A Regression Approach to Image Denoising

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Outline for section 1

1 Background

2 Image Analysis
   • Applications
   • Test Images
   • Denoising

3 Results
   • Gaussian Noise
   • Salt and Pepper Noise
   • Poisson Noise
   • Factor Effects Noise Type, Noise Level and Radius

4 Questions
Image Denoising and Deblurring

The process of processing images to have a better representation of the scene. Make pictures sharp.

\[ z = Ku + \varepsilon \]
The Mathematical Model

\[ z = Ku + \epsilon, \]

where \( z \) is the data, \( K \) is the smoothing operator, \( u \) is the true image, and \( \epsilon \) is the noise.

\[ (Ku)(x) = \int_D k(x, y) \cdot u(y) \, dy, \quad x \in D. \]
Outline for section 2

1. Background

2. Image Analysis
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   - Test Images
   - Denoising

3. Results
   - Gaussian Noise
   - Salt and Pepper Noise
   - Poisson Noise
   - Factor Effects Noise Type, Noise Level and Radius

4. Questions
Golgi Quantization

Copyright: Claudio Aguilar and Swetha Ramadesikan.
Breast Cancer

Standard Mammography

SenoBright CESM
Test Images
Denoising

Consider a sharp image $U$ that undergoes a two-dimensional Gaussian white noise $\eta$, which is added to the original image to produce a noisy image $Z$.

$$Z = I \cdot U + \eta,$$

The noisy image translates into a regression problem as follows

$$Y = X \cdot \beta + \epsilon$$

We want to estimate the parameters $\beta_i$ and the predicted pixel values $\hat{Y}$.

$$\hat{\beta} = (X' \cdot X)^{-1} \cdot X' \cdot Y.$$ 

The predicted values are

$$\hat{Y} = X \cdot (X' \cdot X)^{-1} \cdot X' \cdot Y.$$
Scatterplot Matrix
Types of Noise

- Gaussian Noise, also known as white noise.
- Poisson Noise.
- Gaussian plus Poisson noise.
- Salt & Pepper Noise.
- Speckle Noise.
- Stripped Noise.
Model Assessment

- **Peak signal-to-noise ratio (PSNR)** measure to assess the performance of the proposed model.

\[
\text{SNR} = \frac{\hat{\mu}_{\text{signal}}}{\hat{\sigma}_{\text{noise}}},
\]

where \(\hat{\mu}\) and \(\hat{\sigma}\) are the estimated expected value of the signal and standard deviation of the noise, respectively.

- **Structural similarity (SSIM) index** is a measure used in image analysis to assess the quality between two images, sort of a correlation coefficient.

\[
\text{SSIM}(X, Z) = \frac{(2\mu_X \mu_Z + c_1)(2\sigma_{XZ} + c_2)}{(\mu_X^2 + \mu_Z^2 + c_1)(\sigma_X^2 + \sigma_Z^2 + c_2)},
\]

where \(\mu_X, \mu_Z\) the mean of images \(X\) and \(Z\), respectively; \(\sigma_X^2, \sigma_Z^2\) the variance of images \(X\) and \(Z\), respectively; and \(\sigma_{XZ}\) the covariance between images \(X\) and \(Z\). \(c_1 = 0.01 \cdot L\) and \(c_2 = 0.03 \cdot L\) are constants with \(L = 2^{\text{number of bits per pixel}} - 1\). \(L = 255\).
Outline for section 3

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4 Questions
Gaussian Noise

Figure: From left to right: Exact, noisy and denoised images for a 30% noisy image using radius of 6. The structural similarity changes from 0.14 to 0.73, while PSNR changes from 16.17 to 27.01. The added noise is Gaussian.

- The improvement in the denoising rate is 0.59 increase in the similarity measure and 10.84 increase in the PSNR measure.
- Adjusted $R^2$ value is 0.6235.
- 10% of the pixels were insignificant in predicting the noisy pixel.
- All estimated slopes or weights add up to approximately 1.
Factors: Radius and Noise Level

Figure: Additive Gaussian noise; structural similarity and PSNR versus noise level between the exact and denoised images for different radii. The blue curves reference the exact image and noisy image.
Factors: Radius and Noise Level

Figure: Additive Gaussian noise; structural similarity difference

\[ SSIM(\text{exact,denoised}) - SSIM(\text{exact,noisy}) \]
and PSNR difference

\[ PSNR(\text{exact,denoised}) - PSNR(\text{exact,noisy}) \]

versus noise level for different radii.
Salt & Pepper Noise

Figure: Exact, noisy and denoised images for a 30% noisy image using radius of 6; i.e. 168 pixels. The structural similarity changes from 0.06 to 0.67, while PSNR changes from 11.12 to 22.34. The added noise is salt & pepper.

