A Regression Approach to Image Denoising

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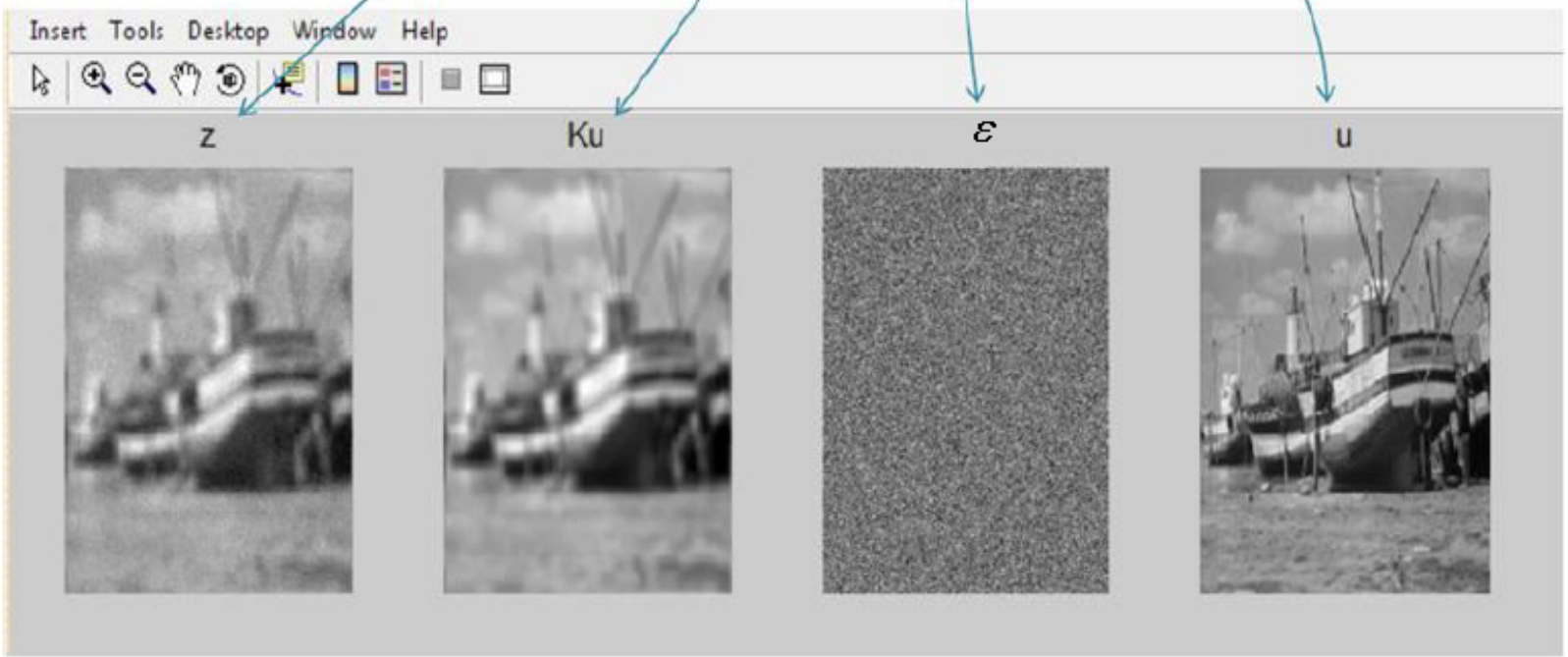
Outline for section 1

Image Denoising and Deblurring

The process of processing images to have a better representation of the scene. Make pictures sharp.

$$z = Ku + \varepsilon$$



The Mathematical Model

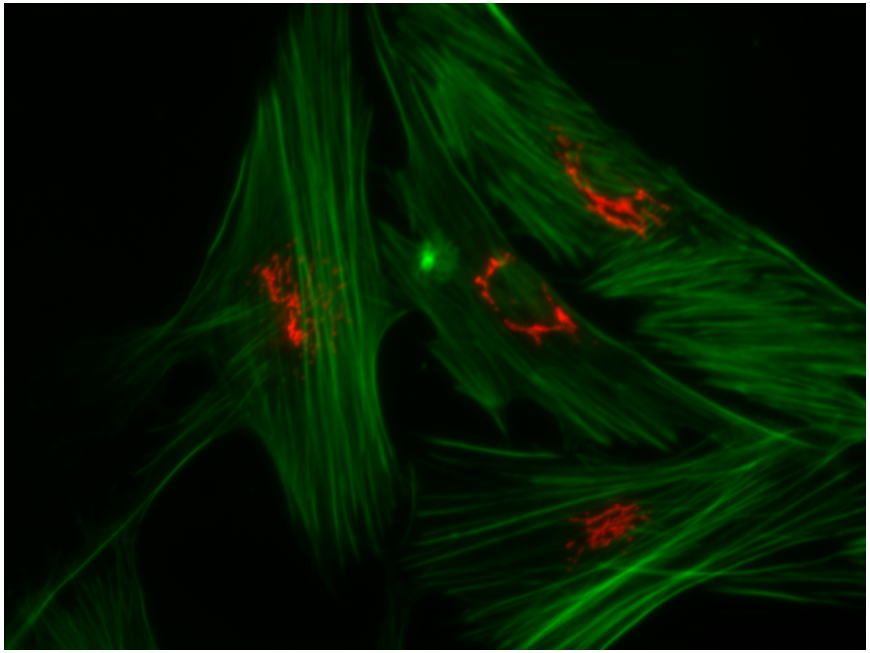
$$z = Ku + \epsilon,$$

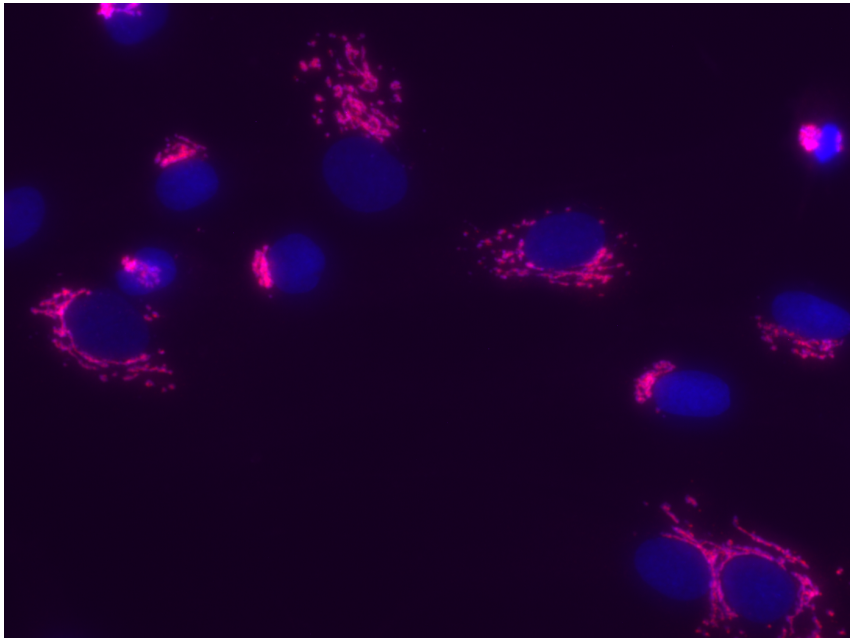
where z is the data, K is the smoothing operator, u is the true image, and ϵ is the noise.

