Experimental and Theoretical Investigations on Performance of Macro-Fiber Composites (MFC).

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Macro-Fiber Composites

- The Macro-Fiber Composite is a seven layered piezoelectric composite.
  - one active layer (PZT fibers embedded in epoxy matrix)
  - two electrode, acrylic and kapton layers.

Advantages

- Flexibility, directional actuation, environmentally sealed package etc.

Applications

- Actuators, sensors, energy harvesters, structural health monitoring etc.

http://www.smart-material.com

$d_{33}$ MFC Elongator

$d_{31}$ MFC Contractor

http://www.smart-material.com

Motivation and Objective

Motivation

- In various applications, the MFCs are subjected to a wide range of temperature due to which the understanding of thermo-electro-elastic behavior of MFCs becomes vital.
- The data available for modeling the MFC is limited.
- Dearth of data available in literature.

Objective

- To develop a simplified micro-mechanical analytical model to study the effective properties of Macro-Fiber Composites (MFC) under thermal environment.
- To develop a numerical model based on finite element calculations.
- To perform the experiments on available MFCs to validate the proposed models.
- To characterize the non-linear hysteretic behavior of Macro-Fiber Composites.
The second phase layer thickness variation is directly proportional to the volume fraction of individual phases. The assumptions are based on the concept of “rule of mixtures” and “series & parallel capacitance theory.”

- Active layer
  - \( V_{act} \)
  - \( S_1, S_3, S_5 \)
  - \( T_2, T_4, T_6 \)
  - \( E_3, \Delta \theta \)

- Electrode Layer (Cu + Epoxy)
  - \( S_2, S_4, S_6 \)
  - \( T_1, T_3, T_5 \)
  - \( D_3 \)

- Acrylic

\[ x^c = x^f = x^m \]

\[ x^c = V^f \cdot x^f + (1 - V^f) \cdot x^m \]

where \( x \) can represent \( S, T, E \) or \( D \).
Analytical model formulation

The strain fields of piezoceramics can be expressed in the equation form as,

\[ S_k^x = s_{kl}^{x} T_l^x + d_{km}^x E_m^x + CTE_k^x \theta^x \]  

(1)

The variables which do not change with the phases \( S_i \) (\( i = 1, 3\&5 \)), \( T_j \) (\( j = 2, 4\&6 \)), \( \Delta \theta \) and \( E_3 \) - independent variables.
The variables \( S_j \), \( T_i \), and \( D_3 \) which change - dependent variables.

The formulation starts with replacing the matrix dependent variables (Eq. (1); \( x = m \)) in-terms of fiber and composite field variables as shown below:

\[ S_i^m = s_{ij}^m ( (T_i^c - T_i^f v_f ) / (1 - v_f ) ) + s_{ij}^m (T_j^c) + d_{3i}^m E_3^c + CTE_i^m \Delta \theta^c \]  

(2)

From the assumptions it is evident that \( S_i^f \) (Eq. (1); \( x = f \)\( )= S_i^m \) (Eq. (2)), which leads to deriving a relation of fiber dependent field variables expressed in-terms of independent field variables as follows:

\[ (s_{ij}^m v_f - s_{ij}^f (1 - v_f )) T_j^f = s_{ij}^m T_j^c + (s_{ij}^m (1 - v_f ) - s_{ij}^f v_f ) T_i^c + (d_{3i}^m (1 - v_f ) - d_{3i}^f v_f ) E_3^c \]  

(3)

Substituting the values for \( i\&j \) and writing in a matrix form, the fiber dependent field variables can be represented in terms of independent variables as shown below:

\[ [T_1^f T_3^f T_5^f]' = [A]^{-1} [B] [T_1^c T_2^c T_3^c T_5^c E_3^c \Delta \theta^c]' \]  

(4)
Substituting the derived fiber dependent field variables in Eq. 1, an equation as a function of independent variables alone is obtained. Finally the electromechanical constants are obtained by equating the coefficients in derived fiber strain field (Eq. 1) and composite strain field (Eq. 4) as follows:

\[
\begin{align*}
\varepsilon_{33}^c &= Z_{13} \varepsilon_{13} + Z_{23} \varepsilon_{33} \\
\delta_{33}^c &= Z_{15} \varepsilon_{13} + Z_{25} \varepsilon_{33} + \delta_{33}^f \\
CTE_{33}^c &= Z_{16} \varepsilon_{13} + Z_{26} \varepsilon_{33} + CTE_{33}^f
\end{align*}
\]

where, the required constants $Z_{mn}$ are functions of volume fraction of constituents and their material properties.
A unit-cell is the smallest part of a periodic composite which contains sufficient information on the geometrical and material parameters at the microscopic level to deduce the effective properties of the composite.

