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**ANALYTIC THEORY OF
LOW THRESHOLD
LASING IN PHOTONIC
SPIRAL MEDIA**

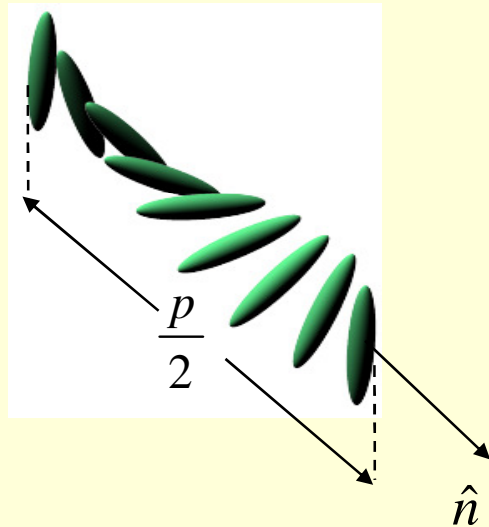
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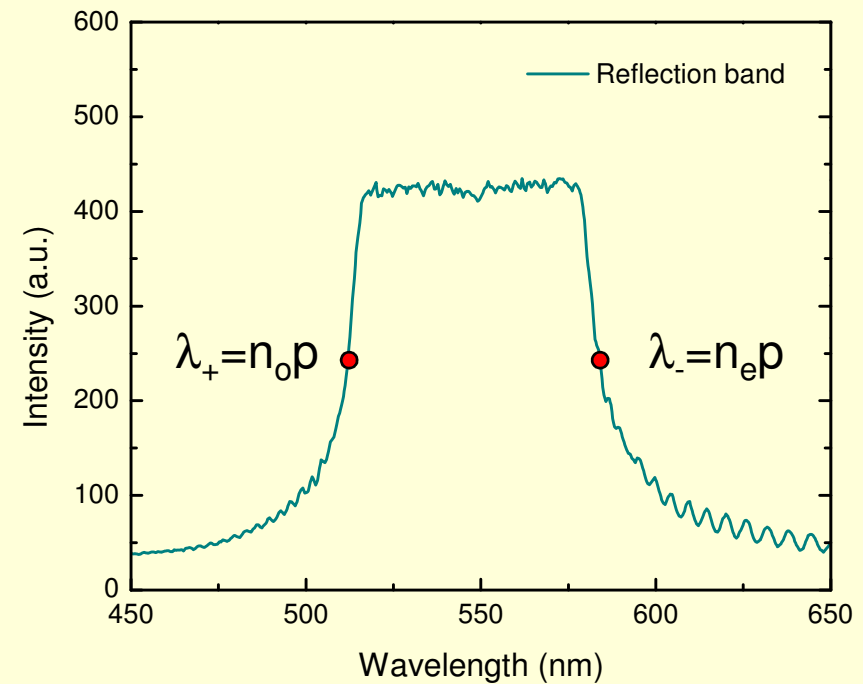
LOCALIZED OPTICAL MODES IN CLC: APPLICATION TO LASING AND OTHER OPTICAL PHENOMENA

- 1. INTRODUCTION**
- 2. EIGEN WAVES IN CHIRAL LC**
- 3. BOUNDARY PROBLEM (EDGE MODE)**
- 4. ABSORBING LC**
- 5. AMPLIFYING LC**
- 6. OPTIMIZATION OF PUMPING**
- 7. DEFECT MODES**
- 8. ACTIVE DEFECT LAYER (BIREFRINGENT,
AMPLIFYING, ABSORBING and so on)**
- 9. CONCLUSION**

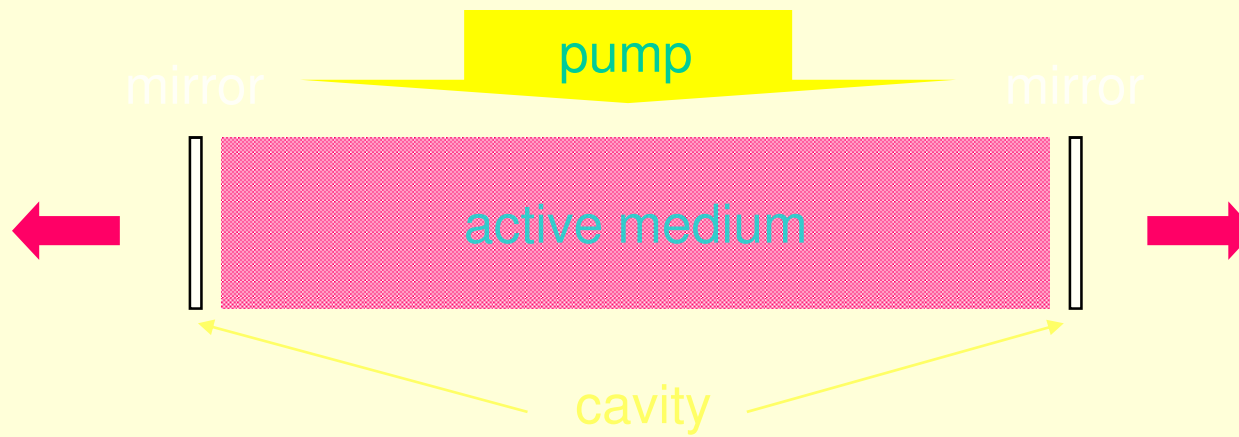
Cholesteric Liquid Crystal



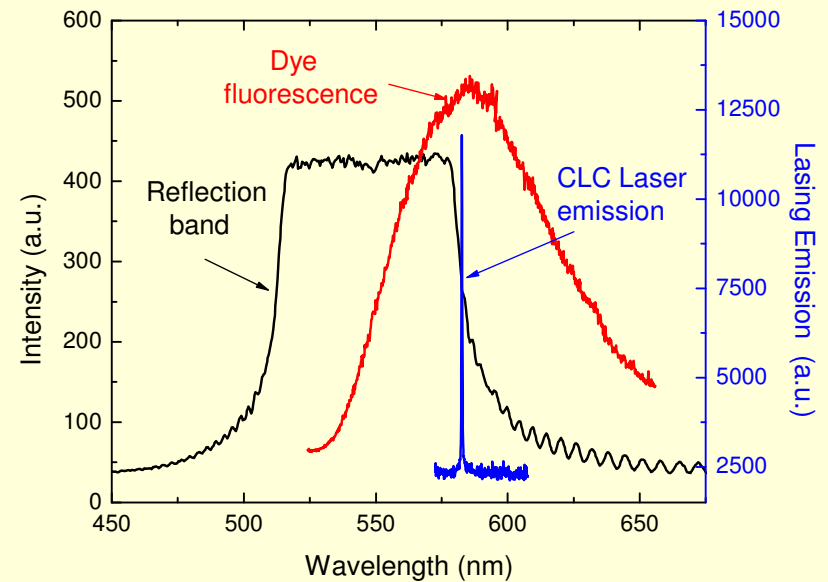
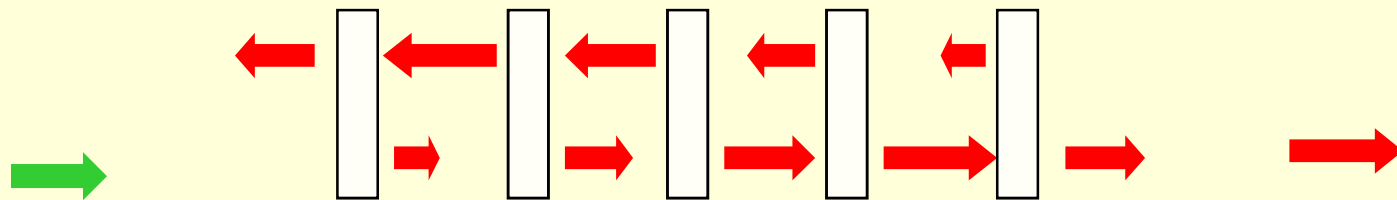
$$\lambda = n(T)p(T)$$



Lasing



Cholesteric Liquid Crystal Laser

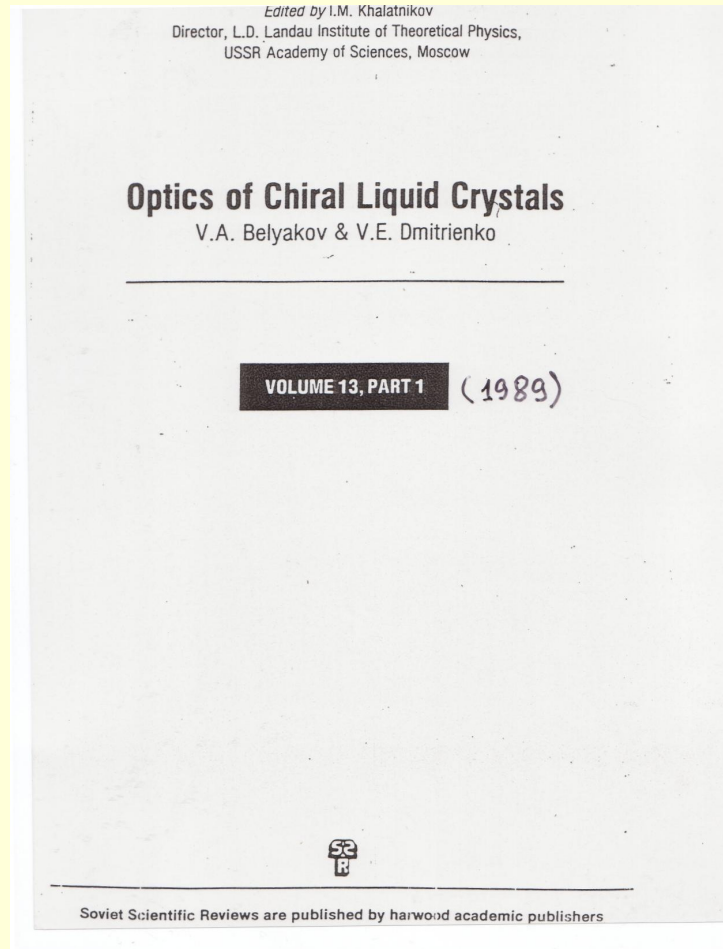


Mirrorless laser

MAIN PUBLICATIONS

1. Il'chishin I. P., Tikhonov E. A., Tishchenko V. G., and Shpak M. T.: Generation of tunable radiation by impurity cholesteric liquid crystals; *JETP Lett.* 1980; **32**: 24.
2. Kopp V. I., Zhang Z.-Q., and Genack A. Z.: *Prog. Quant. Electron.* 2003; **27(6)**: 369.
3. Yang Y.-C., Kee C.-S., Kim J.-E. et al. : Defect Mode in Cholesterics. *Phys.Rev.E*, 1999 **60**, 6852.
4. Kopp V. I., and Genack A. Z.: Phase Jump Defect Mode in Cholesterics. *Phys.Rev Lett.* 2003; **89**: 033901.
5. Schmidtke J., Stille W., and Finkelmann H.: Observation of Phase Jump Defect Mode in Cholesterics. *Phys.Rev Lett.* 2003; **90**: 083902.
6. Kogelnik H. and Shank C.V.: Band Edge Modes in Periodic Media. *J.Appl.Phys.* 1972; **43**: 2327.
7. Becchi M., Ponti S., Reyes J.A., and Oldano C.: Defect Modes in Cholesterics: Analytic Approach. *Phys.Rev.E* 2004; **70**: 033103.
8. Matsushita Y., Huang Y., Zhou Y., Wu S. et al.: Low Threshold and High Efficiency Lasing upon Band-edge Excitation in a Cholesteric Liquid Crystal. *Appl.Phys.Lett* 2007; **90**: 091114.
9. Humar M., Ravnik M., Pajk S., Musevich I.: Electrically tunable liquid crystal optical microresonators, *NATURE PHOTONICS*, 2009, **3**, 595.
10. V.A.Belyakov, Localized Optical Modes in optics of Chiral Liquid Crystals in “New Developments in Liquid Crystals and Applications”, 2013 (Ed. P.K. Choudry, Nova Publishers, New York), Chp. 7, p.199.