$$(Ku)(x) = \int_D k(x,y) \cdot u(y)\ dy, \quad x \in D.$$

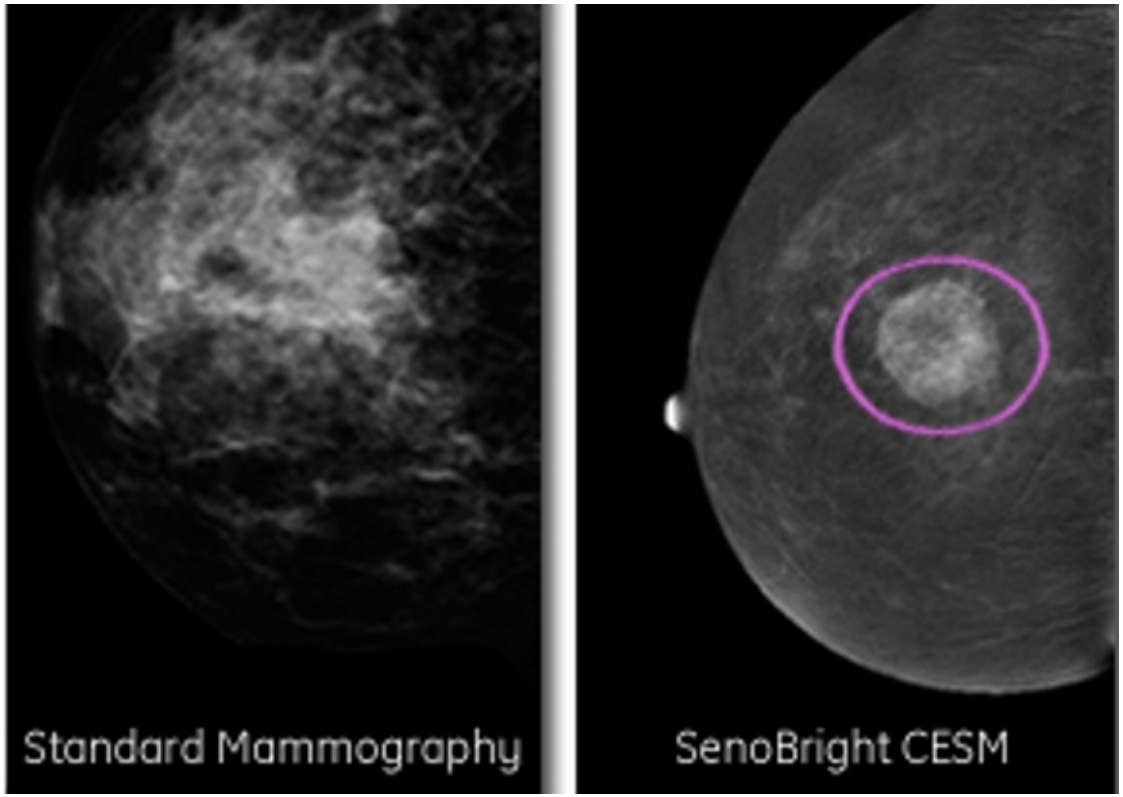
Outline for section 2

Golgi Quantization

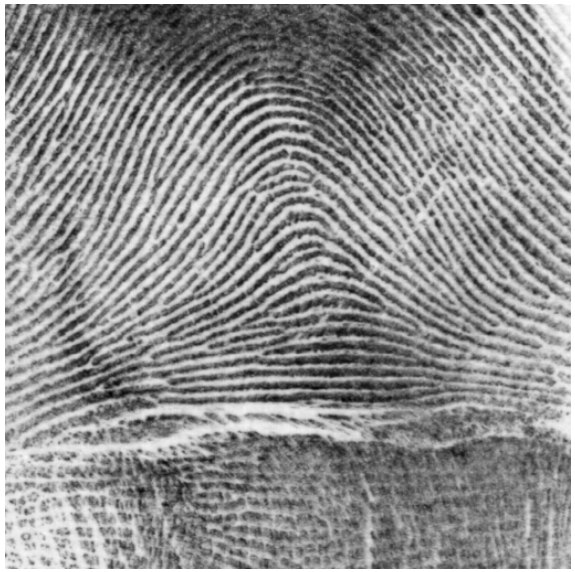


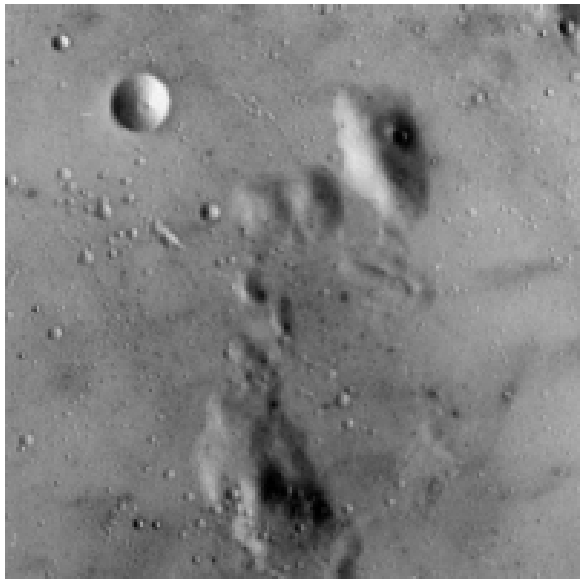


Breast Cancer



Test Images





Denoising

Consider a sharp image U that undergoes a two-dimensional Gaussian white noise η, which is added to the original image to produce a noisy image Z.

$$Z = I \cdot U + \eta,$$

The noisy image translates into a regression problem as follows

$$Y = X \cdot \beta + \epsilon$$

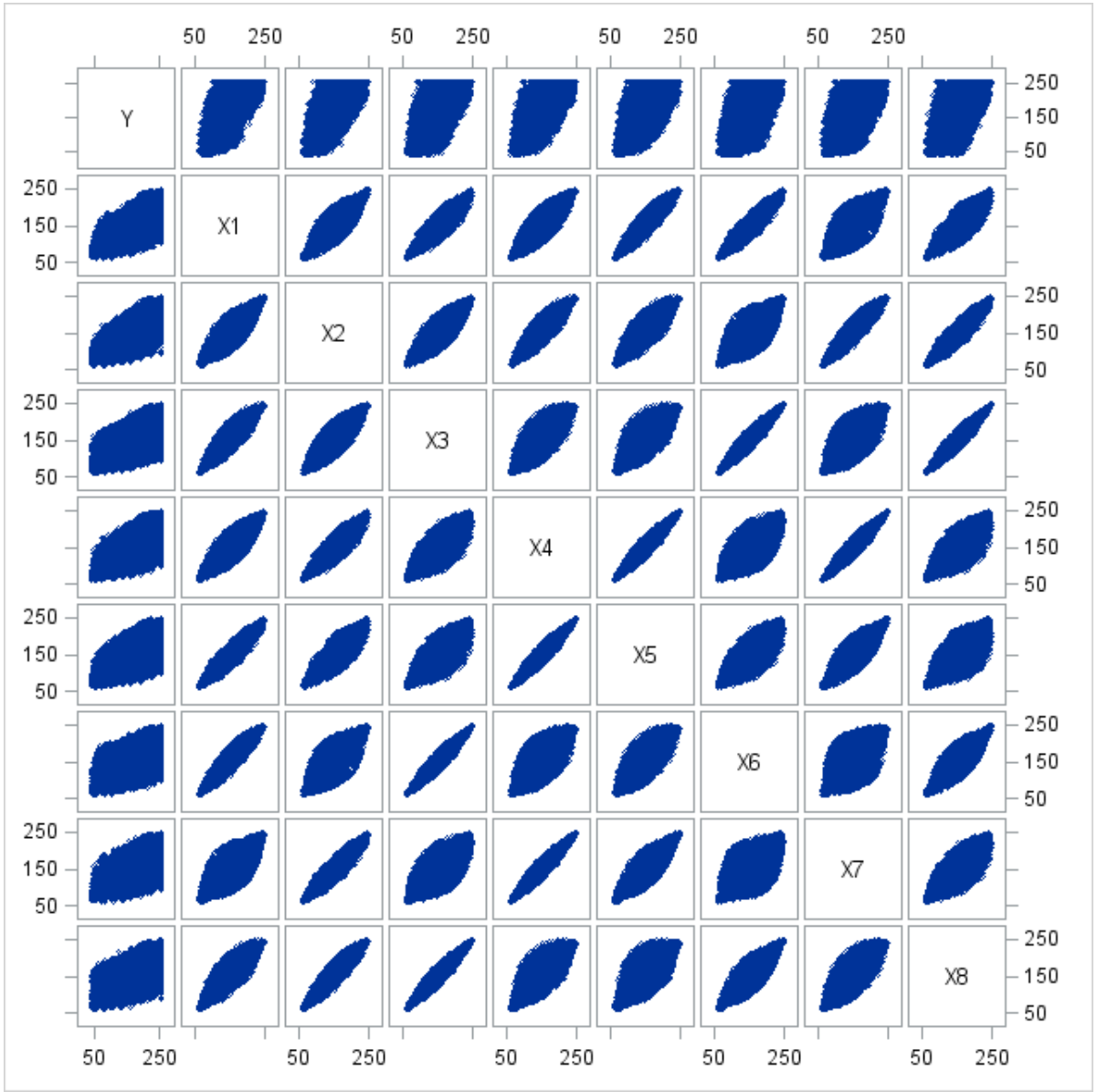
We want to estimate the parameters β_i and the predicted pixel values $\hat{Y}$.

$$\hat{\beta} = \left(X' \cdot X\right)^{-1} \cdot X' \cdot Y.$$

The predicted values are

$$\hat{Y} = X \cdot \left(X' \cdot X\right)^{-1} \cdot X' \cdot Y.$$

Scatterplot Matrix



Types of Noise

- Gaussian Noise, also known as white noise.
- Poisson Noise.
- Gaussian plus Poisson noise.
- Salt & Pepper Noise.
- Speckle Noise.
- Stripped Noise.

Model Assessment

- **Peak signal-to-noise ratio (PSNR)** measure to assess the performance of the proposed model.

$$\mathrm{SNR} = \frac{\hat{\mu}_{\text{signal}}}{\hat{\sigma}_{\text{noise}}},$$

where $\hat{\mu}$ and $\hat{\sigma}$ are the estimated expected value of the signal and standard deviation of the noise, respectively.

- **Structural similarity (SSIM) index** is a measure used in image analysis to assess the quality between two images, sort of a correlation coefficient.

$$\mathrm{SSIM}(X, Z) = \frac{(2\mu_X\mu_Z + c_1)(2\sigma_{XZ} + c_2)}{(\mu_X^2 + \mu_Z^2 + c_1)(\sigma_X^2 + \sigma_Z^2 + c_2)},$$

where μ_X, μ_Z the mean of images X and Z, respectively; σ_X^2, σ_Z^2 the variance of images X and Z, respectively; and σ_{XZ} the covariance between images X and Z. $c_1 = 0.01 \cdot L$ and $c_2 = 0.03 \cdot L$ are constants with $L = 2^{(\text{number of bits per pixel})} - 1$. $L = 255$.