**Periodic Boundary Condition**

- To ensure continuous deformation and to avoid overlap between unit-cells.
- In a cubic unit cell, the displacements of a pair of opposite boundary surfaces can be represented as [5]:

\[
\begin{align*}
    u_i^{S+} &= S_{ij}^{avg} x_j^{S+} + u_i^p; \\
    u_i^{S-} &= S_{ij}^{avg} x_j^{S-} + u_i^p
\end{align*}
\]  

(6)

- The periodic boundary condition for mechanical loading is represented as:

\[
    u_i^{S+} - u_i^{S-} = S_{ij}^{avg} (x_j^{S+} - x_j^{S-})
\]  

(7)

- The applied and induced mechanical and electrical fields are evaluated using total volume average technique.

\[
\begin{align*}
    S_{ij}^{avg} &= \frac{1}{\text{vol}} \int_{\text{vol}} S_{ij} d\text{vol}; \\
    T_{ij}^{avg} &= \frac{1}{\text{vol}} \int_{\text{vol}} T_{ij} d\text{vol}; \\
    E_i^{avg} &= \frac{1}{\text{vol}} \int_{\text{vol}} E_i d\text{vol}; \\
    D_i^{avg} &= \frac{1}{\text{vol}} \int_{\text{vol}} D_i d\text{vol}
\end{align*}
\]  

(8)
Numerical model cont.

### Periodic Boundary Condition

<table>
<thead>
<tr>
<th>Cases</th>
<th>Electrical PBC</th>
<th>Mechanical PBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ((S_1))</td>
<td>(u_1^{A+} - u_1^{A-} = k)</td>
<td>(u_2^{B+} - u_2^{B-} = 0)</td>
</tr>
<tr>
<td>2 ((S_2))</td>
<td>(u_1^{A+} - u_1^{A-} = 0)</td>
<td>(u_2^{B+} - u_2^{B-} = k)</td>
</tr>
<tr>
<td>3 ((S_3))</td>
<td>(u_1^{A+} - u_1^{A-} = 0)</td>
<td>(u_2^{B+} - u_2^{B-} = 0)</td>
</tr>
<tr>
<td>4 ((S_4))</td>
<td>(\phi^{A+} - \phi^{A-} = 0)</td>
<td>(u_1^{A+} - u_1^{A-} = 0)</td>
</tr>
<tr>
<td>5 ((S_5))</td>
<td>(\phi^{B+} - \phi^{B-} = 0)</td>
<td>(u_3^{A+} - u_3^{A-} = k)</td>
</tr>
<tr>
<td>6 ((S_6))</td>
<td>(u_2^{A+} - u_2^{A-} = k)</td>
<td>(u_1^{B+} - u_1^{B-} = k)</td>
</tr>
<tr>
<td>7 ((E_3))</td>
<td>(u_1^{A+} - u_1^{A-} = 0)</td>
<td>(u_2^{B+} - u_2^{B-} = 0)</td>
</tr>
</tbody>
</table>

### Thermal loading

- The nodes in the A-, B- and C- planes are constrained along x, y and z directions, respectively.
- To get uniform displacement, the nodes in the A+, B+ and C+ planes are coupled along respective directions.
- Reference temperature is set as \(0^0\)C and \(1^0\)C temperature is applied on all nodes.
Experimental characterization cont.

Photograph and schematic of experimental setup

Heater with Specimen holder
Results

(a) $E_L$ (GPa) vs. Temperature ($^\circ$C)

(b) CTE$_L$ ($\mu 0^\circ$C$^{-1}$) vs. Temperature ($^\circ$C)

(c) $d_{33}$ (pC/N) vs. Temperature ($^\circ$C)

(d) $\varepsilon^T$ (nF/m) vs. Temperature ($^\circ$C)
The piezoelectric materials exhibit a hysteretic loop even when subjected to moderate cyclic electric field.

\[ f(t) = \int \int_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha \beta} u(t) d\alpha d\beta \]

where, hysteresis operators - \( \gamma_{\alpha \beta} u(t) \), Preisach weighting function - \( \mu(\alpha, \beta) \), \( \alpha \) and \( \beta \) represent switching values of the input.
Geometric Interpretation of Preisach Model

Experemental procedure
Evaluation of Preisach weighing function Cont.

Numerical formulation

\[
\int \int_{T} \mu(\alpha, \beta) d\alpha d\beta = F(\alpha_0, \beta_0) \tag{10}
\]

\[
\int \int_{S^+ (t)} \mu(\alpha, \beta) d\alpha d\beta = \sum_{k=1}^{n(t)} \int \int_{Q_k(t)} \mu(\alpha, \beta) d\alpha d\beta = \sum_{k=1}^{n(t)} \int \int_{T(M_k, m_{k-1})} \mu(\alpha, \beta) d\alpha d\beta - \int \int_{T(M_k, m_k)} \mu(\alpha, \beta) d\alpha d\beta \tag{11}
\]

\[
f(t) = -F(\alpha_0, \beta_0) + 2 \sum_{k=1}^{n(t)} [F(M_k, m_{k-1}) - F(M_k, m_k)] \tag{12}
\]
Derivation of Preisach Model Cont.
Summary

- An analytical model based on equivalent layered approach is devised. To account geometric properties and non-uniform electric field a FE approach with PBC is proposed. Experiments are performed on commercially available MFCs.

- A comparative study is carried out between proposed models and the experimental results which are in good agreement.

- The simulated results show that the effect of temperature is significant on the coupling and electrical constants comparative to mechanical constants.

- Based on the devised numerical model 10°C temperature rise results in ≈ 3.5% increase in coupling constants.

- In addition to the linear characterization, a classical Preisach model is developed to predict the output of a MFC subjected to a cyclic excitation voltage.
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Thank you