FITTING THEORY AND EXPERIMENT



V.A. Belyakov

Diffraction Optics of Complex-Structured Periodic Media

With 86 Illustrations



Springer-Verlag, 1992.

New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

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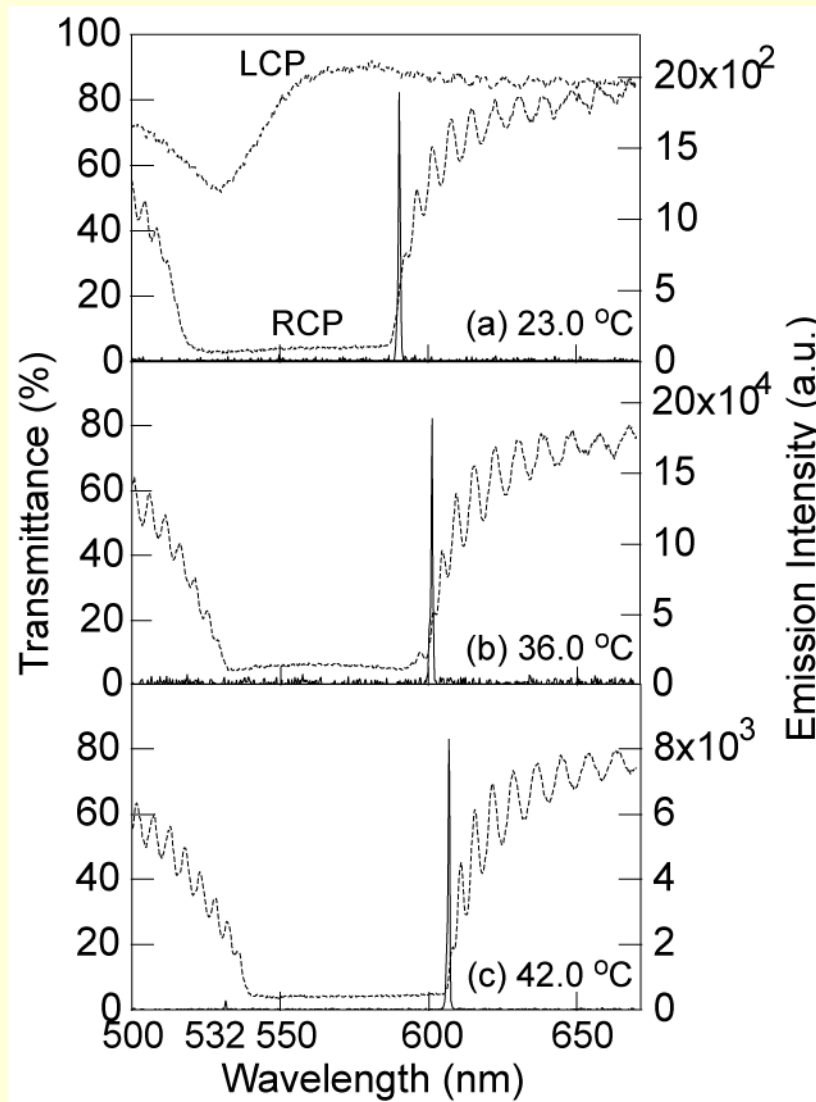


Fig. 1, Matsuhisa et al, Appl. Phys. Lett. 90, 091114 (2007)

EIGEN WAVES IN CHIRAL LC

As it is known [7,8,9] the eigenwaves corresponding to propagation of light in chiral LC along a spiral axes, i.e. the solution of the Maxwell equation

$$\partial^2 \mathbf{E} / \partial z^2 = c^{-2} \boldsymbol{\varepsilon}(z) \partial^2 \mathbf{E} / \partial t^2 \quad (28)$$

are presented by a superposition of two plane waves of the form

$$\mathbf{E}(z,t) = e^{-i\omega t} [\mathbf{E}^+ \mathbf{n}_+ \exp(i\mathbf{K}^+ z) + \mathbf{E}^- \mathbf{n}_- \exp(i\mathbf{K}^- z)] \quad (29)$$

where ω is the light frequency, \mathbf{n}_\pm are the two vectors of circular polarizations, $\boldsymbol{\varepsilon}(z)$ is the dielectric tensor of the chiral liquid crystal [7 - 10], c is the light velocity and the wave vectors \mathbf{K}^\pm satisfy to the condition

$$\mathbf{K}^+ - \mathbf{K}^- = \boldsymbol{\tau}, \quad (30)$$

where $\boldsymbol{\tau}$ is the reciprocal lattice vector of the LC spiral ($\tau = 4\pi/p$, where p is the cholesteric pitch).

The wave vectors \mathbf{K}^\pm in the four eigen solutions (29) are determined by the eq.(30) and the following formulas

$$\mathbf{K}_j^+ = \tau/2 \pm \kappa \{ 1 + (\tau/2\kappa)^2 \pm [(\tau/\kappa)^2 + \delta^2]^{1/2} \}^{1/2}, \quad (31)$$

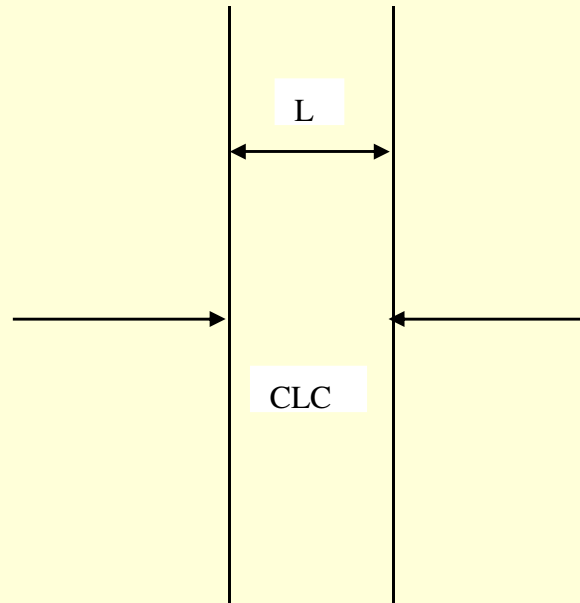
Where j numerates the eigen solutions with the ratio of amplitudes (E^-/E^+) given by the expression

$$\xi_j = (E^-/E^+)_j = \delta / [(\mathbf{K}_j^+ - \tau)^2 / \kappa^2 - 1], \quad (32)$$

where $\kappa = \omega \epsilon_0^{1/2} / c$, $\epsilon_0 = (\epsilon_{\parallel} + \epsilon_{\perp}) / 2$, $\delta = (\epsilon_{\parallel} - \epsilon_{\perp}) / (\epsilon_{\parallel} + \epsilon_{\perp})$ is the dielectric anisotropy, and ϵ_{\parallel} , ϵ_{\perp} are the principal values of the LC dielectric tensor [8 - 10]. Define the ratio of the dielectric constant imaginary part to the real part as γ , i.e.

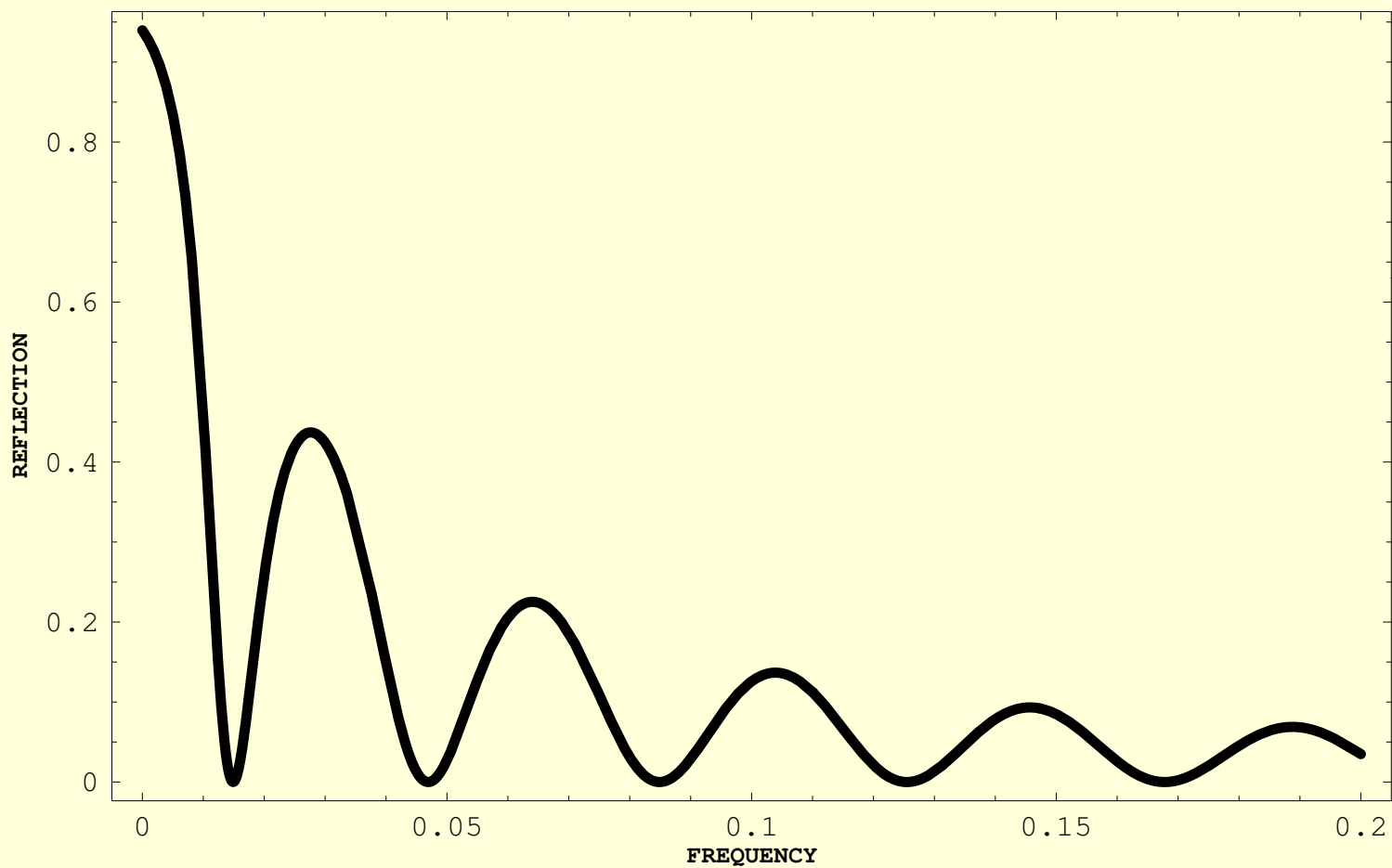
$$\epsilon = \epsilon_0 (1 + i\gamma).$$

Schematic of the boundary problem for edge modes



UNABSORBING LC LAYER

$\gamma=0$ ($\delta(\nu-1)$ is plotted at the abscissa, see below)



EM frequencies

It occurs that for nonabsorbing LC layers the real parts of EM frequencies are coinciding with the positions of the beats minima of the reflection coefficient R.