Outline for section 3

Gaussian Noise

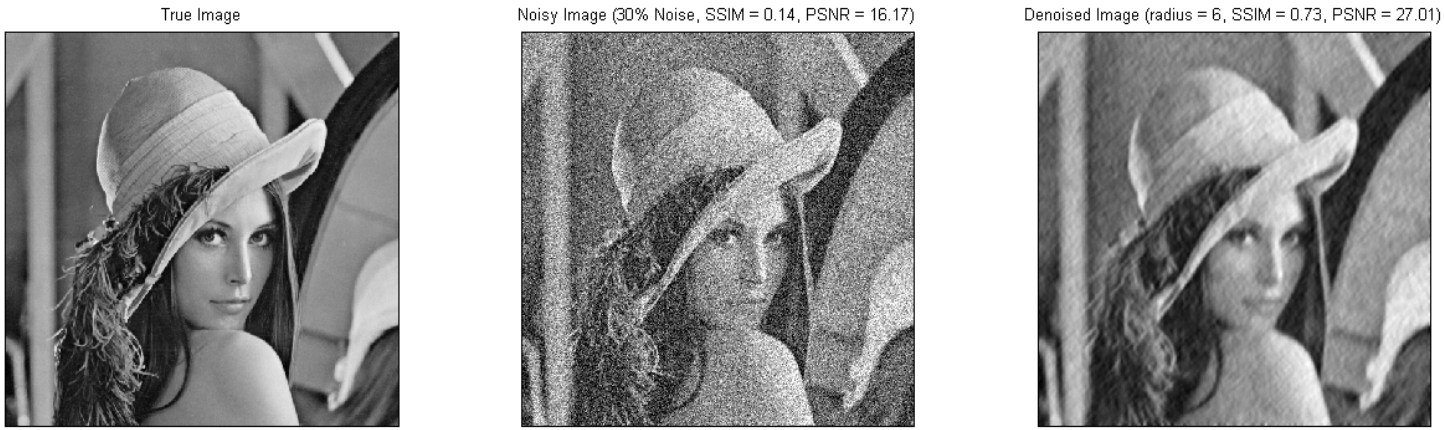
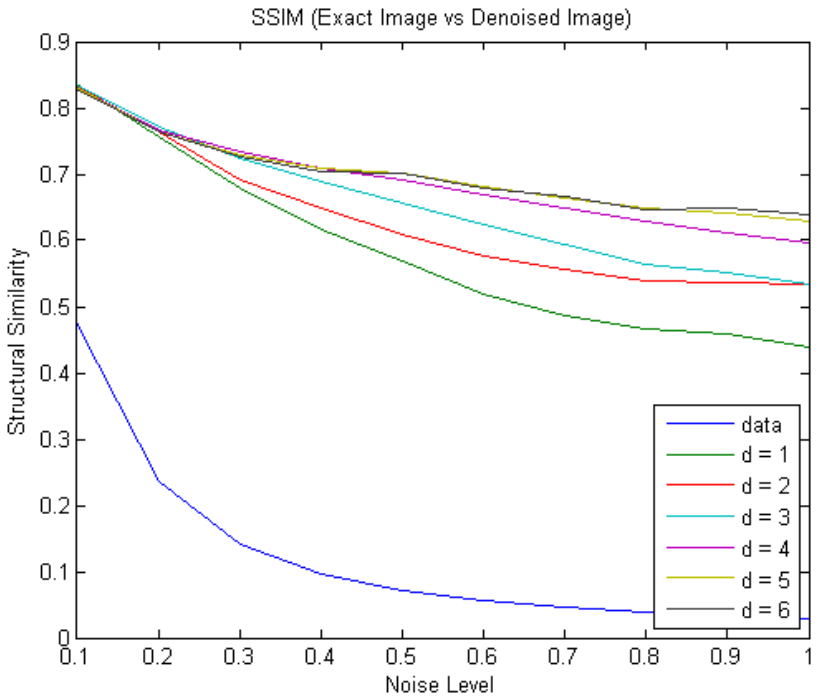


Figure : From left to right: Exact, noisy and denoised images for a 30% noisy image using radius of 6. The structural similarity changes from 0.14 to 0.73, while PSNR changes from 16.17 to 27.01. The added noise is Gaussian.

- The improvement in the denoising rate is 0.59 increase in the similarity measure and 10.84 increase in the PSNR measure.
- Adjusted R^2 value is 0.6235.
- 10% of the pixels were insignificant in predicting the noisy pixel.
- All estimated slopes or weights add up to approximately 1.

Factors: Radius and Noise Level



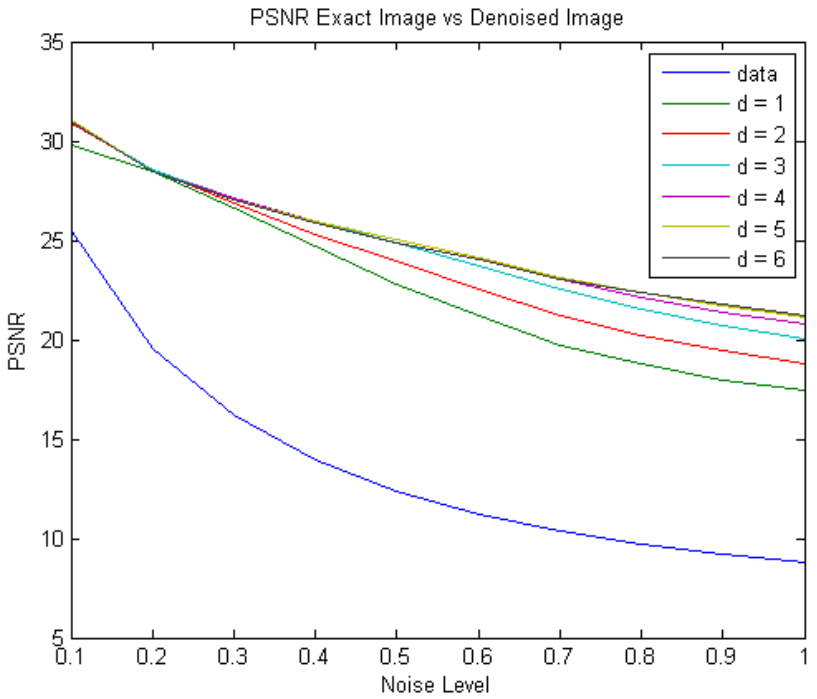
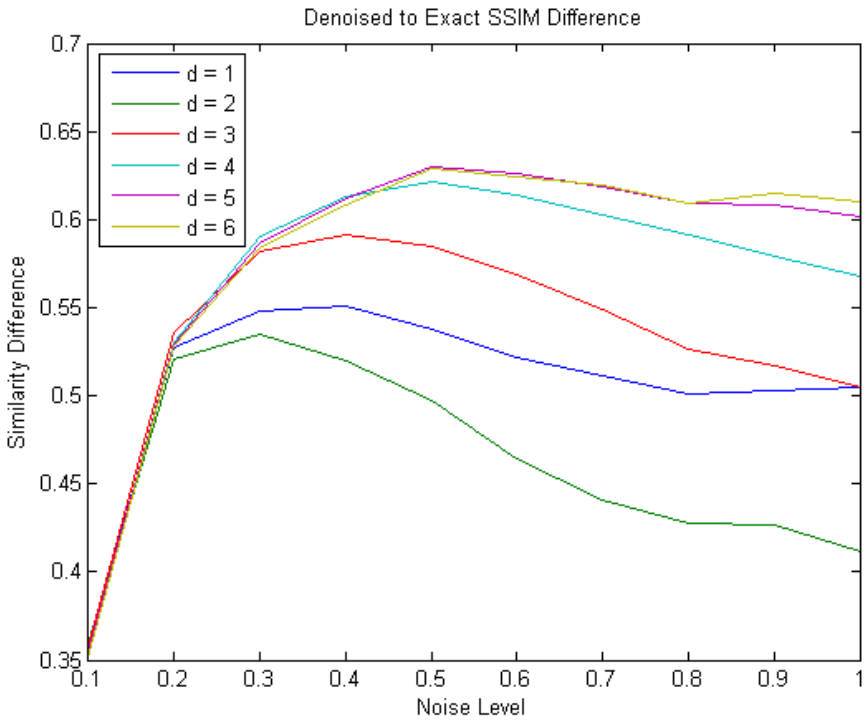


Figure : Additive Gaussian noise; structural similarity and PSNR versus noise level between the exact and denoised images for different radii. The blue curves reference the exact image and noisy image.