- The improvement in the denoising rate is 0.61 increase in the similarity measure and 11.22 increase in the PSNR measure.
- Adjusted $R^2$ value is 0.2316.
- 18.5% of the 168 pixels were statistically insignificant.
- Salt & pepper perturbation is 0 or 255. The mean is greatly impacted by the binary nature of the noise.
Salt & Pepper Noise

**Figure**: From left to right: Exact, noisy and denoised images for a 30% noisy image using radius of 6; i.e. 168 pixels. The structural similarity changes from 0.14 to 0.73, while PSNR changes from 16.17 to 27.01. The added noise is salt & pepper.
Factors: Radius and Noise Level

Figure: Additive salt & pepper noise; structural similarity and PSNR versus noise level between the exact and denoised images for different radii. The blue curves reference the exact image and noisy image.
Factors: Radius and Noise Level

Figure: Additive salt & pepper noise; structural similarity difference
\[ SSIM^{(exact,\text{denoised})} - SSIM^{(exact,\text{noisy})} \]
and PSNR difference
\[ PSNR^{(exact,\text{denoised})} - PSNR^{(exact,\text{noisy})} \]
versus noise level for different radii.
**Poisson Noise**

**Figure**: From left to right: Exact, noisy and denoised images for a 30% noisy image using 168 pixels. The structural similarity changes from 0.23 to 0.74, while PSNR changes from 16.24 to 18.48. The added noise is Poisson.

- Poisson noise is skewed, which affects both the mean and variance.
- Adjusted $R^2$ value is 0.7808.
- 9.5% of the 168 pixels were found to be statistically insignificant in predicting the noisy pixel.
- The weights add up to 0.9895.
Factors: Radius and Noise Level

Figure: Additive Poisson noise; structural similarity and PSNR versus noise level between the exact and denoised images for different radii. The blue curves reference the exact image and noisy image.
Factors: Radius and Noise Level

**Figure:** Additive Poisson noise; structural similarity difference

\[ SSIM(\text{exact}, \text{denoised}) - SSIM(\text{exact}, \text{noisy}) \]

and PSNR difference

\[ PSNR(\text{exact}, \text{denoised}) - PSNR(\text{exact}, \text{noisy}) \]

versus noise level for different radii.
Two-way ANOVA for SSIM reveals that both noise type and radius are statistically significant factors. $R^2$ for this model is 0.3252.

Two-way ANOVA for PSNR reveals that only noise type is a statistically significant factor. $R^2$ for this model is 0.4402.
Two-way ANOVA for the SSIM shows that both noise type and noise level are statistically significant. $R^2$ for this model is 0.817.

Two-way ANOVA for PSNR shows that both factors and their interaction are statistically significant. $R^2$ for this model is 0.9799.
Factor Effects Noise Type, Noise Level and Radius

Figure: SSIM and PSNR means plots for radius categorized by noise level.

- Two-way ANOVA for SSIM reveals that both noise level and radius are statistically significant factors. $R^2$ for this model is 0.7238.
- Two-way ANOVA for PSNR reveals that only noise level is statistically significant. $R^2$ for this model is 0.5518.
Factor: Noise Type

SSIM plot shows that Gaussian and Poisson noise are almost identical, but higher than salt & pepper noise.

PSNR plot shows that Gaussian noise has higher PSNR overall compared to salt & pepper and Poisson.
Gaussian Noise (Revisited)
Results

Factor Effects Noise Type, Noise Level and Radius

True Image

Noisy Image (30% Noise, SSIM = 0.14, PSNR = 16.17)

Denoised Image (radius = 6, SSIM = 0.73, PSNR = 27.01)

True Image

Noisy Image (30% Noise, SSIM = 0.45, PSNR = 15.48)

Denoised Image (radius = 6, SSIM = 0.62, PSNR = 24.04)

True Image

Noisy Image (30% Noise, SSIM = 0.29, PSNR = 23.67)

Denoised Image (radius = 6, SSIM = 0.63, PSNR = 30.00)
Median Filtering

Denoised Image (radius = 6, SSIM = 0.66, PSNR = 25.49)

Denoised Image (radius = 6, SSIM = 0.73, PSNR = 22.00)

Denoised Image (radius = 6, SSIM = 0.77, PSNR = 25.90)
Outline for section 4

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4. Questions
Thank You!

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