The frequency positions of the beats minima of the reflection coefficient R correspond to

$$qL = \pi n, \quad \pm \nu = 1 + (\pi n/a)^2/2, \quad n = 1, 2, 3, \dots,$$

$$\nu = 2(\omega - \omega_B)/\delta\omega_B, \quad \omega_B = c\tau/2e_0^{1/2}, \quad a = \delta L\tau/4.$$

The complex frequencies are determined by the dispersion equation (the solvability condition of the homogeneous Eqs.(8)) :

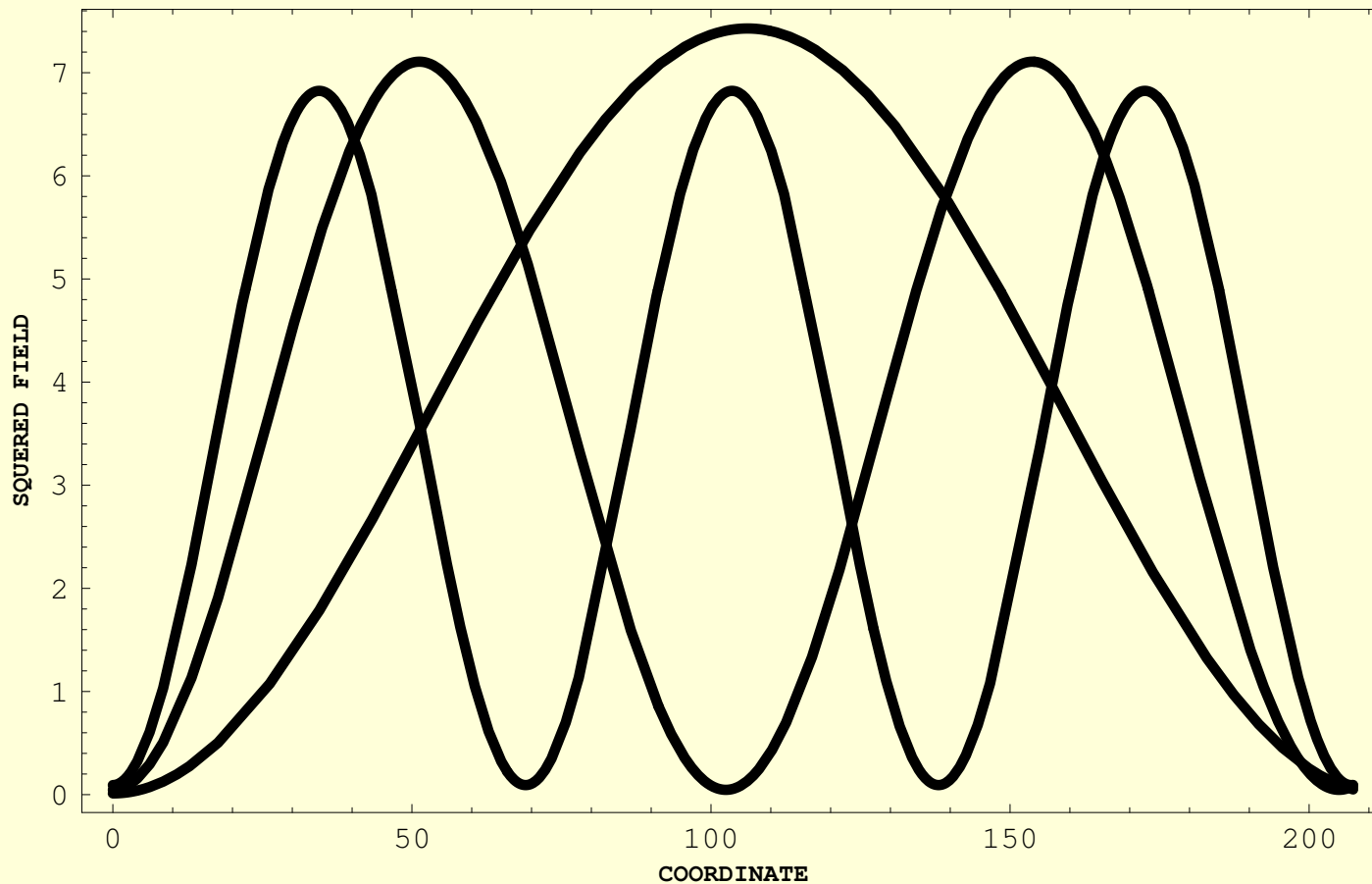
$$\operatorname{tg} qL = i(q\tau/\kappa^2)/[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1] \quad (12)$$

In a general case the solution of Eq.(12) determining the EM frequencies $\omega_{EM} = \omega_{EM}^0(1+i\Delta)$ may be found only numerically. For a sufficiently small Δ ensuring the condition $L\operatorname{Im}q \ll 1$ an analytic solution exists:

$$\Delta = -1/2 \delta(n\pi)^2/(\delta L\tau/4)^3.$$

The corresponding EM life-time is $\diamond = (L/c)(\delta L/pn)^2$

Calculated EM coordinate (in the dimensionless units $z\tau$) energy distribution inside the CLC layer for the three first edge modes ($\delta=0.05$, $N=33$, $n=1,2,3$).



anomalously strong absorption (1-R-T)

(at the edge mode frequencies (a) $\gamma=0.001$, (b) $\gamma=0.005$)

Fig.1a

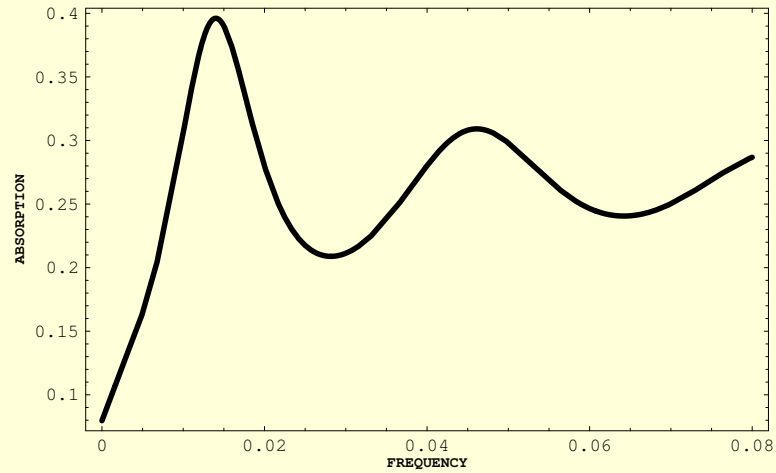
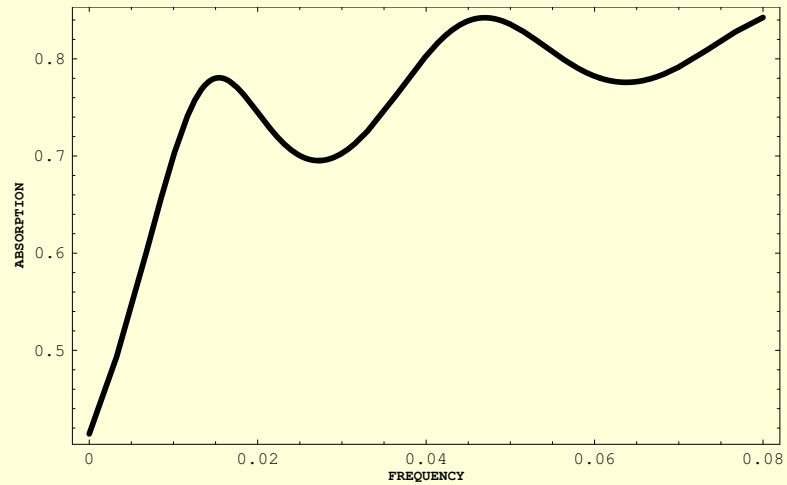
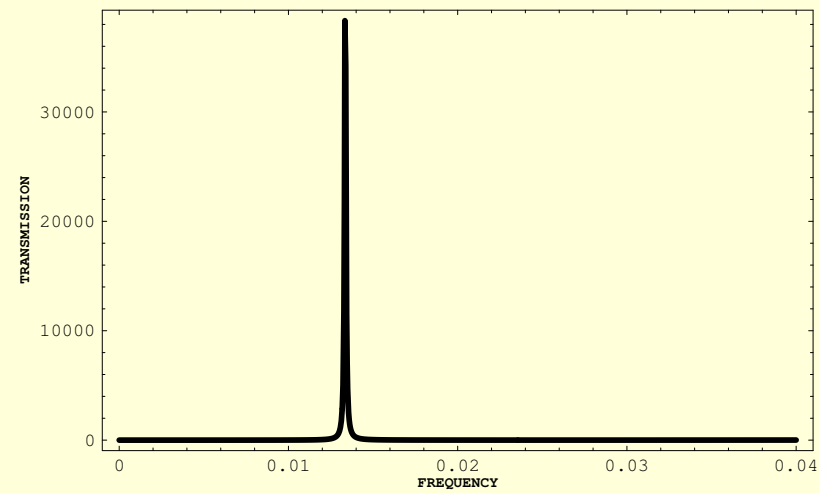
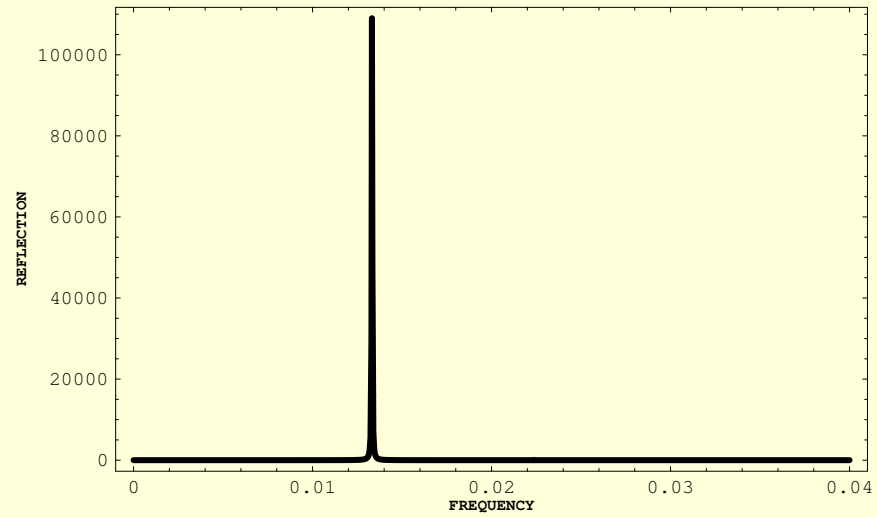