Factors: Radius and Noise Level



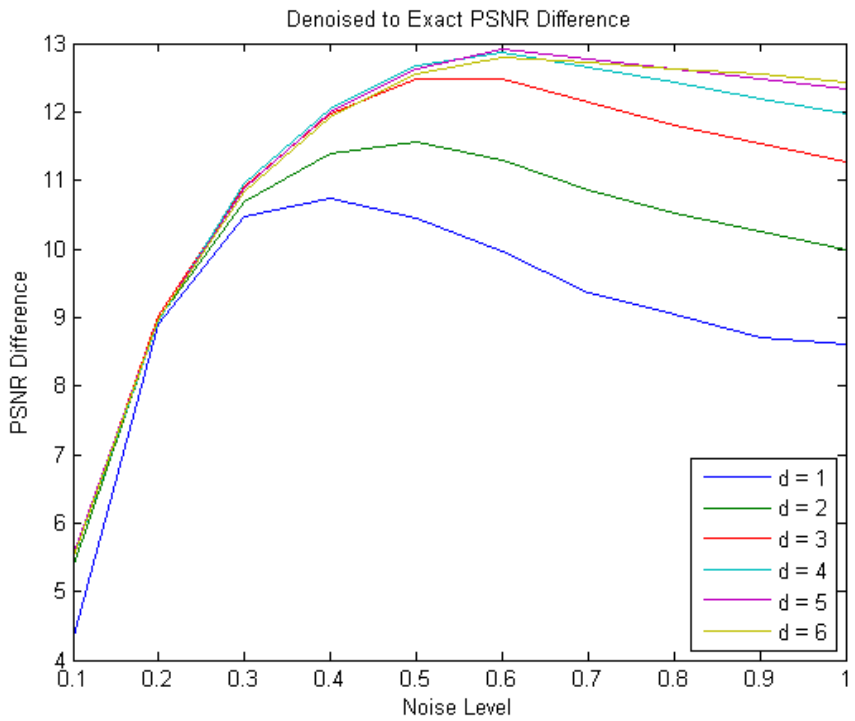


Figure : Additive Gaussian noise; structural similarity difference $= SSIM(exact, denoised) - SSIM(exact, noisy)$ and PSNR difference $= PSNR(exact, denoised) - PSNR(exact, noisy)$ versus noise level for different radii.

Salt & Pepper Noise

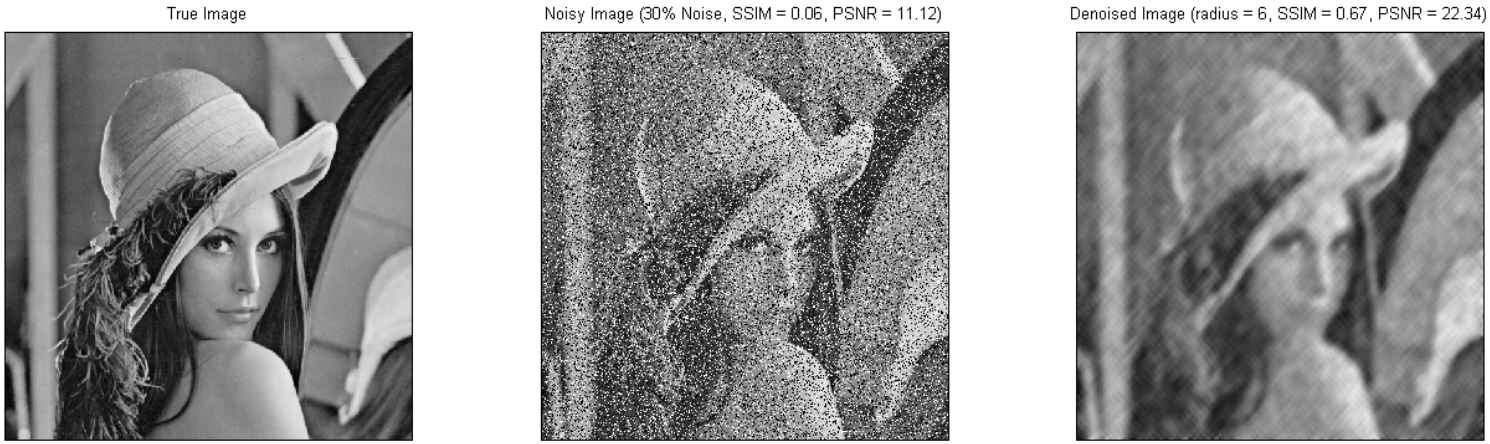


Figure : Exact, noisy and denoised images for a 30% noisy image using radius of 6; i.e. 168 pixels. The structural similarity changes from 0.06 to 0.67, while PSNR changes from 11.12 to 22.34. The added noise is salt & pepper.

- The improvement in the denoising rate is 0.61 increase in the similarity measure and 11.22 increase in the PSNR measure.
- Adjusted R^2 value is 0.2316.
- 18.5% of the 168 pixels were statistically insignificant.
- Salt & pepper perturbation is 0 or 255. The mean is greatly impacted

Salt & Pepper Noise

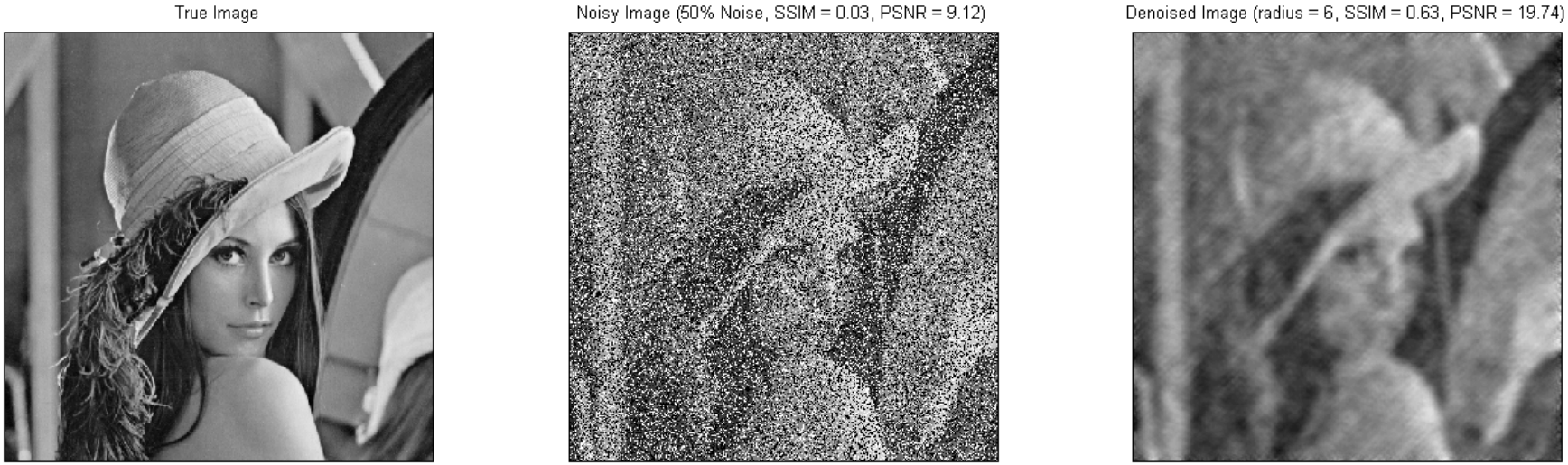
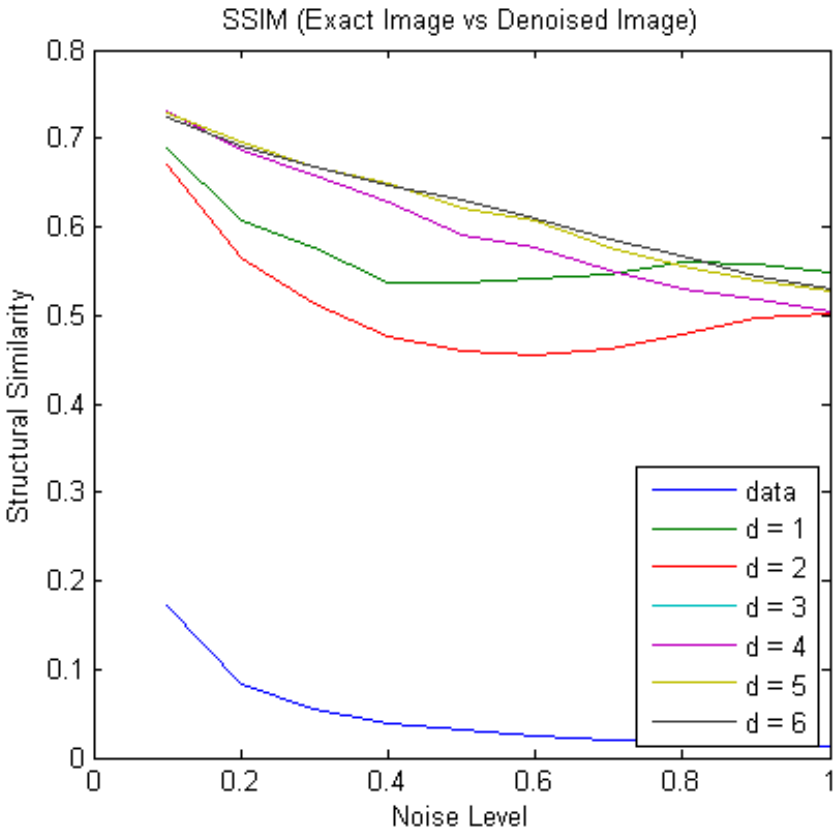


Figure : From left to right: Exact, noisy and denoised images for a 30% noisy image using radius of 6; i.e. 168 pixels. The structural similarity changes from 0.14 to 0.73, while PSNR changes from 16.17 to 27.01. The added noise is salt & pepper.

