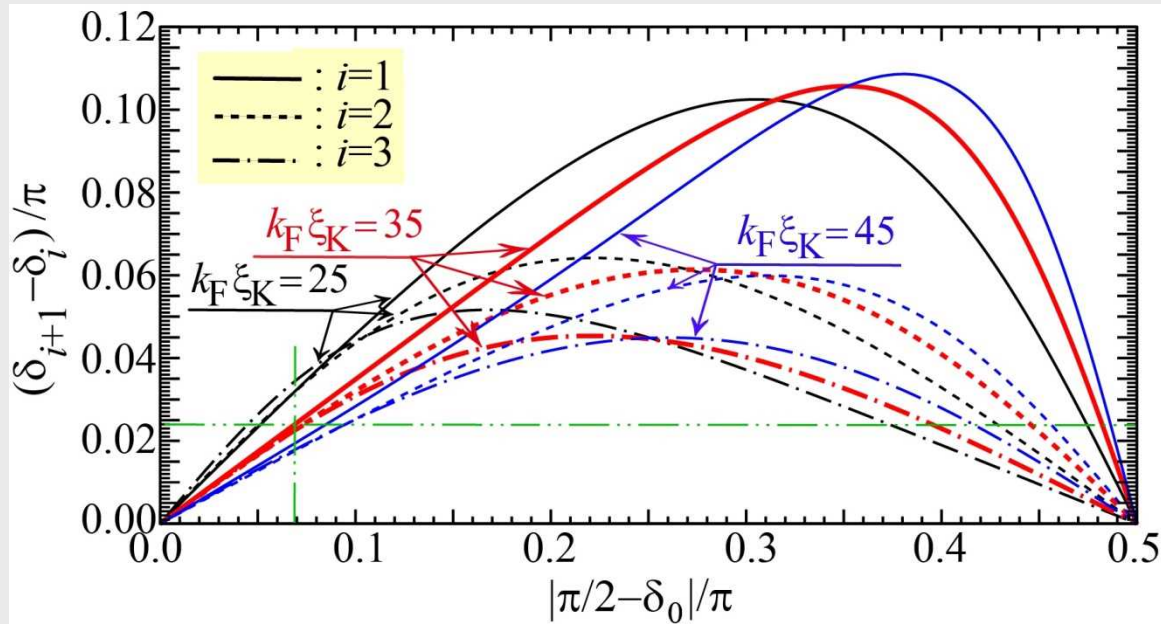
○ $\delta(r)$ is a monotonically increasing function of r over ξ_K

Write r_i as i th zero of $n(r)-n_0$: $2k_F r_i + \delta_0(r_i) = i\pi + \pi/2$

Write r_{i+1} as $(i+1)$ th zero of $n(r)-n_0$: $2k_F r_{i+1} + \delta_0(r_{i+1}) = (i+1)\pi + \pi/2$

$$\rightarrow r_{i+1} - r_i = \pi \{1 - [\delta(r_{i+1}) - \delta(r_i)]/\pi\} / 2k_F < \pi/2k_F$$

Shortening of the half period of the Friedel oscillation by the factor of $[\delta(r_{i+1}) - \delta(r_i)]/\pi$!

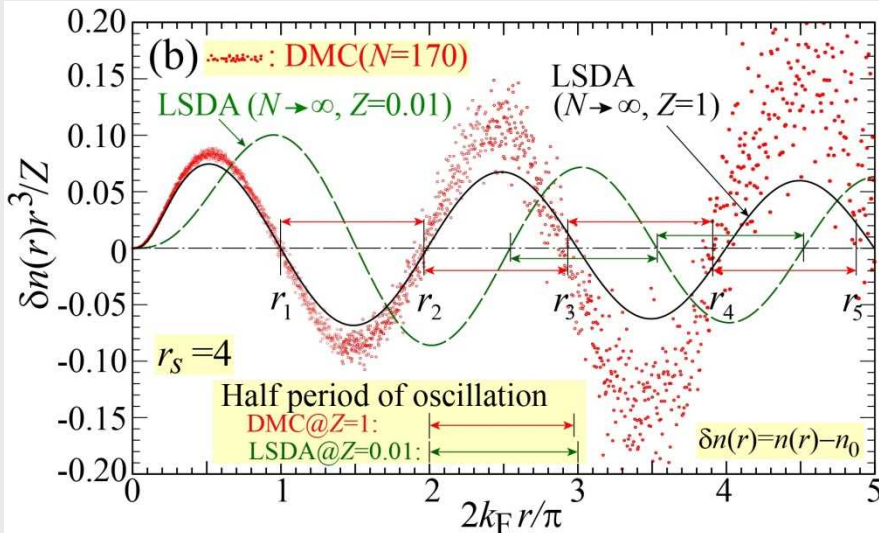
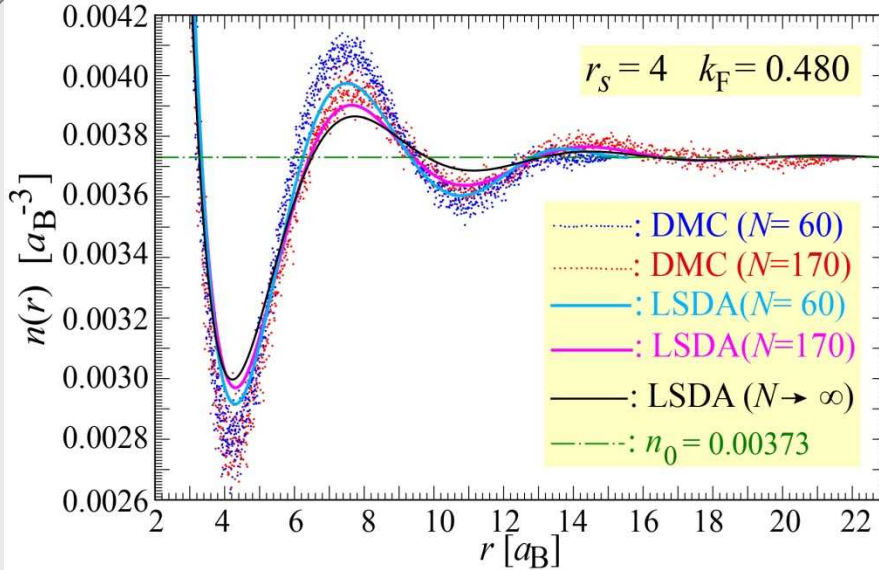