Fig.1b



REFLECTION AND TRANSMISSION CLOSE TO FIRST EDGE LASING MODE

$$\gamma = -0.00565 \quad (\delta = 0.05, 4\pi L/p = 300)$$



Edge lasing modes

The equation determining the edge lasing modes (at $\gamma < 0$) is given by the following expression:

$$\operatorname{tg}qL = i(q\tau/\kappa^2)/[\tau/2\kappa)^2 + (q/\kappa)^2 - 1] \quad (37)$$

In general case, this equation has to be solved numerically. However for a very small negative γ the frequency values of the edge lasing modes are pinned to the frequencies of zero value of reflection coefficient in its frequency beats outside of the stop band edge for the same layer with zero imaginary part of the dielectric tensor [10,13]. It is why for this limiting case for a small $|\gamma|$ and $L|\operatorname{Im}q| \ll 1$ the threshold values of the gain (γ) for the edge lasing modes may be found analytically:

$$\gamma = -\delta(n\pi)^2/a^3 = -\delta(n\pi)^2/(\delta L\tau/4)^3 \quad (38)$$

The threshold values of $|\gamma|$ are inversely proportional to the third power of the layer thickness and a minimal value of $|\gamma|$ corresponds to $n=1$.

OPTIMIZATION OF PUMPING

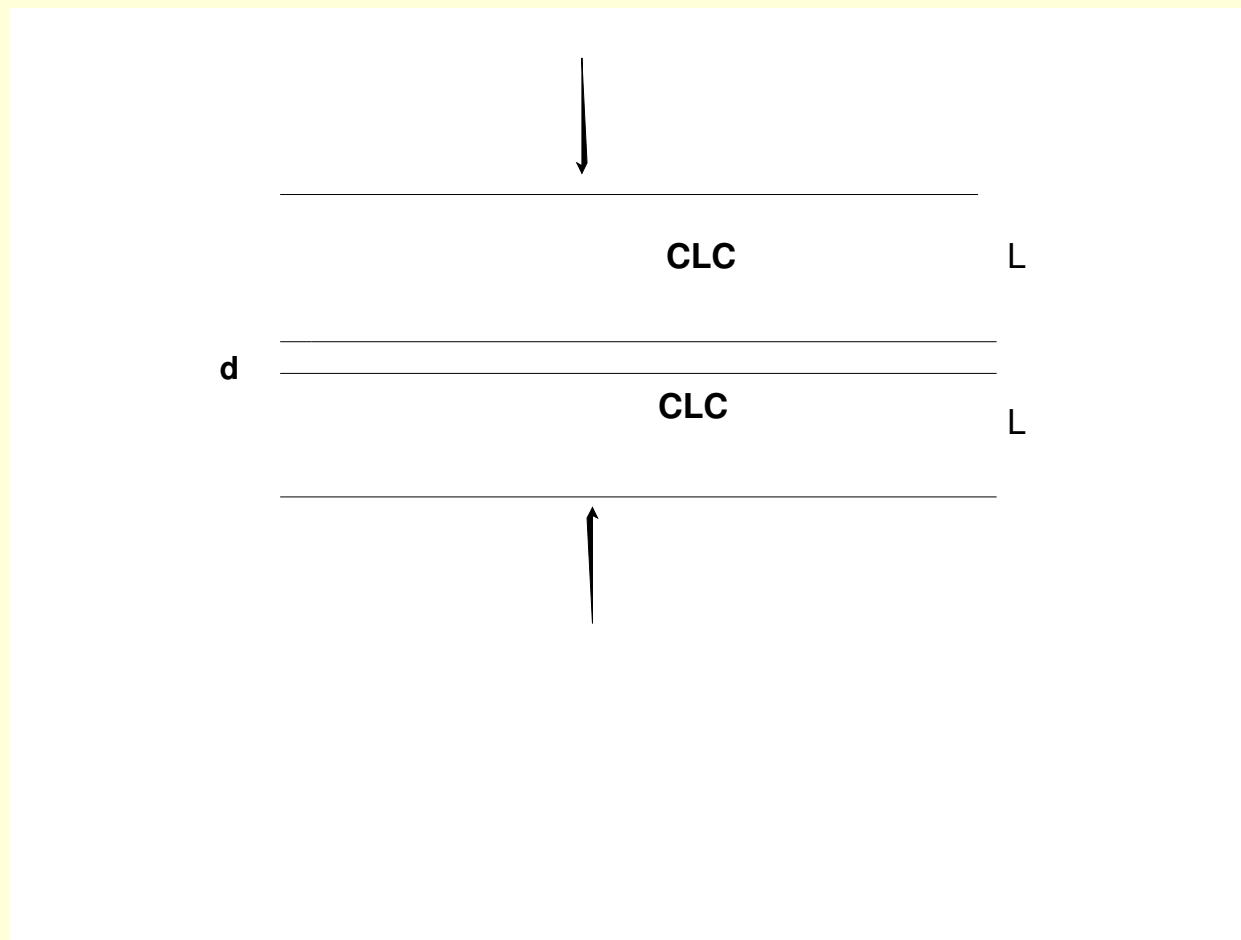
The highest efficiency of the pumping and the lowest value of the lasing threshold gain may be reached if the lasing occurs at the first EM frequency and the pumping wave is under conditions of the anomalously strong absorption effect. These may occur in a collinear geometry, however it demands a very special choice of the CLC parameters. A regular way to reach the optimization is to use a non collinear pumping [11,14]. The corresponding value of the angle between the spiral axis and the pumping wave propagation direction is determined approximately as:

$$\theta = \arccos[\omega_l / \omega_p],$$

where ω_l and ω_p are the lasing and pumping frequency, respectively.