Factors: Radius and Noise Level



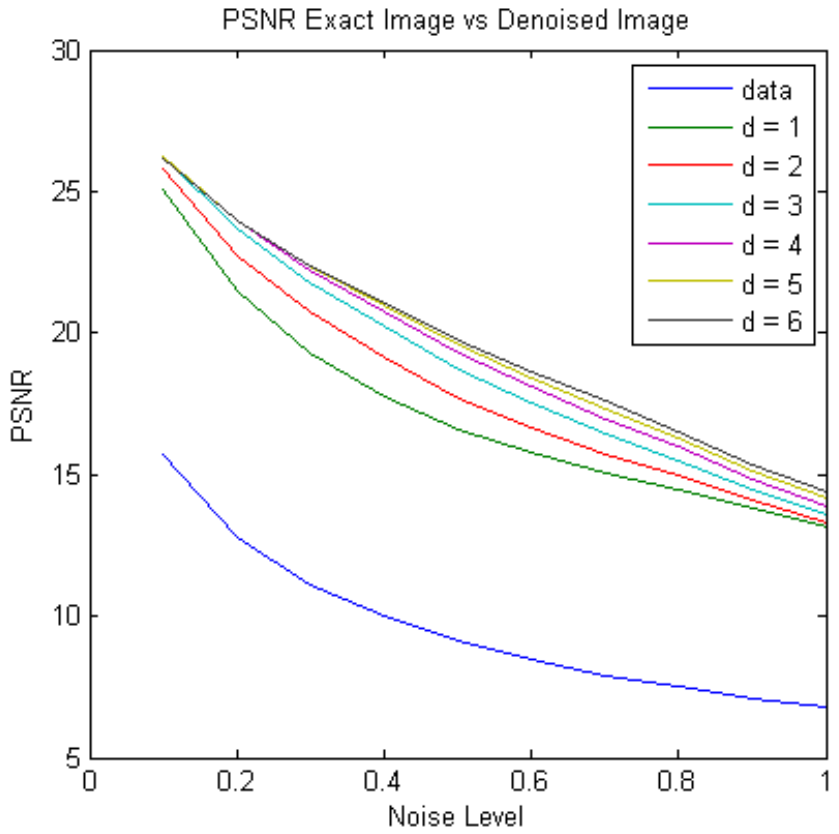
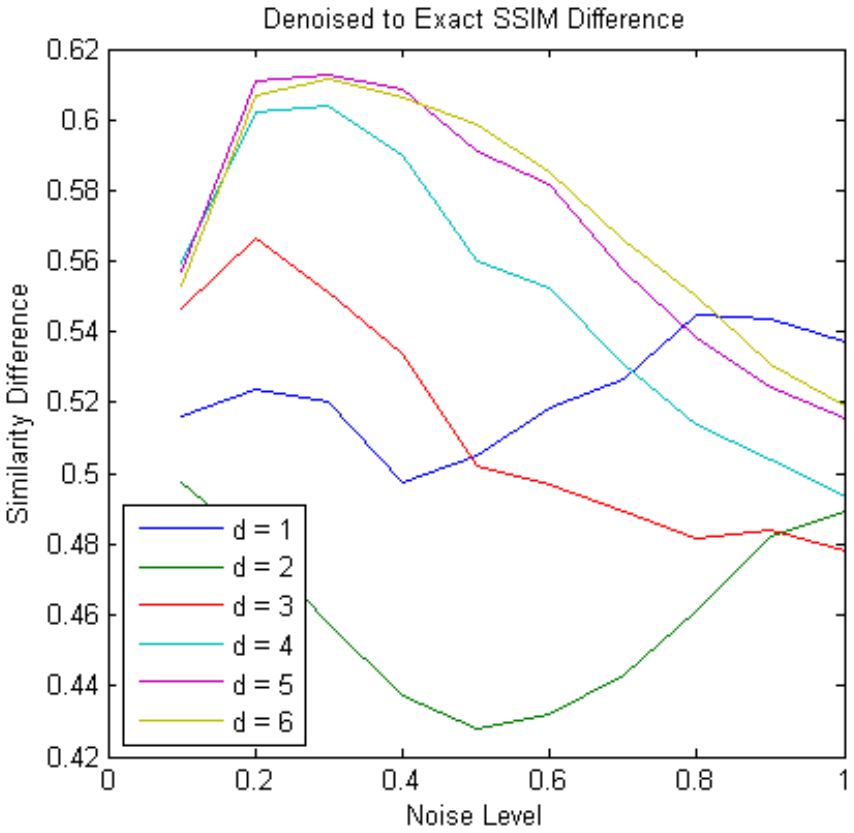


Figure : Additive salt & pepper noise; structural similarity and PSNR versus noise level between the exact and denoised images for different radii. The blue curves reference the exact image and noisy image.

Factors: Radius and Noise Level



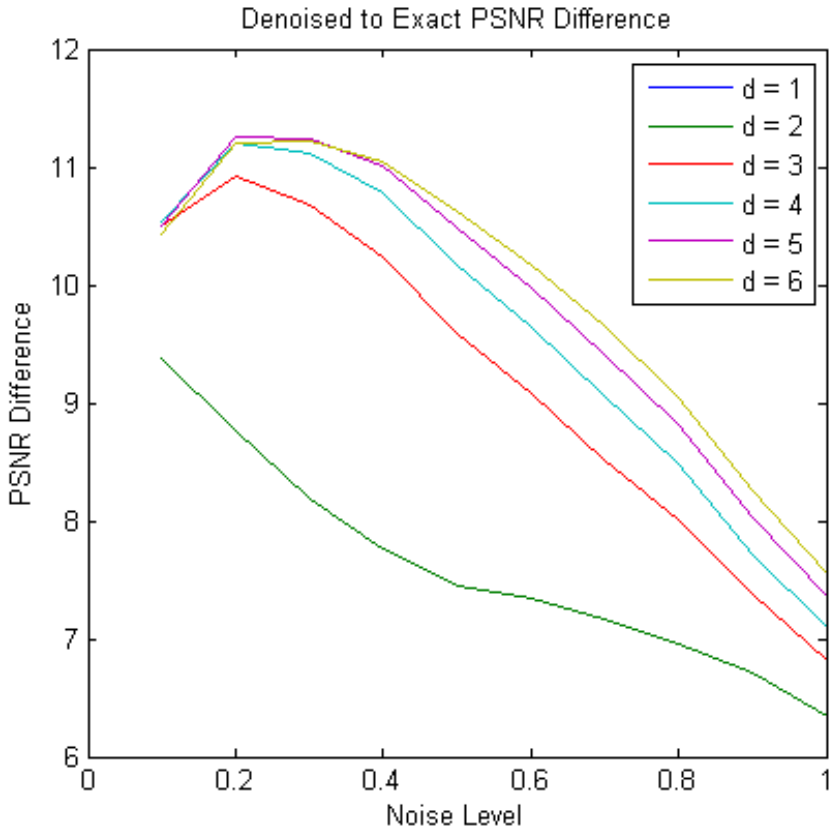


Figure : Additive salt & pepper noise; structural similarity difference $= SSIM(exact, denoised) - SSIM(exact, noisy)$ and PSNR difference $= PSNR(exact, denoised) - PSNR(exact, noisy)$ versus noise level for different radii.

Poisson Noise

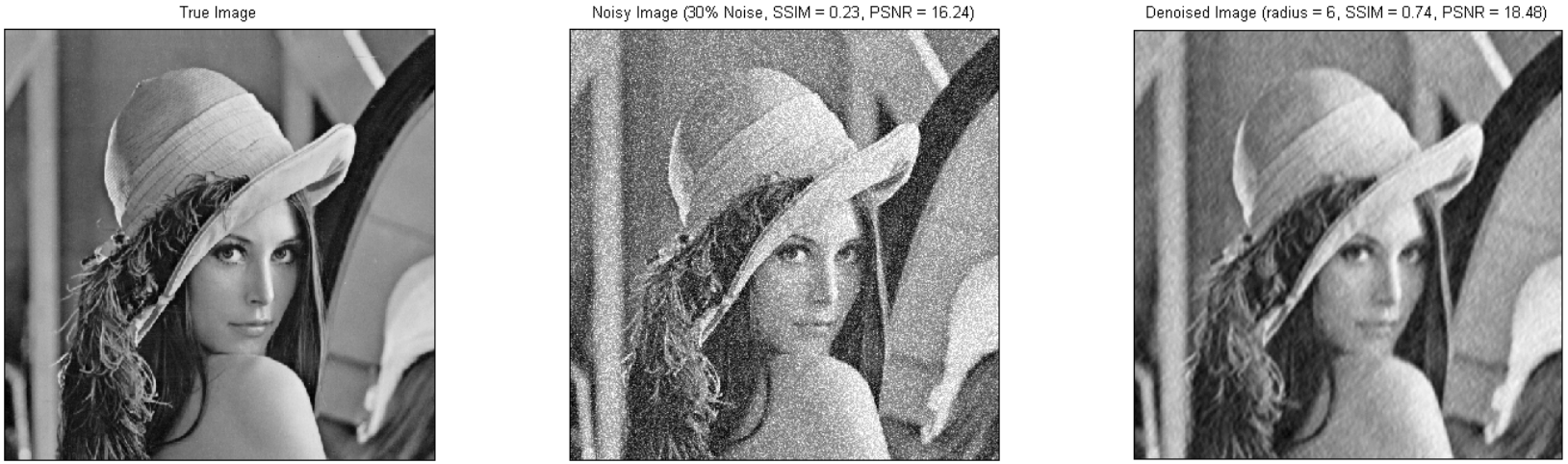
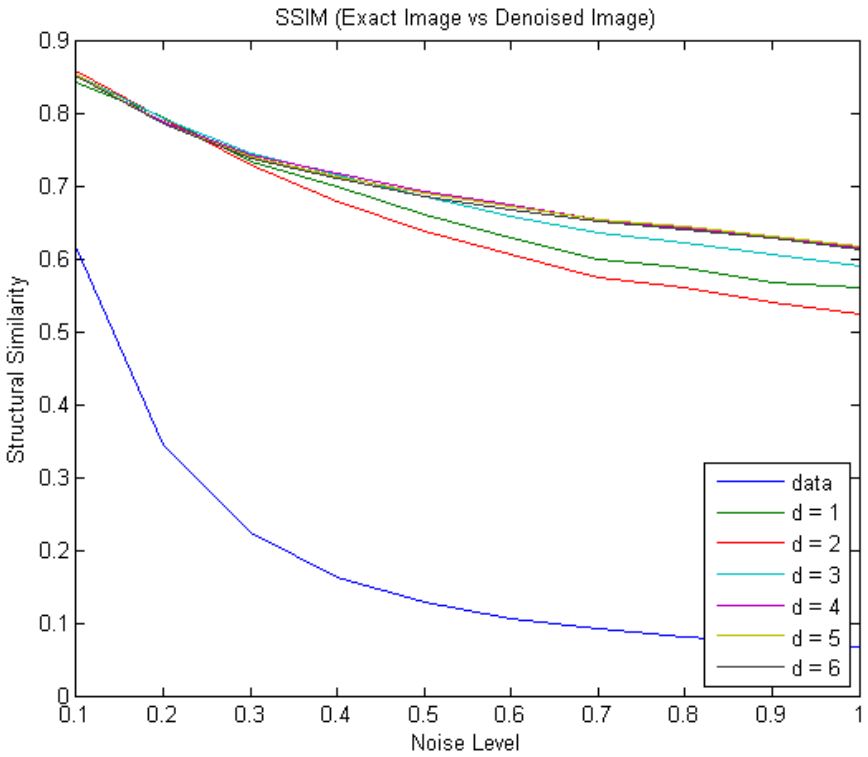


