Dark Energy in General Relativity

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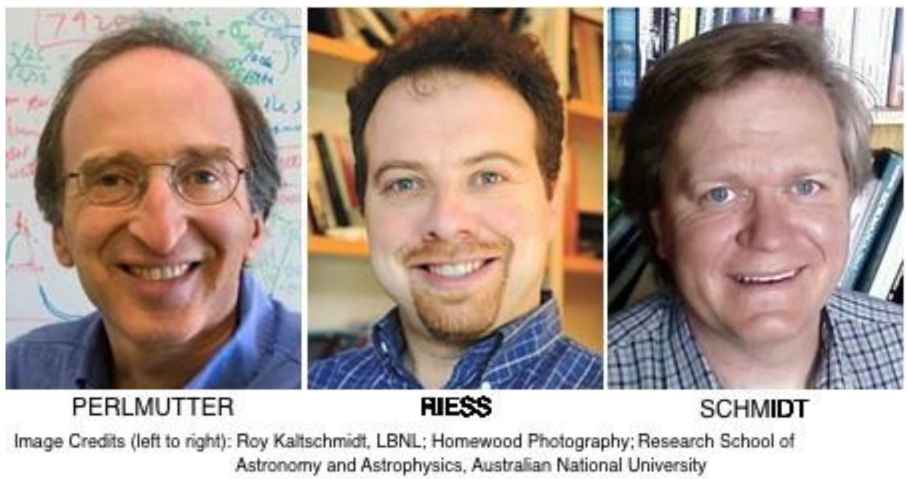
Contents

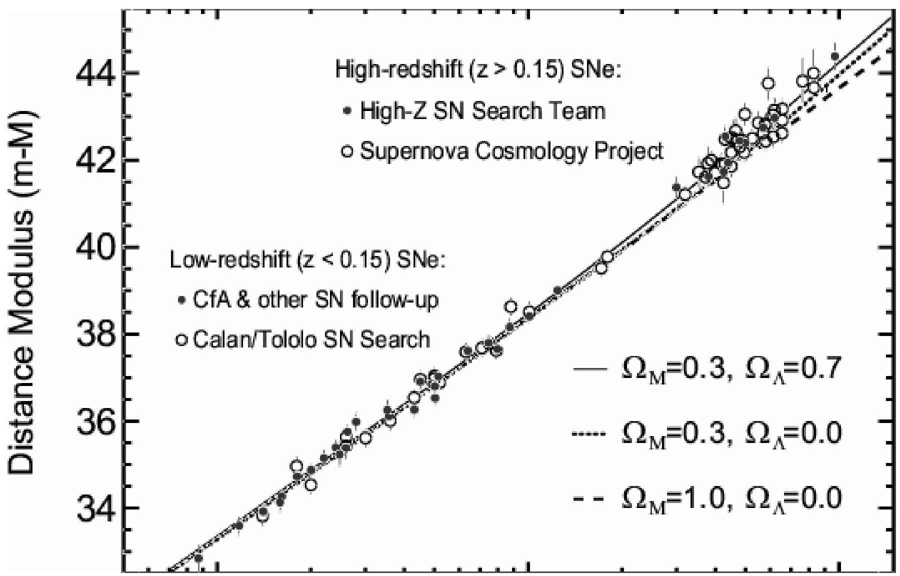
- Acceleration of Universe expansion, dark energy
- Experimental test of General Relativity (GR)
- The Physical Metric (PM)
- Nature of black hole (BH), extended horizon
- Masses of BH merger in GW150914
- Coordinate transformation from PM to FRW (Friedmann-Robertson-Walker) metric
- Inside-acceleration, Outside-deceleration
- The end of the universe expansion

Acceleration of Universe expansion



The universe expansion is accelerated. Physics Nobel Prizes are given in 2011.

Acceleration



Cosmological constant, Λ

- Acceleration always.
- $a(\tau)$: expansion factor
- $[(da(\tau)/d\tau)/a(\tau)]^2=8\pi G\rho/3 + \Lambda/3$, Λ=constant
- For $\rho=0$, $a(\tau) = A \exp((\Lambda/3)^{1/2} * \tau)$

- Accelerating expansion!! $a''(\tau) > 0$

Experimental test of GR
Time delay experiment of Solar system

- I. Shapiro et al, Bertotti et al.
- $\Delta t = (r_s / c) \ln (r/b)$ fits the data well (1/10^5 accuracy), where b is the impact parameter and r_s is the Schwarzschild radius (SR hereafter).

$$r_s = 2Gm/c^2$$

- The Schwarzschild metric of GR gives the prediction (See Weinberg's Gravitation, p.202)

$$\Delta t = (r_s / c) \left[\ln (r/b) + \tfrac{1}{2} \left((r-b)/(r+b)\right)^{1/2}\right]$$

- This does not fit the data.