SCHEMATIC OF A DEFECT MODE STRUCTURE

Fig.1



**T(d) versus the frequency for a nonabsorbing CLC, $\delta=0.05$,
 $N=33$, $d/p=0.1$**

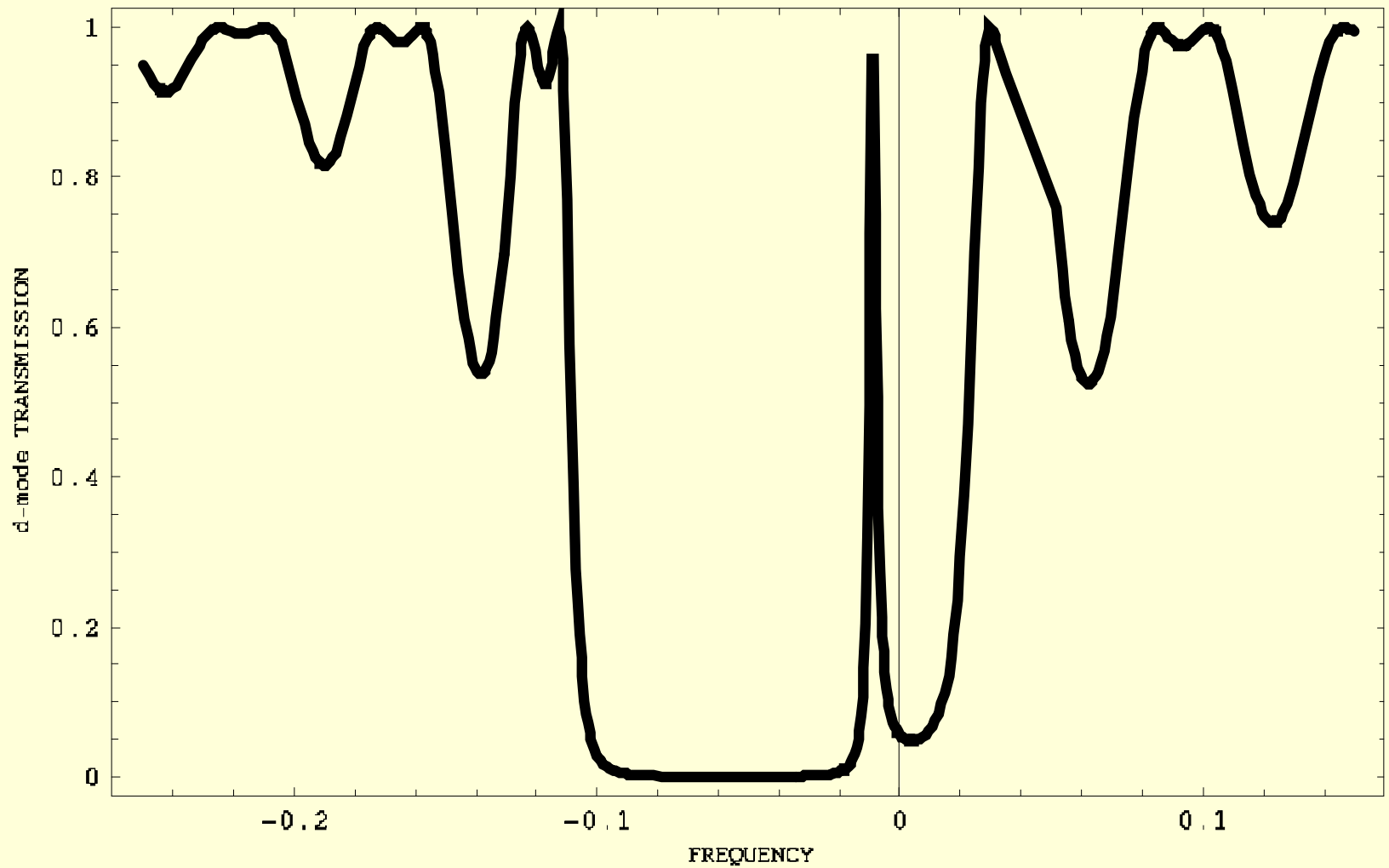
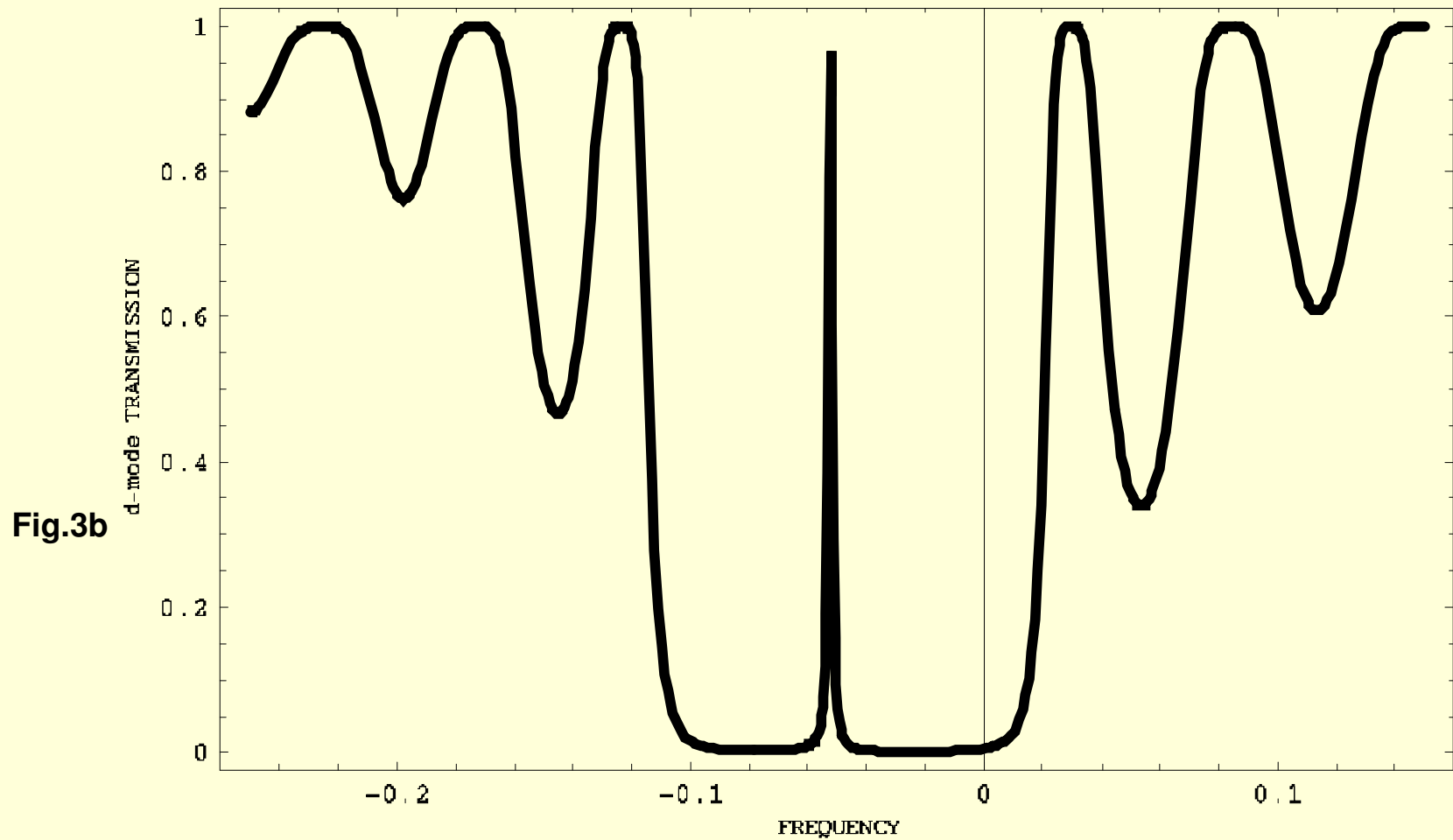


Fig.3a

**T(d) versus the frequency for a nonabsorbing CLC, $\delta=0.05$,
 $N=33$. $d/p=0.25$**



**R(d) versus the frequency for a nonabsorbing CLC at
 $d/p=200.1$; $\delta=0.05$, $N=33$.**

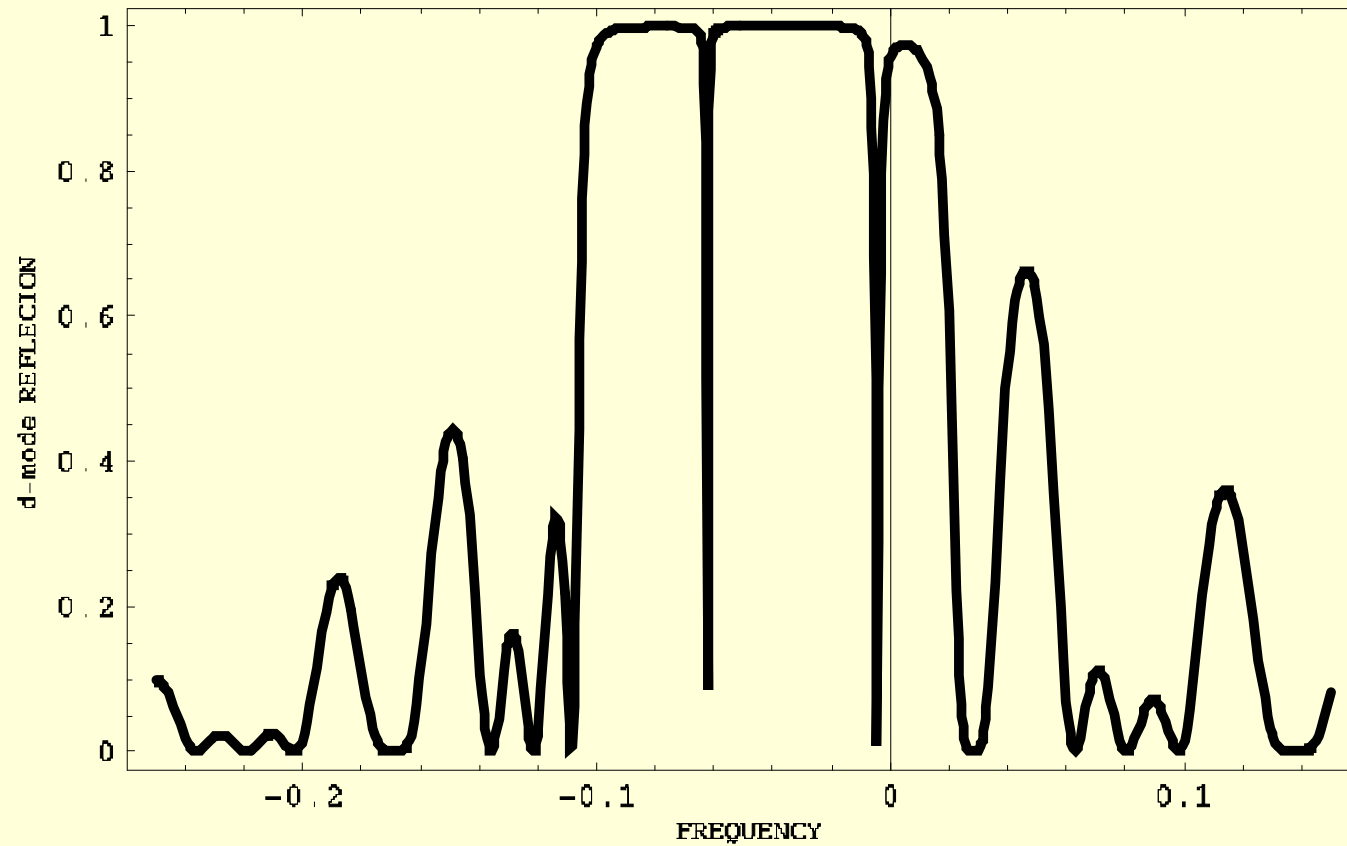
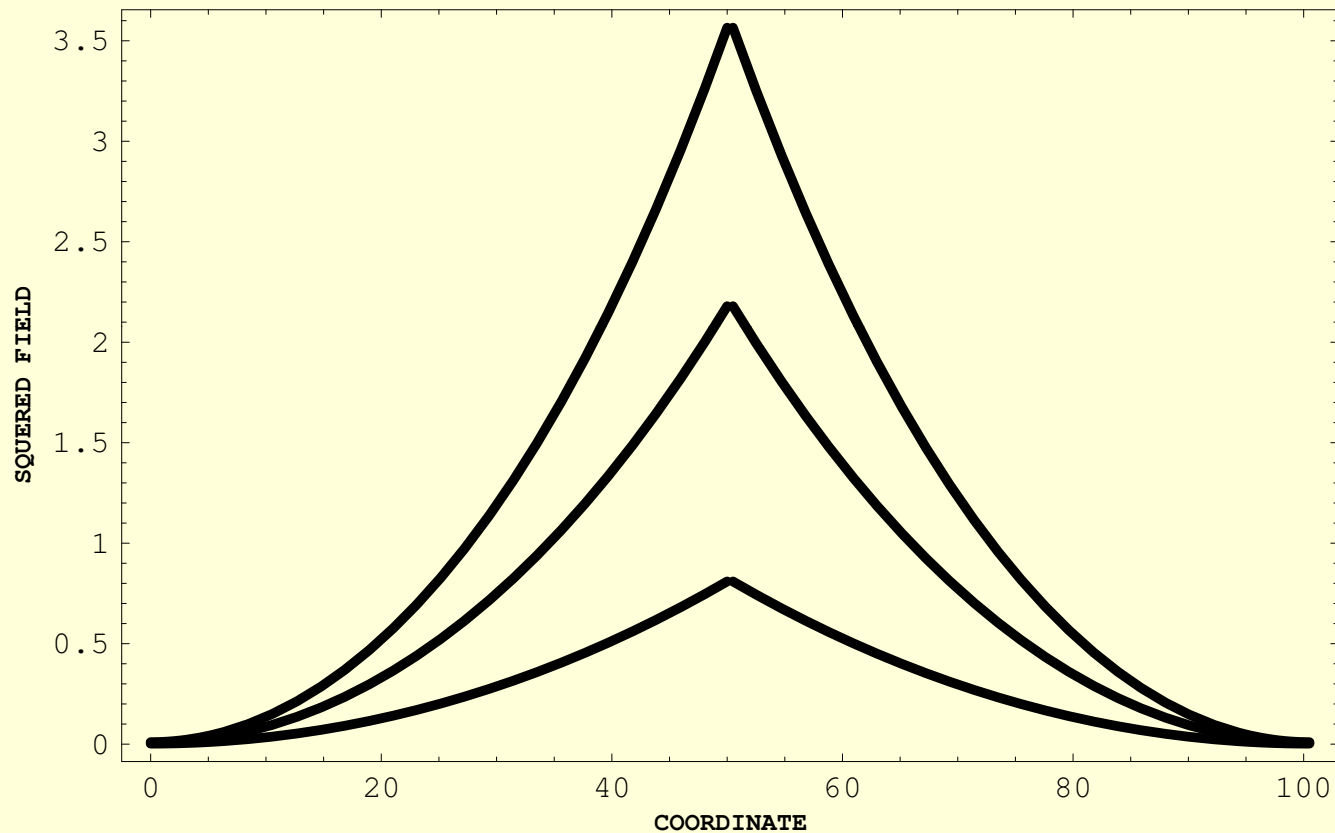


Fig.4a

Calculated distribution of the squared field modulus in the CLC layers versus the distance from the defect layer centre ($x=z/p$) ($\delta=0.05, 0.04, 0.025$ from the top curve to the bottom, respectively); $d/p=1/4$, $N=50$.