Figure : From left to right: Exact, noisy and denoised images for a 30% noisy image using 168 pixels. The structural similarity changes from 0.23 to 0.74, while PSNR changes from 16.24 to 18.48. The added noise is Poisson.

- Poisson noise is skewed, which affects both the mean and variance.
- Adjusted R^2 value is 0.7808.
- 9.5% of the 168 pixels were found to be statistically insignificant in predicting the noisy pixel.
- The weights add up to 0.9895.

Factors: Radius and Noise Level



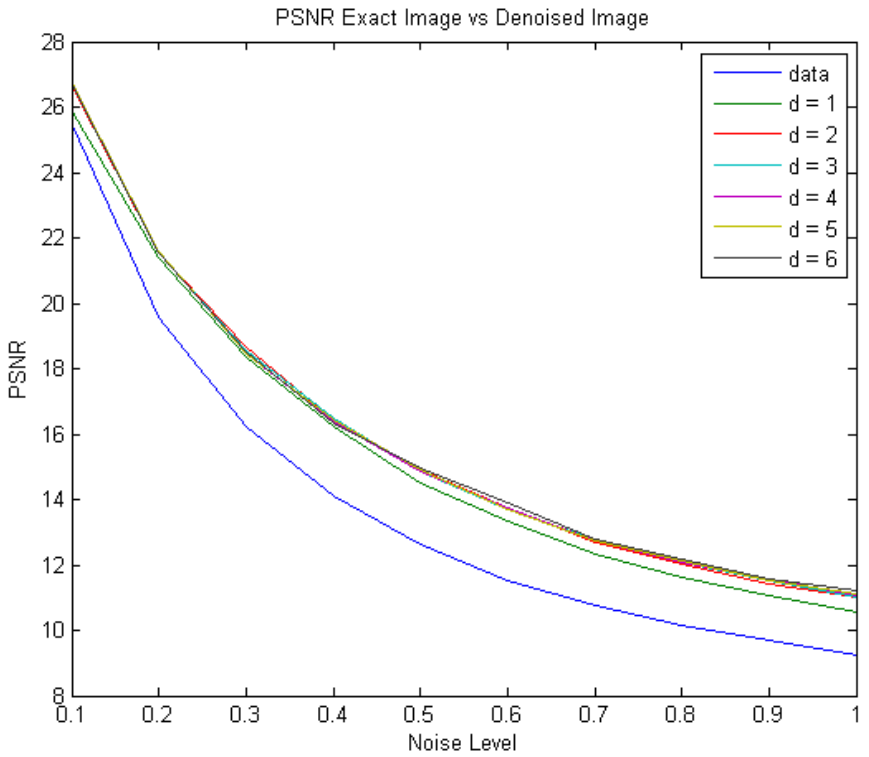
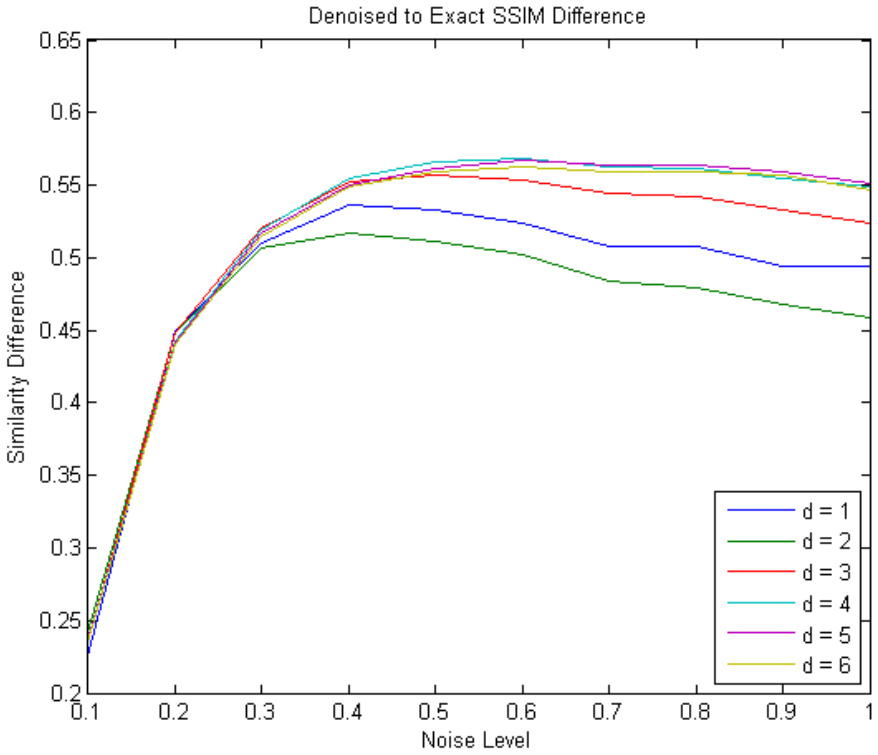


Figure : Additive Poisson noise; structural similarity and PSNR versus noise level between the exact and denoised images for different radii. The blue curves reference the exact image and noisy image.

Factors: Radius and Noise Level



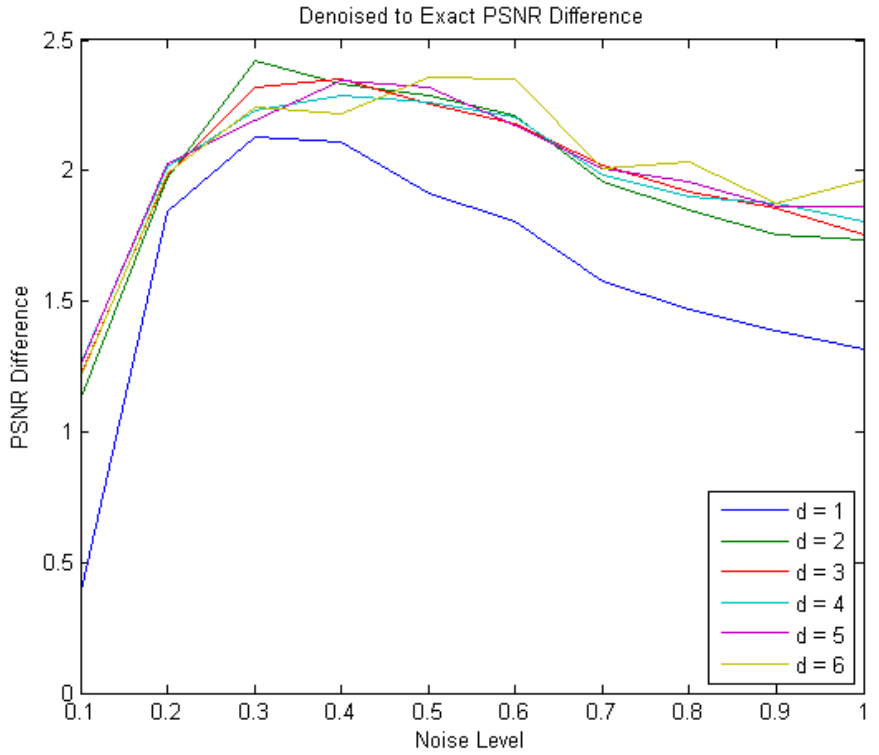


Figure : Additive Poisson noise; structural similarity difference $= SSIM(exact, denoised) - SSIM(exact, noisy)$ and PSNR difference $= PSNR(exact, denoised) - PSNR(exact, noisy)$ versus noise level for different radii.