Lesson in GR

- The Schwarzschild metric is the exact solution of the Einstein equation, but it does not fit the experimental data.
- ds^2=exp(a(r))*dt^2 – exp(b(r))*dr^2 – r^2*exp(c(r)*dΩ
- If exp(a(r)=exp(c(r)), one gets the correct formula for time delay experiment, Δt=(r_s/c) * ln (r/b).

The physical metric (PM)

- $\exp(a(r)) = \exp(c(r)) = \omega$
- Schwarzschild metric, $r' = r * \omega$ ^1/2
- $1 - r_s/(r * \omega$ ^1/2$) = \omega$
- Solving for r_s/r, one gets

  $$r_s/r = \omega^{1/2} * (1 - \omega)$$

  This terminates at

  $$R = (3*3^{1/2})/2 \; r_s = 2.60 \; r_s$$

- This gives the size of BH, called the extended horizon.
- $\exp(b(r)) = (d(r*\omega^{1/2})/dr)^2/\omega = (2\omega/(3\omega-1))^2$

PM(Inside solution)

- Inside the extended horizon, using the internal solution of the Schwarzschild metric one gets

  $D (r_s/r) = \omega^{1/2} (A\omega - 1)$

- Connecting the inside and outside solutions at the extended horizon, R, one gets

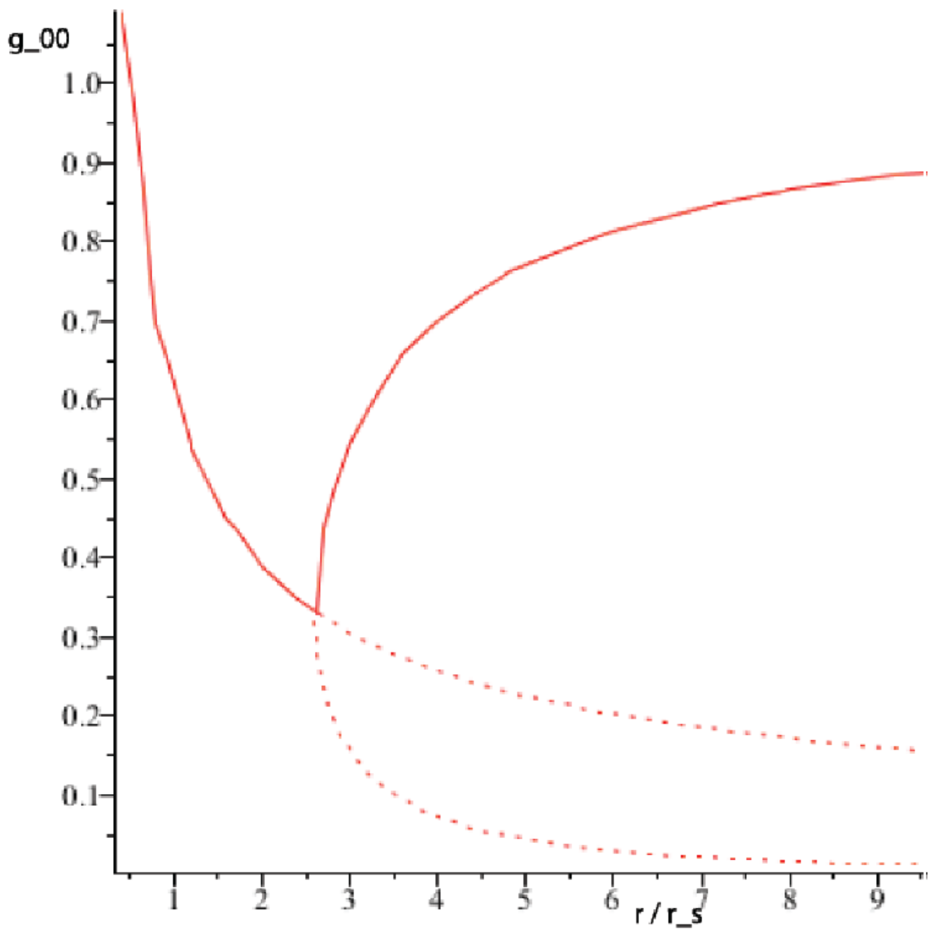
  $A = 2D + 3$

- The choice of $D > 0, A > 3$

  makes all the metric functions positive definite.

- $\exp(b(r)) = A*(2\omega/(3A\omega-1))^2$
- The inside solution is repulsive. (See the graph in the next page. This is a new feature!)