ABSORBING AND AMPLIFYING LC (Thick CLC layers)

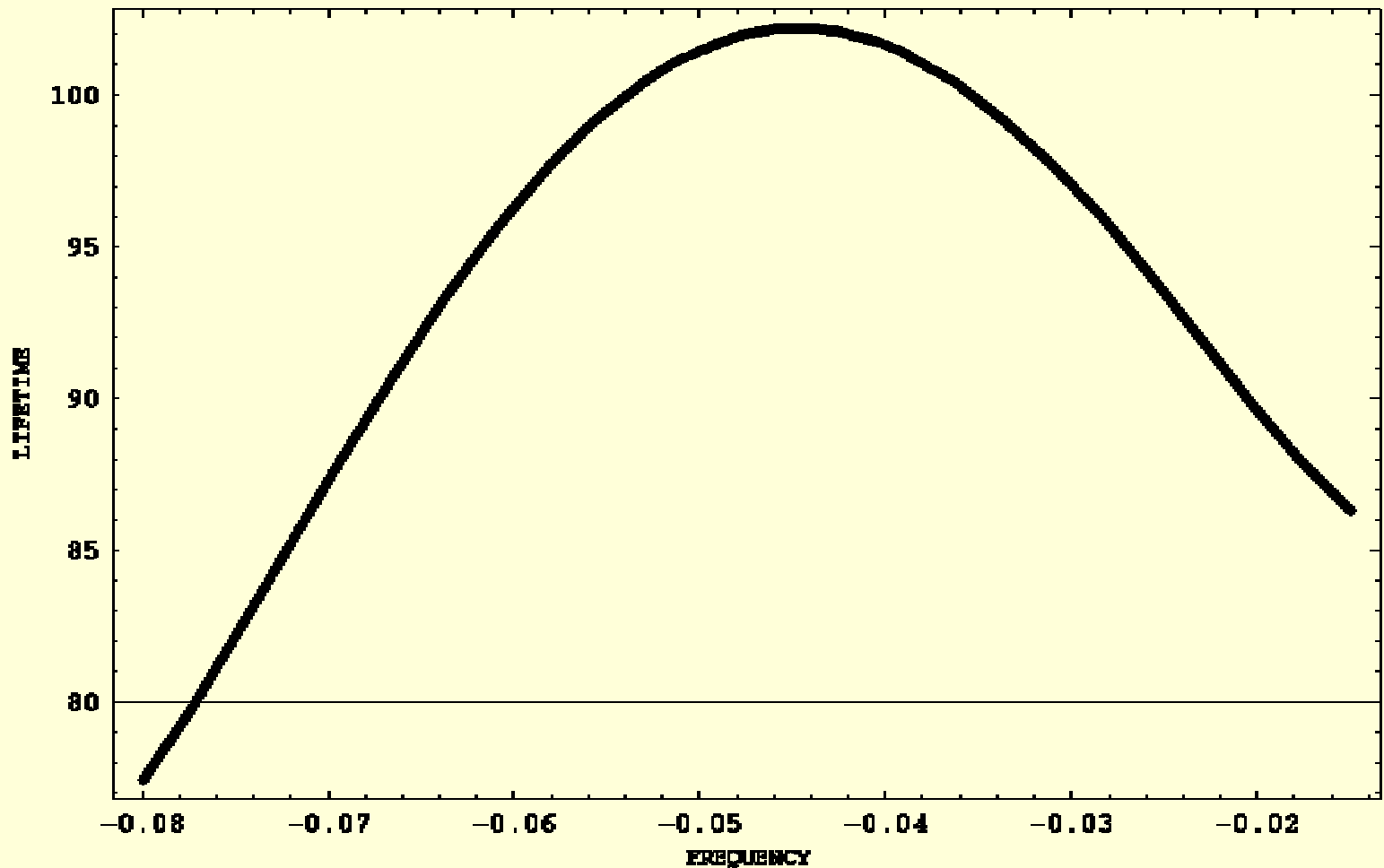
Assume for simplicity that the absorption in LC is isotropic, i.e. $\epsilon = \epsilon_0(1+i\gamma)$. For thick CLC layers an analytic solution for γ ensuring maximal absorption may be found. For the position of ω_D just in the middle of the stop band the expression for γ reduces to

$$\gamma = (4/3\kappa)(p/L) \exp[-2\kappa\delta(L/p)] .$$

For thick CLC amplifying layers an analytic solution for γ (gain) corresponding to the lasing threshold may be found. For the position of ω_D just in the middle of the stop band the expression for γ is given by the formula

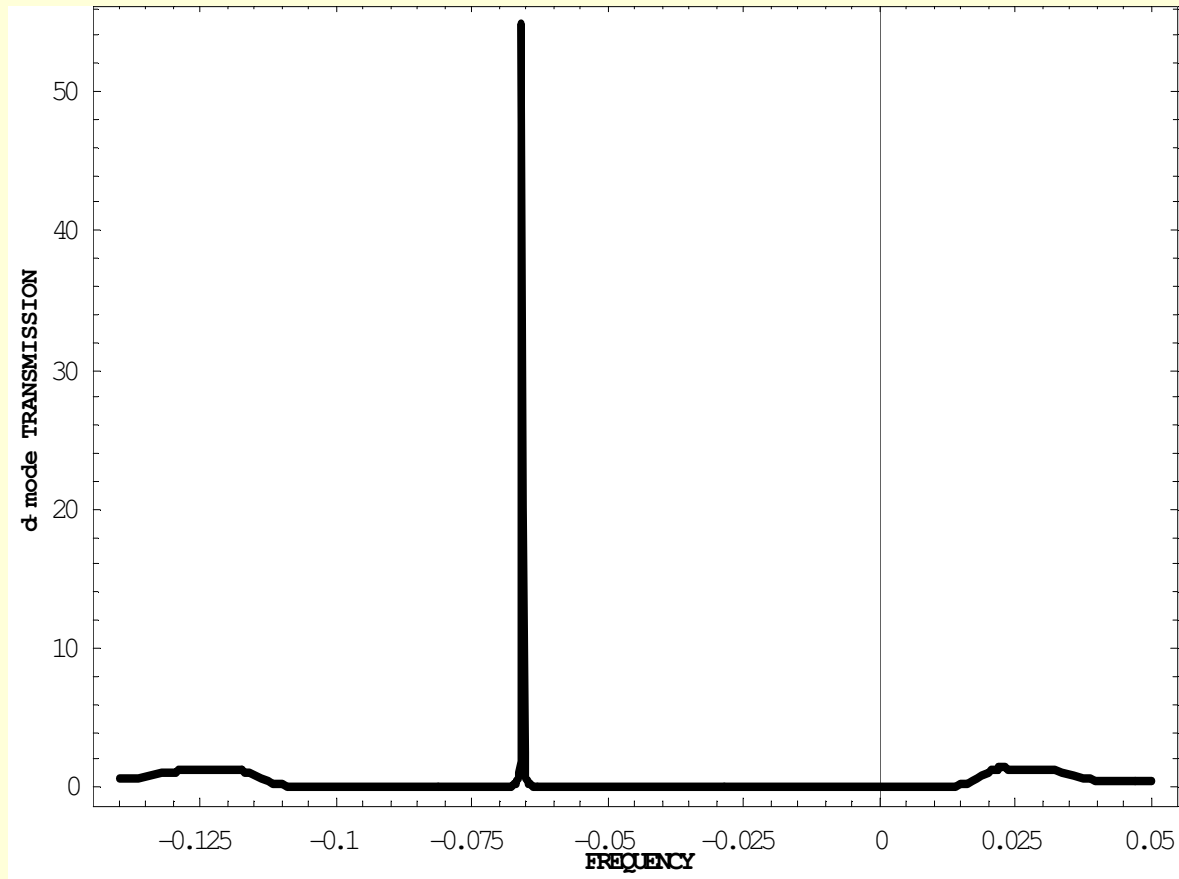
$$\gamma = -(4/3\kappa)(p/L) \exp[-2\kappa\delta(L/p)] .$$

DM lifetime (normalized by the time of light flight through DMS) dependence on the DM frequency location inside stop-band calculated for thick CLC layers ($\delta=0.05$, $N=40$)



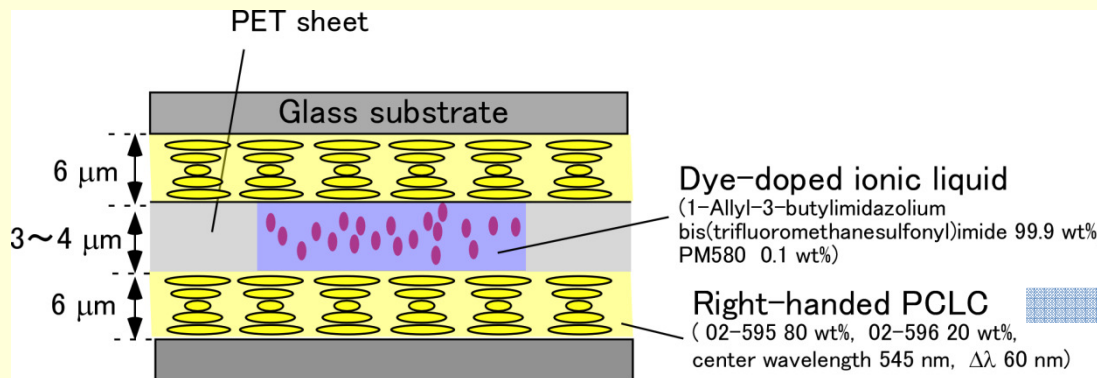
Transmission coefficient $|T(d,L)|^2$

for $\gamma = -0.000675$ ($\delta=0.05$, $N=33$, $d/p=2.25$)