Factor Effects Noise Type, Noise Level and Radius

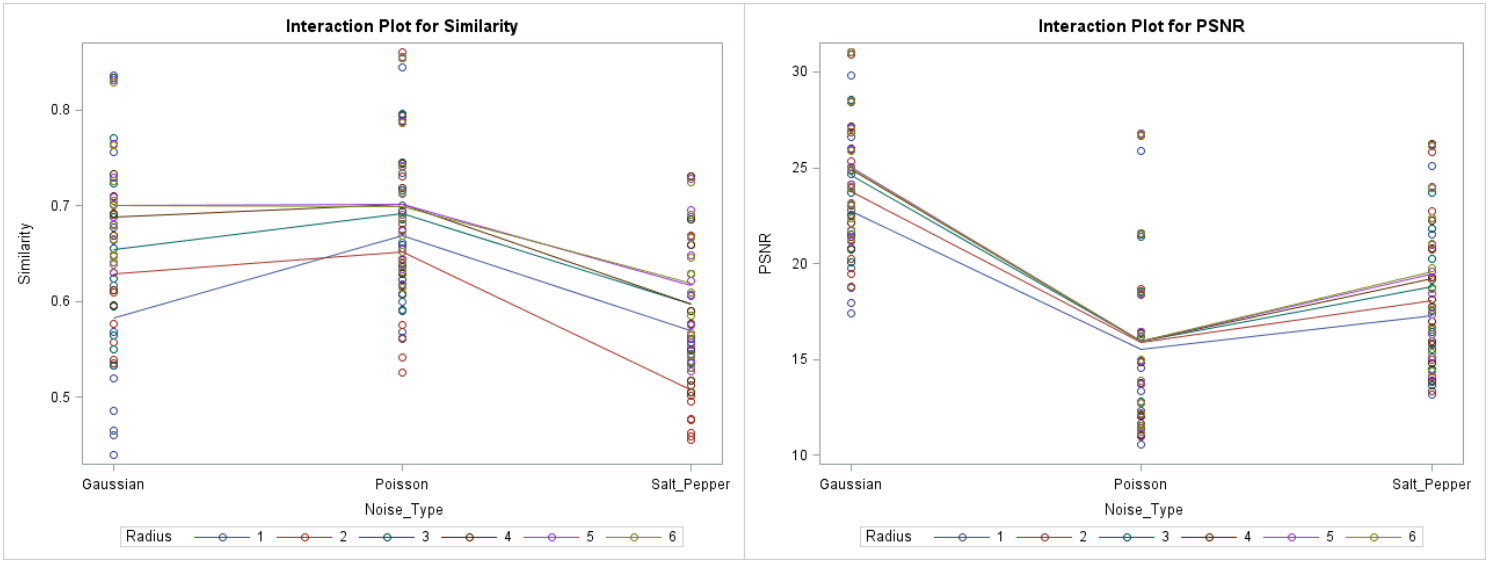


Figure : SSIM and PSNR means plots for noise type categorized by radius value.

- Two-way ANOVA for SSIM reveals that both noise type and radius are statistically significant factors. R^2 for this model is 0.3252.
- Two-way ANOVA for PSNR reveals that only noise type is a statistically significant factor. R^2 for this model is 0.4402.

Factor Effects Noise Type, Noise Level and Radius

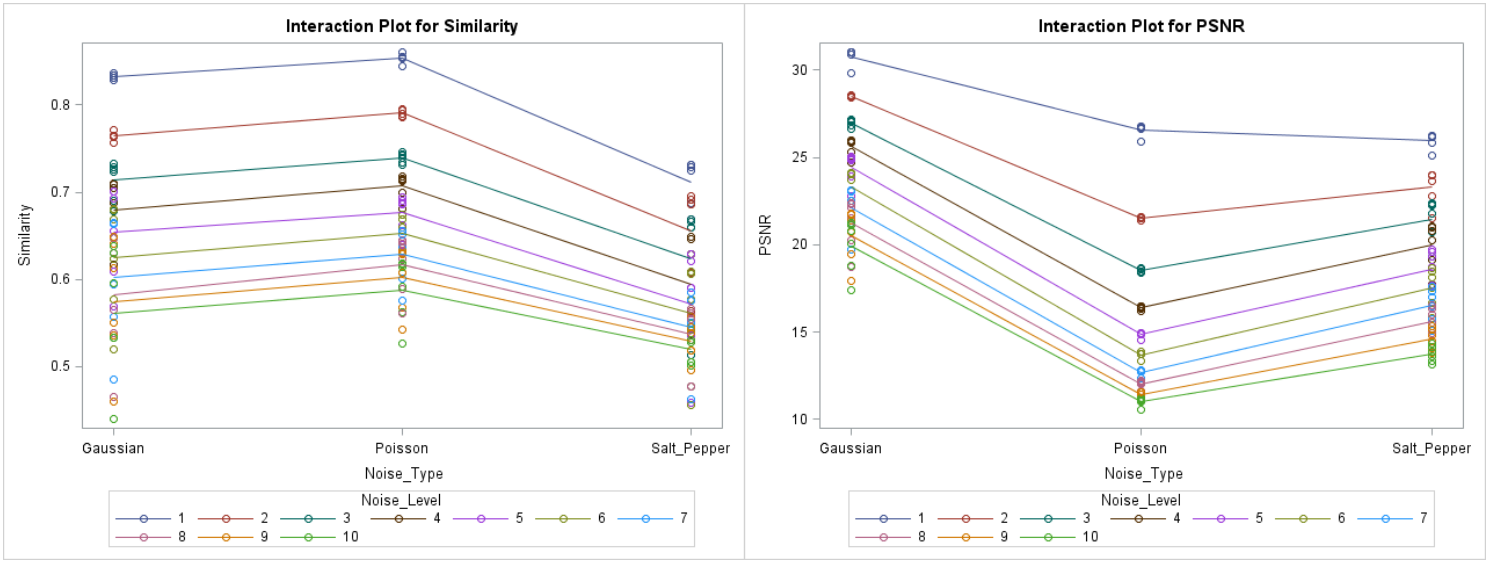


Figure : SSIM and PSNR means plots for noise type categorized by noise level.

- Two-way ANOVA for the SSIM shows that both noise type and noise level are statistically significant. R^2 for this model is 0.817.
- Two-way ANOVA for PSNR shows that both factors and their interaction are statistically significant. R^2 for this model is 0.9799.

Factor Effects Noise Type, Noise Level and Radius

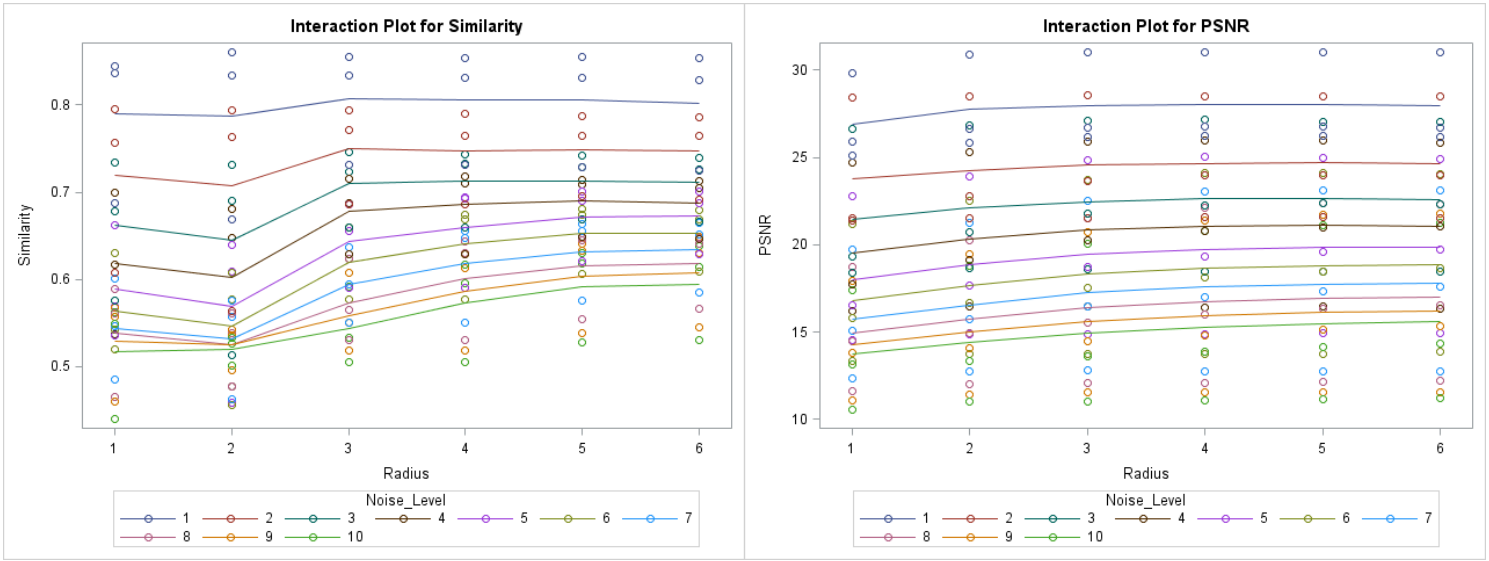


Figure : SSIM and PSNR means plots for radius categorized by noise level.