Physical metric (ω=g_00)



Gravitational redshift at R

- On the surface of BH, i. e. at $r = R$,

  $g\_00(R) = 1/3.$
- Then the gravitational redshift at R is

  $1 + z = 1/(g\_00(R))^1/2 =3^1/2 = 1.732$

PM with constant density, ρ [inside]

- For $r < R = 2.60\ r_s$,

  $\omega = 1/(B+(8\pi G\rho r^2)/3))$

  $\exp(b(r)) = B\ \omega^2$

- This is a repulsive gravity.
- Connecting ω at r = R, one gets

  $B = 3 - 2/3 * 3^{1/2} = 2.615$

Nature of BH and Neutron Stars

- The radius is 2.60 times SR. This is a surprise!
- The gravitational red shift at R is

  $1+z = 1/\omega^{1/2} = 3^{1/2} = 1.732$
- The temperature of BH and NS is very high.
- $T \approx d\omega/dr$

  outside of R   positive temperature

  at R   infinity

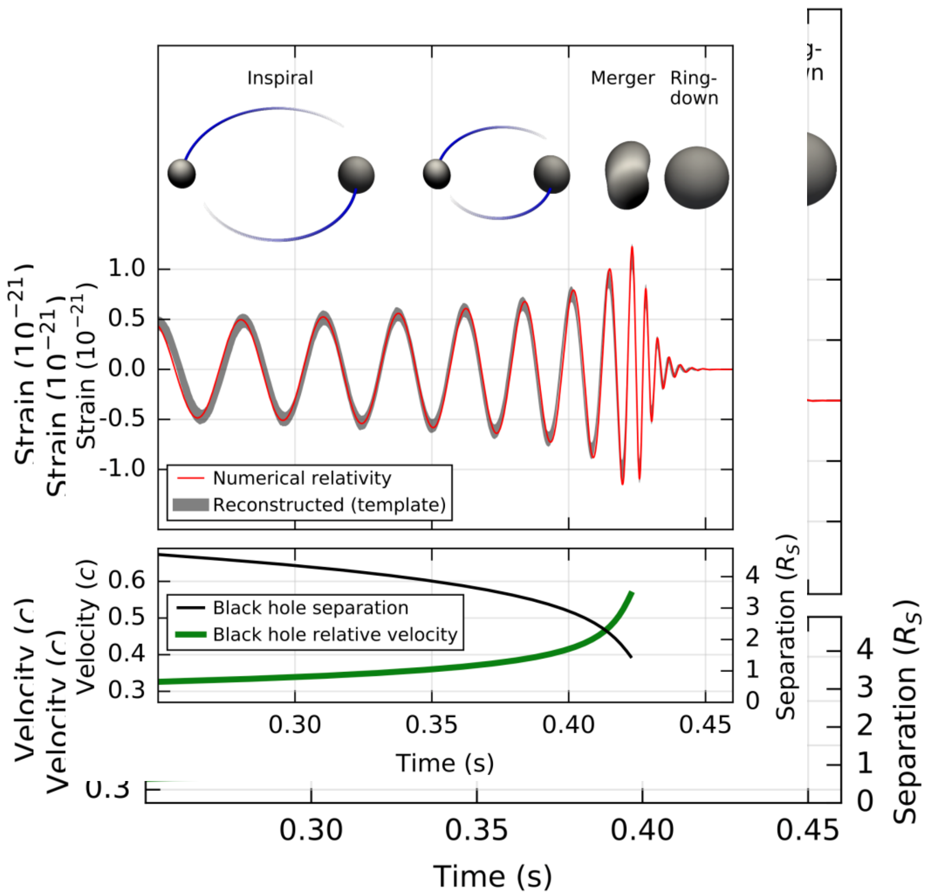
  inside of R   negative temperature

  (higher than any positive temperature)

  Boltzman Probability $\approx \exp(-E/kT)$

  For $T<0$, high E state is more probable.

Black hole merger in GW150914



Black hole masses in GW150914

- The numerical simulation indicates the masses of black hole merger to be
- M_1 = 29 (+4, -4) M_ʘ
- M_2 = 36 (+5, -4) M_ʘ➔M_f = 62 (+4, -4)M_ʘ
- These must be changed to (divided by 2.60), since the size of black hole is increased by 2.60
- M_1 = 11.2 M_ʘ
- M_2 = 13.8 M_ʘ ➔ M_f = 23.8 M_ʘ

Away from the Merger Contact

- The change of energy by GW emission,

  $-dE/dt = F (32G/5c^5)*(m1\ m2 /(m1+m2))^2 *$

  $r^4 *\omega^6,$

where $F= (2.60)^2 * (1+z)^2$

is the enhancement factor by the size of BH and the gravitational red shift, $1+z = \sqrt{3} =1.732$.