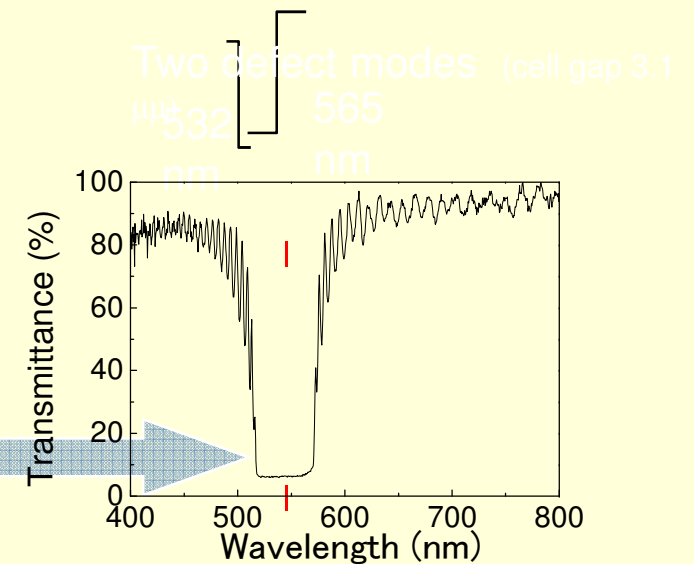


ACTIVE DEFECT LAYER (Device Configuration)

- The cell was sandwiched by two polymer cholesteric liquid crystal (PCLC) mirror (reflection bandwidth 60 nm, center 545 nm)
- The cell gap was 3~4 μm , where only two defect-modes (532 nm and 565 nm) exist in reflection band for defect-mode excitation and defect-mode lasing
- Pyrromethene 580 (laser dye, Exciton) was used to obtain laser action at the wavelength of 565 nm



Excitation light
 (Nd:YAG 532 nm, 1 ns, 10 Hz)



Transmittance of right-handed PCLC irradiated with right-handed circularly polarized light

Fig.2a Intensity transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency (Here and at all other figures “frequency” $\blacksquare = \delta[2(\omega - \omega_B)/(\delta\omega_B) - 1]$) for diffracting incident and exiting polarizations at the birefringent phase shift at the defect layer thickness $\Delta\phi = \pi/20$ at $d/p=0.25$; $Lt=2pN$, where here and at all other figures $\delta=0.05$ and $N=33$ is the director half-turn number at the CLC layer thickness L .

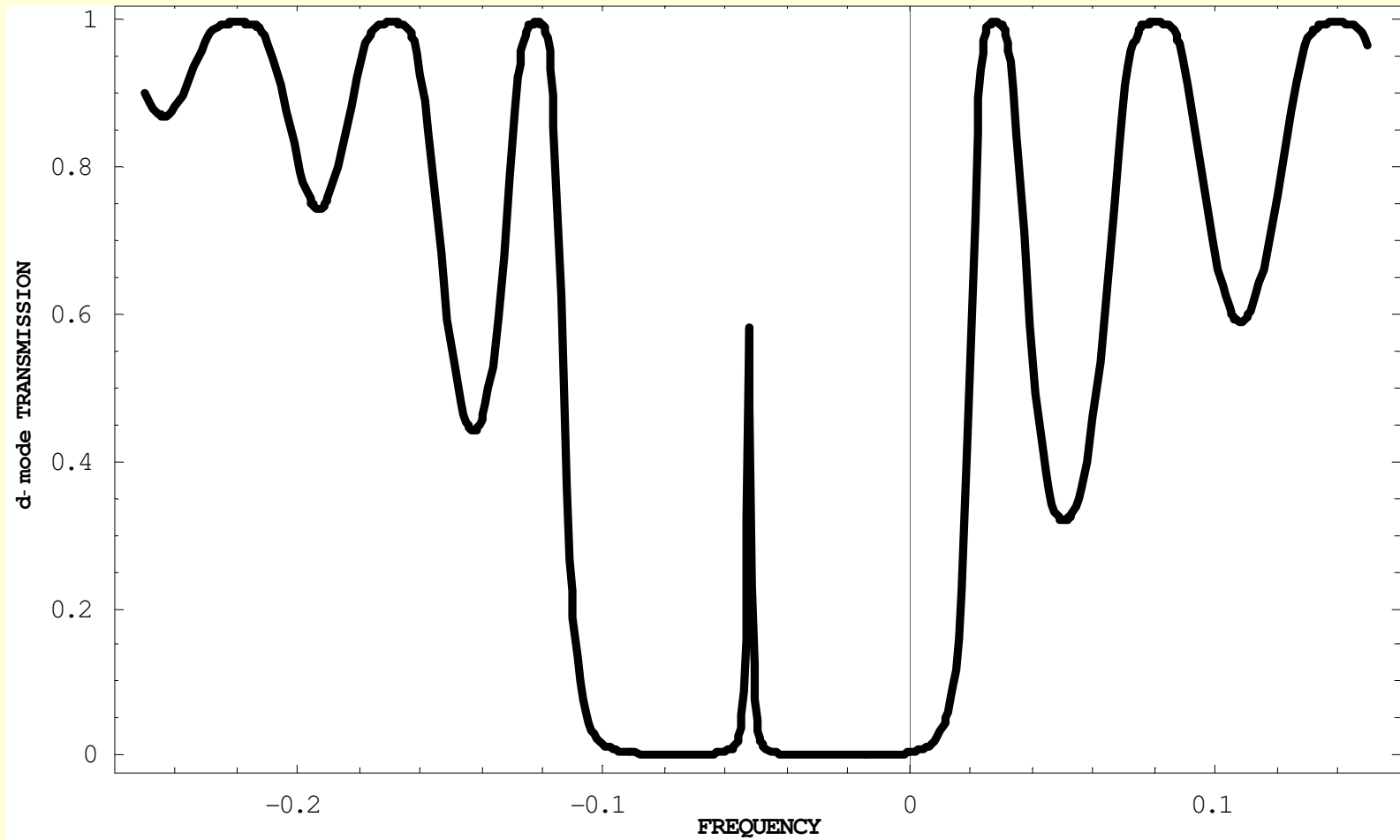


Fig.2b Intensity transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency for **diffracting incident and exiting polarizations** at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/16$

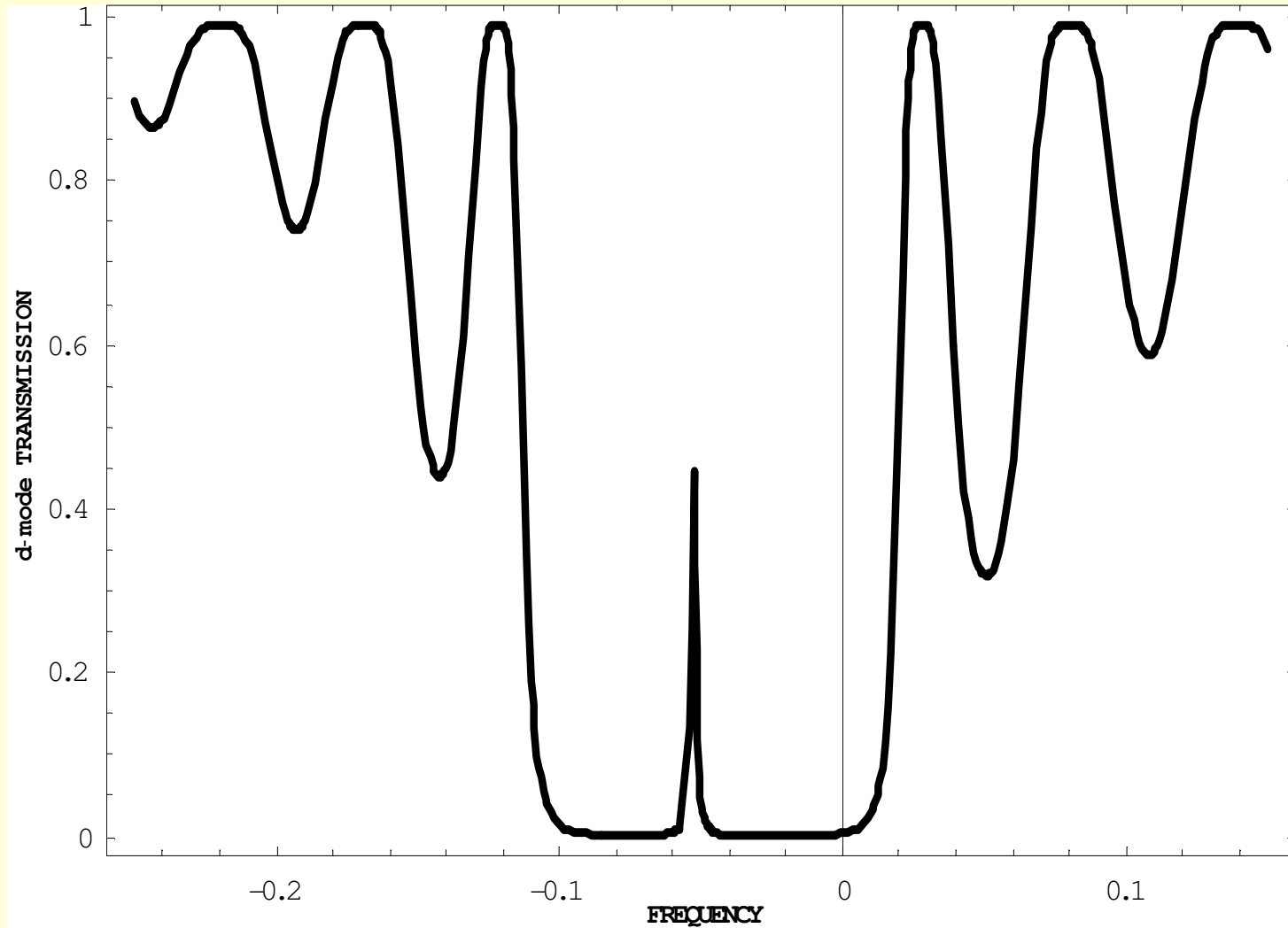


Fig.2c Transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency **for diffracting incident and exiting polarizations** at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/12$