An example of the shortening factor plotted as a function of δ_0 for various $k_F \xi_K$.

First few shortening factors are enough to uniquely determine δ_0 and $k_F \xi_K$.



Shortening of the Oscillation Period in DMC



- Size correction in DMC:

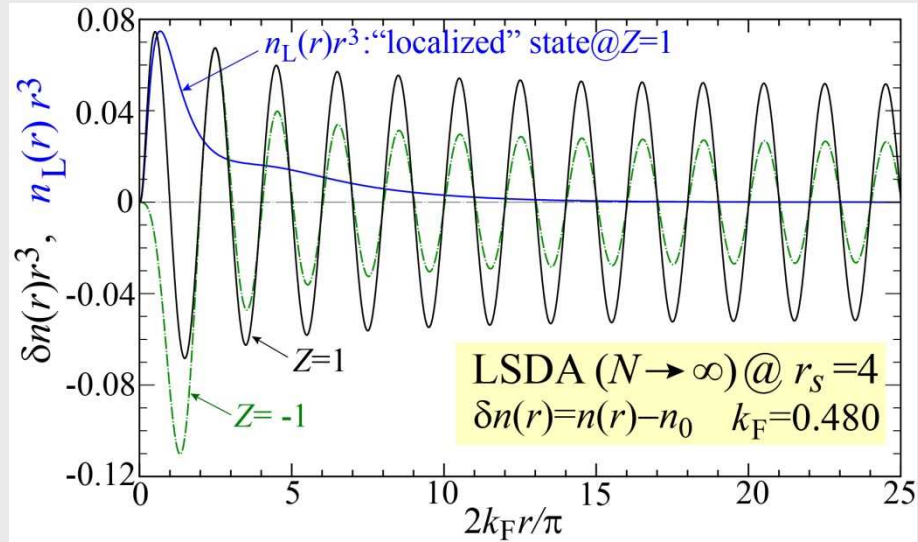
$$n(\mathbf{r}) = n(\mathbf{r})_{Z=1, N} - \lambda^3 n(\lambda \mathbf{r})_{Z=0, N-2} + n_0$$

with $\lambda = [(N-1)/(N-2)]^{1/3}$

- For $r < 13$, oscillation nodes are converged.
- The node positions are:
 - $2k_F r_1/\pi = 0.994 \pm 0.005$
 - $2k_F r_2/\pi = 1.970 \pm 0.008$
 - $2k_F r_3/\pi = 2.949 \pm 0.013$
 - $2k_F r_4/\pi = 3.927 \pm 0.019$
- The corresponding shortening factors $[\delta_{i+1} - \delta_i]/\pi$ are 0.024, 0.021, and 0.022 for $i=1, 2$, and 3, respectively.
- These factors are consistent with the choice of $k_F \xi_K \sim 35$.



Shortening of the Oscillation Period in LDA



- There appears a **very shallow** (extending about $r \sim 80$ or $k_F r \sim 38$) **1s bound state** in LSDA, but this is **not a real physical state**, but it appears only to correctly reproduce $n(r)$ in the Kondo spin-singlet resonance state.
- The shortening of the oscillation period appears as long as this shallow-state density $n_L(r)$ exists, as seen by comparison with the oscillation induced by the point charge of $Z = -1$ at the origin.
- The Kondo temperature $T_K \sim (2/k_F \xi_K) E_F$ is estimated as:
 $\sim 2100\text{K}$ in DMC and $\sim 1900\text{K}$ in LSDA at $r_s = 4$. Thus we cannot expect to observe the $\ln T$ anomaly in the usual experiments.

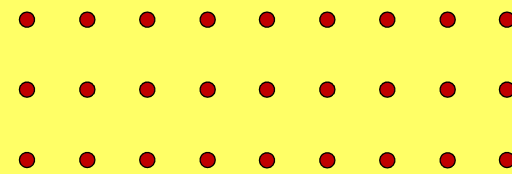


Summary

- 1⁰ Two sequential transitions in the ground state with intervening Kondo spin-singlet resonance state are clarified and identified for the proton embedding: **With decreasing the metallic density, H^+ (charge resonance) \rightarrow H (Kondo resonance) \rightarrow H^- (negative ion)**
- 2⁰ This result enriches the paradigm for metallic screening to a point charge of Z in the sense that, depending on the metallic density and Z , the spin resonance takes the place of the charge resonance.
- 3⁰ By referring to heavy-fermion superconductors and by considering the astonishingly high Kondo temperature in the proton-embedded metal in the intermediate-density region, we may expect **room-temperature superconductivity** in a metallic hydrogen alloy, if the electron density is appropriately given.

$$r_s = (3/4\pi n)^{1/3}$$

• H



Single-hydrogen Kondo system

Hydrogen Kondo lattice