- Two-way ANOVA for SSIM reveals that both noise level and radius are statistically significant factors. R^2 for this model is 0.7238.
- Two-way ANOVA for PSNR reveals that only noise level is statistically significant. R^2 for this model is 0.5518.

Factor: Noise Type

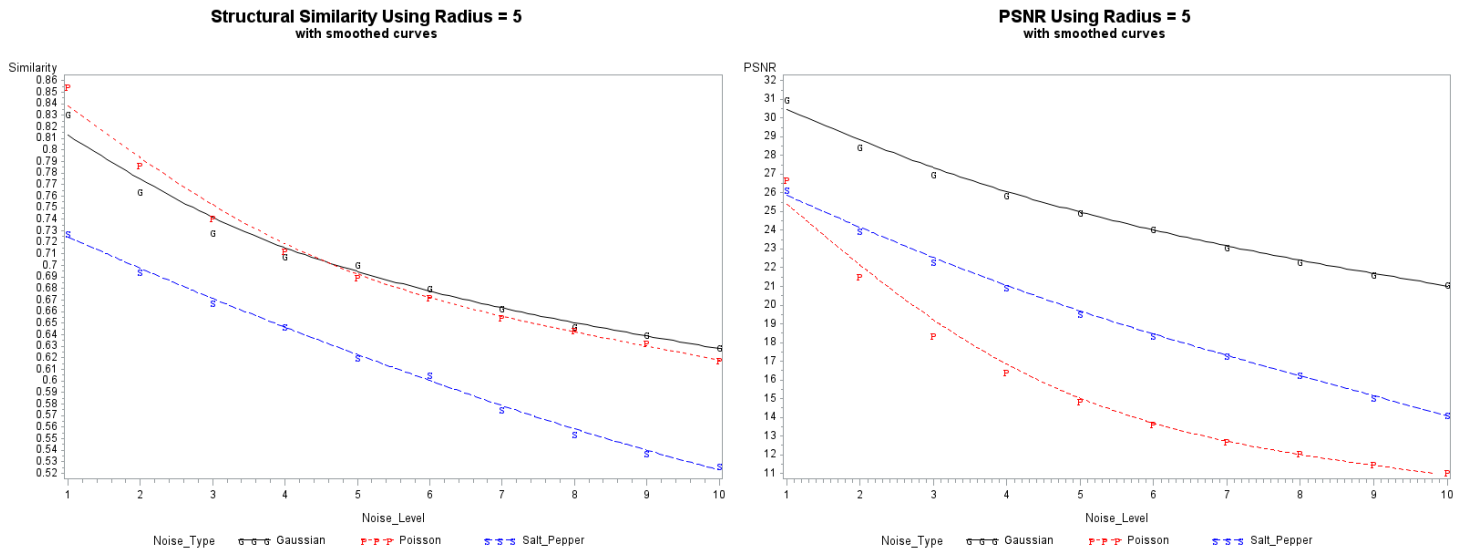


Figure : SSIM and PSNR versus noise level categorized by noise type for $d = 5$.

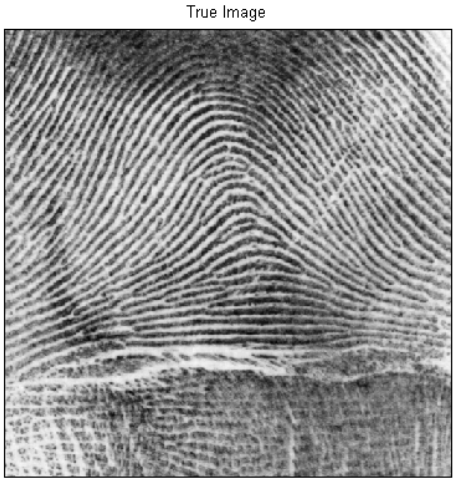
- SSIM plot shows that Gaussian and Poisson noise are almost identical, but higher than salt & pepper noise.
- PSNR plot shows that Gaussian noise has higher PSNR overall compared to salt & pepper and Poisson.

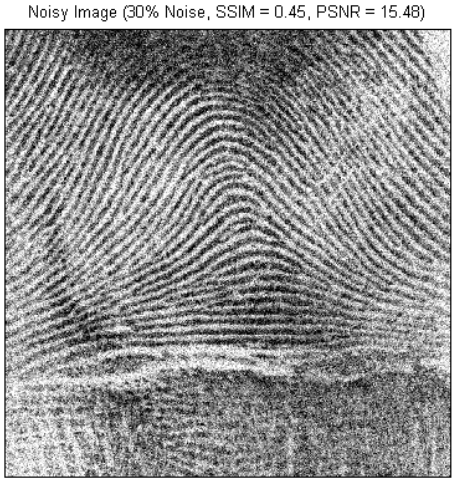
Gaussian Noise (Revisited)


True Image

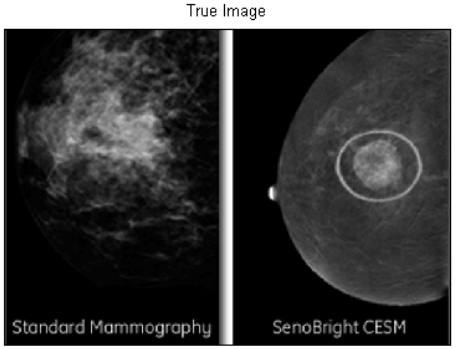

Noisy Image (30% Noise, SSIM = 0.14, PSNR = 16.17)

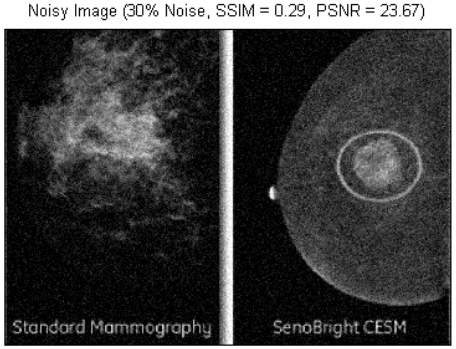

Denoised Image (radius = 6, SSIM = 0.73, PSNR = 27.01)

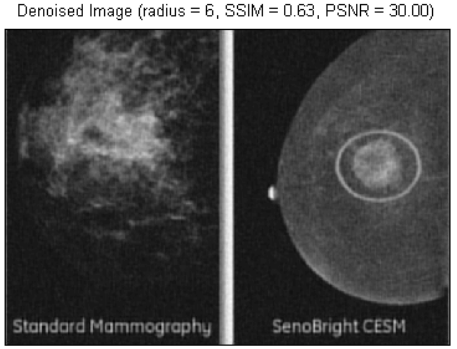

True Image


Noisy Image (30% Noise, SSIM = 0.45, PSNR = 15.48)


Denoised Image (radius = 6, SSIM = 0.82, PSNR = 24.04)


True Image
Standard Mammography
SenoBright CESM

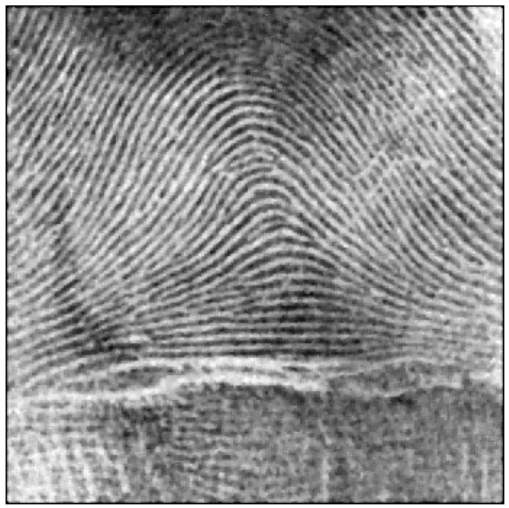

Noisy Image (30% Noise, SSIM = 0.29, PSNR = 23.67)
Standard Mammography
SenoBright CESM


Denoised Image (radius = 6, SSIM = 0.63, PSNR = 30.00)
Standard Mammography
SenoBright CESM

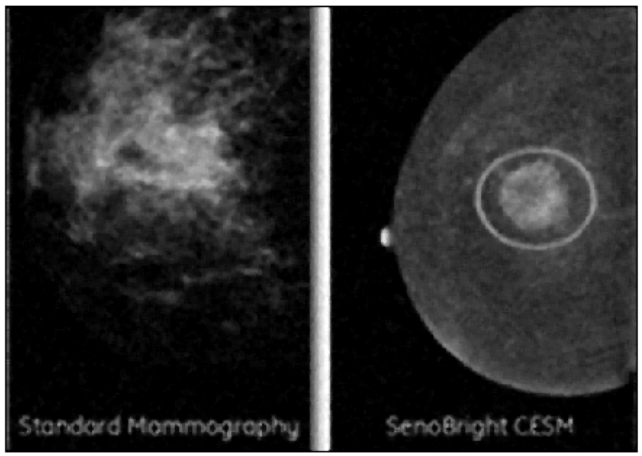
Median Filtering

Denoised Image (radius = 6, SSIM = 0.56, PSNR = 25.49)

Denoised Image (radius = 6, SSIM = 0.73, PSNR = 22.00)



Denoised Image (radius = 6, SSIM = 0.77, PSNR = 25.50)



Outline for section 4

Thank You!