- This factor F is good enough to fit the data by smaller masses, m1 = 13.8 and m2 = 11.2 SM.

Coordinate transformation from PM to FRW (Friedman- Robertson- Walker)

- FRW

  $ds^2=d\tau^2 - a(\tau)^2*(d\xi^2/(1-k\xi^2) + \xi^2*d\Omega)$,

  where $k = 0, \pm 1$.

- PM

  $ds^2 = \omega*dt^2 - \exp(b(r))*dr^2 - r^2*\omega*d\Omega$

  $r < R$, $\omega = 1/(B + (8\pi G\rho r^2)/3)$

  $\exp(b(r)) = B\,\omega^2$, $B=2.615$

  $r > R$, $r_s/r = \omega^{1/2} * (1 - \omega)$

  $\exp(b(r)) = (2\omega/(3\omega-1))^2$

Coordinate transformation [1]

- $t = t(\tau,\xi)$, $r = r(\tau,\xi)$
- $dt = (\partial t/\partial \tau)d\tau + (\partial t/\partial \xi)d\xi$
- $dr = (\partial r/\partial \tau)d\tau + (\partial r/\partial \xi)d\xi$
- $a(\tau)*\xi = r*\omega\text{^}1/2$
- $\omega(\partial t/\partial \tau)\text{^}2 – \exp(b(r))*(\partial r/\partial \tau)\text{^}2 = 1$
- $\exp(b(r))*(\partial r/\partial \xi)\text{^}2 – \omega*(\partial t/\partial \xi)\text{^}2 = a(\tau)\text{^}2/(1-k\xi\text{^}2)$
- $\omega*(\partial t/\partial \tau) (\partial t/\partial \xi) – \exp(b(r))(\partial r/\partial \tau) (\partial r/\partial \xi) = 0$

Coordinate transformation [2]

- Eliminating $\partial t/\partial\tau$ and $\partial t/\partial\xi$, and using the explicit formulae for ω and $\exp(b(r))$, one gets

  $(a'(\tau)\xi)^2 = 1 - k\xi^2 - B/(B + (8\pi G\rho r^2)/3)$

  for $r < R$, and

  $(a'(\tau)\xi)^2 = 1 - \omega - k\xi^2$ for $r > R$

- The right hand sides of the both equations are positive definite for $k < 0$ or $k=0$ or for small value of ξ.

Coordinate transformation [3]

- Differentiating the equations in the previous page wrt τ, one gets

$a''(\tau) = (8\pi G\rho r/3)/(B + (8\pi G\rho r^2)/3)^{1/2}$ (1)

for $r < R$, and

$a''(\tau) = -(1 - \omega)/(2*r*\xi*\omega^{1/2})$ (2)

for $r > R$

- $a''(\tau)$ in Eq. (1) is positive and that in Eq. (2) is negative. These are the results from $d\omega/dr < 0$

for $r < R$ (repulsive) and $d\omega/dr > 0$ for $r > R$ (attractive).

Dark energy by PM

- By coordinate transformation from PM to FRW metric, the author has shown that the universe expansion is accelerated inside the extended horizon (R) and decelerated outside R.
- This is the result from the nature of the gravity being repulsive/attractive for inside/outside of R in PM.
- The difference from Cosmological Constant is clear, since the latter gives acceleration everywhere.

The end of the universe expansion [1]

- For simplicity, let us assume $k = 0$. (The observation is consistent with this assumption.) Then,
- $a'(\tau)*\xi = (1 - \omega)^{1/2}$ for $r>R$.
- Since $(1 - \omega)^{1/2} = (r_s/r\omega^{1/2})^{1/2}$
  $= (r_s/a(\tau)\xi)^{1/2}$, one gets
- $a(\tau)^{1/2} * a'(\tau) = (r_s/\xi^3)^{1/2}$ and it's solution is
- $a(\tau) = (H(\xi)* \tau + K(\xi))^{2/3}$, where
- $H(\xi) = (3/2)*(r_s/\xi^3)^{1/2}$, and $K(\xi)$ is an integration constant.

The end of the universe expansion [2]

- For $r > R$,

  $$a(\tau) = (H(\xi) * \tau + K(\xi))^{2/3}$$

- This means that although outside the extended horizon the universe expansion is decelerated, the expansion reaches at the infinite distance.

The most important comment

- The property of BH in PM is entirely different from that in the Schwarzschild metric. Almost everything based on the latter is wrong.
- One has to review all the observable properties of BH and Neutron Stars (Compact Objects) based on the new metric (PM).