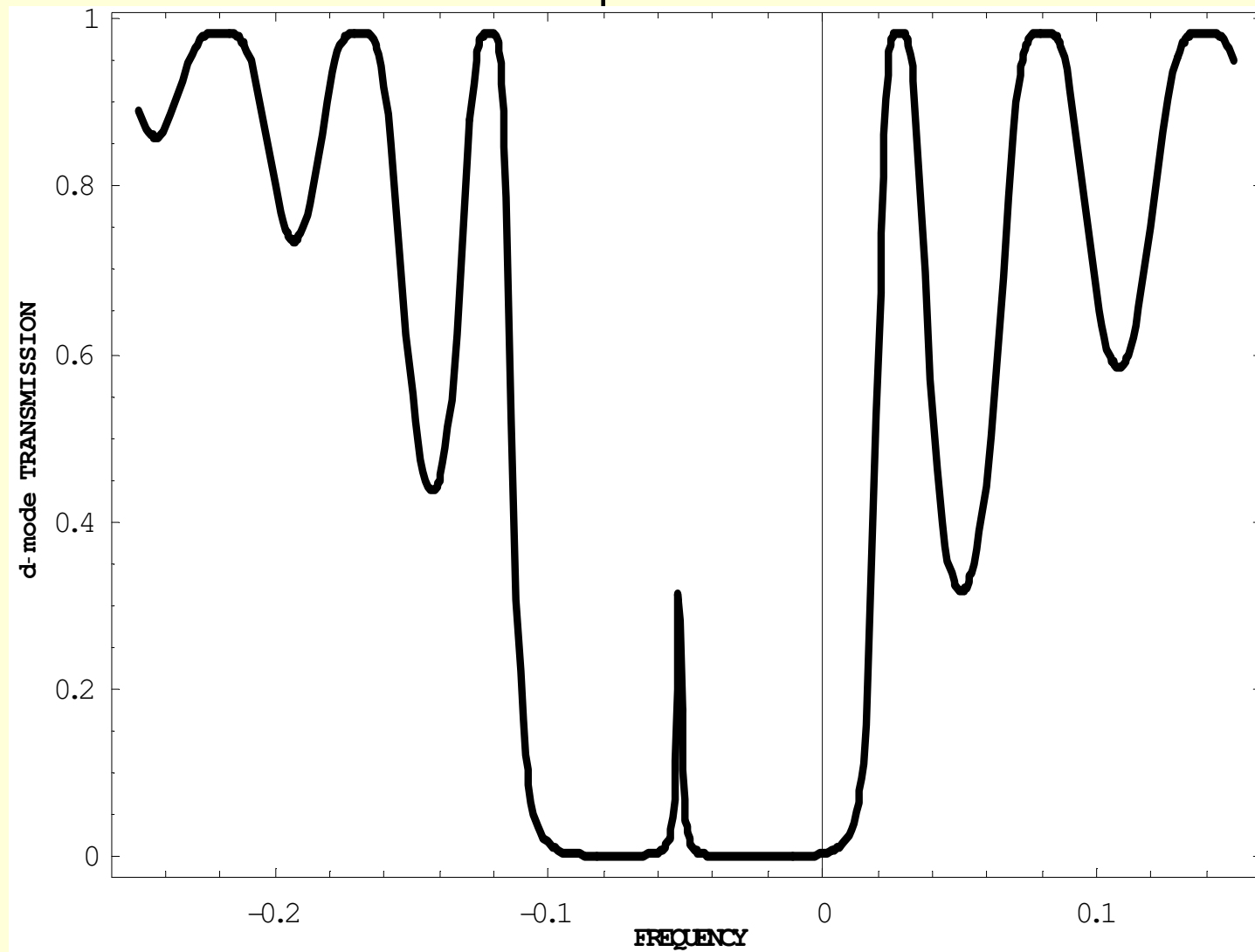


Fig.2d Transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency **for diffracting incident and exiting polarizations** at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/8$

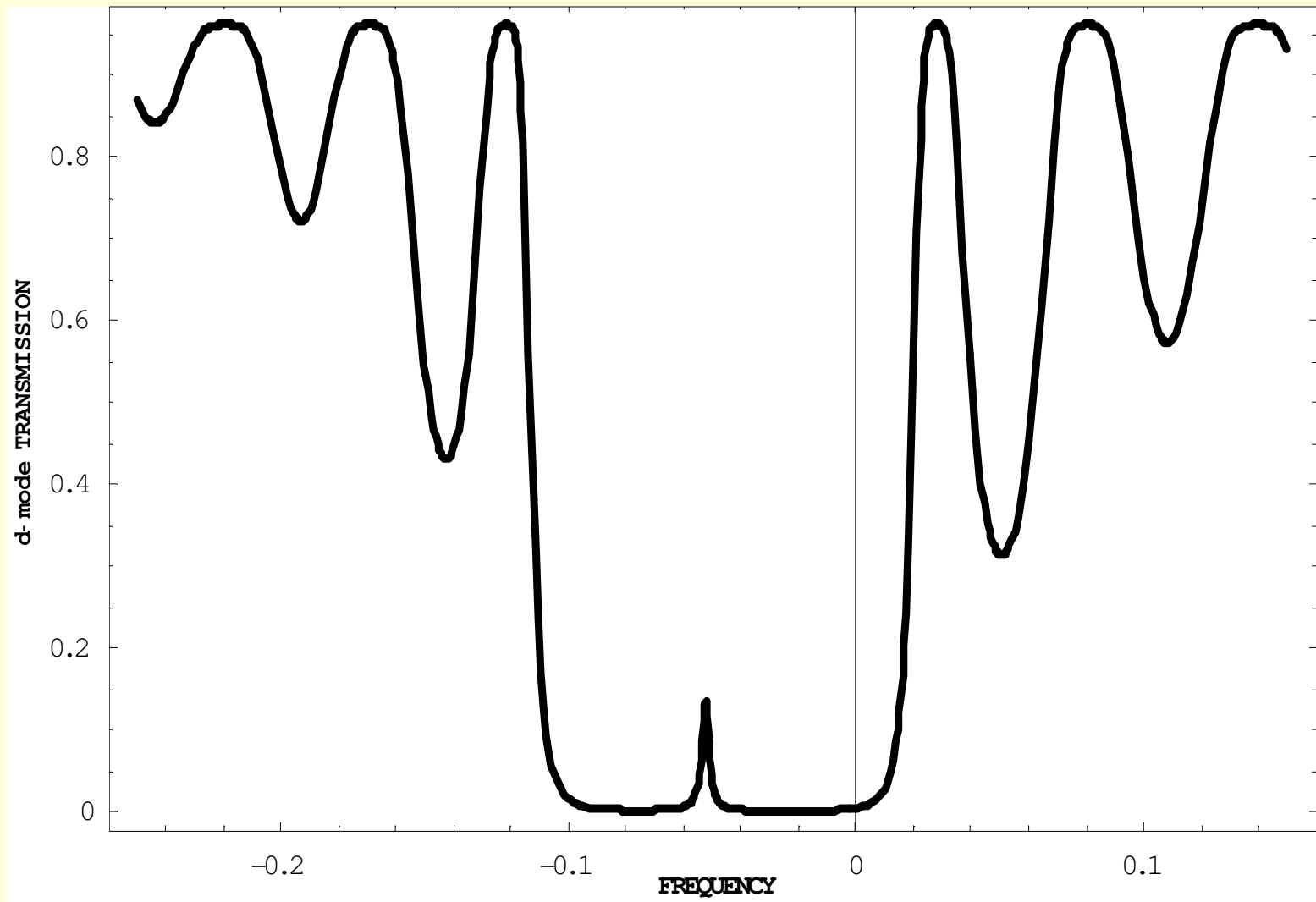


Fig.2e Transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency **for diffracting incident and exiting polarizations** at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/6$

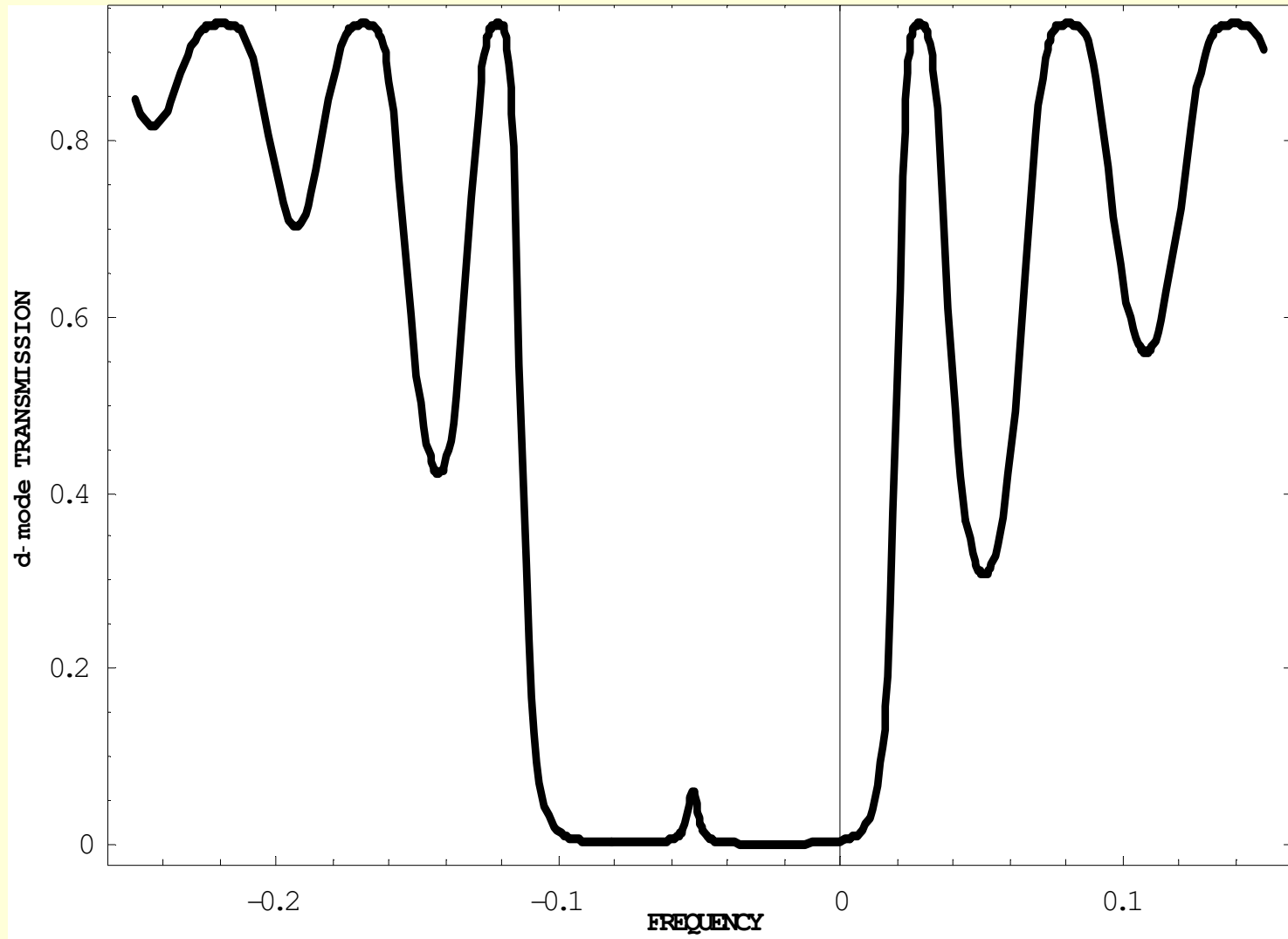


Fig.2f Transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency **for diffracting incident and exiting polarizations** at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/4$

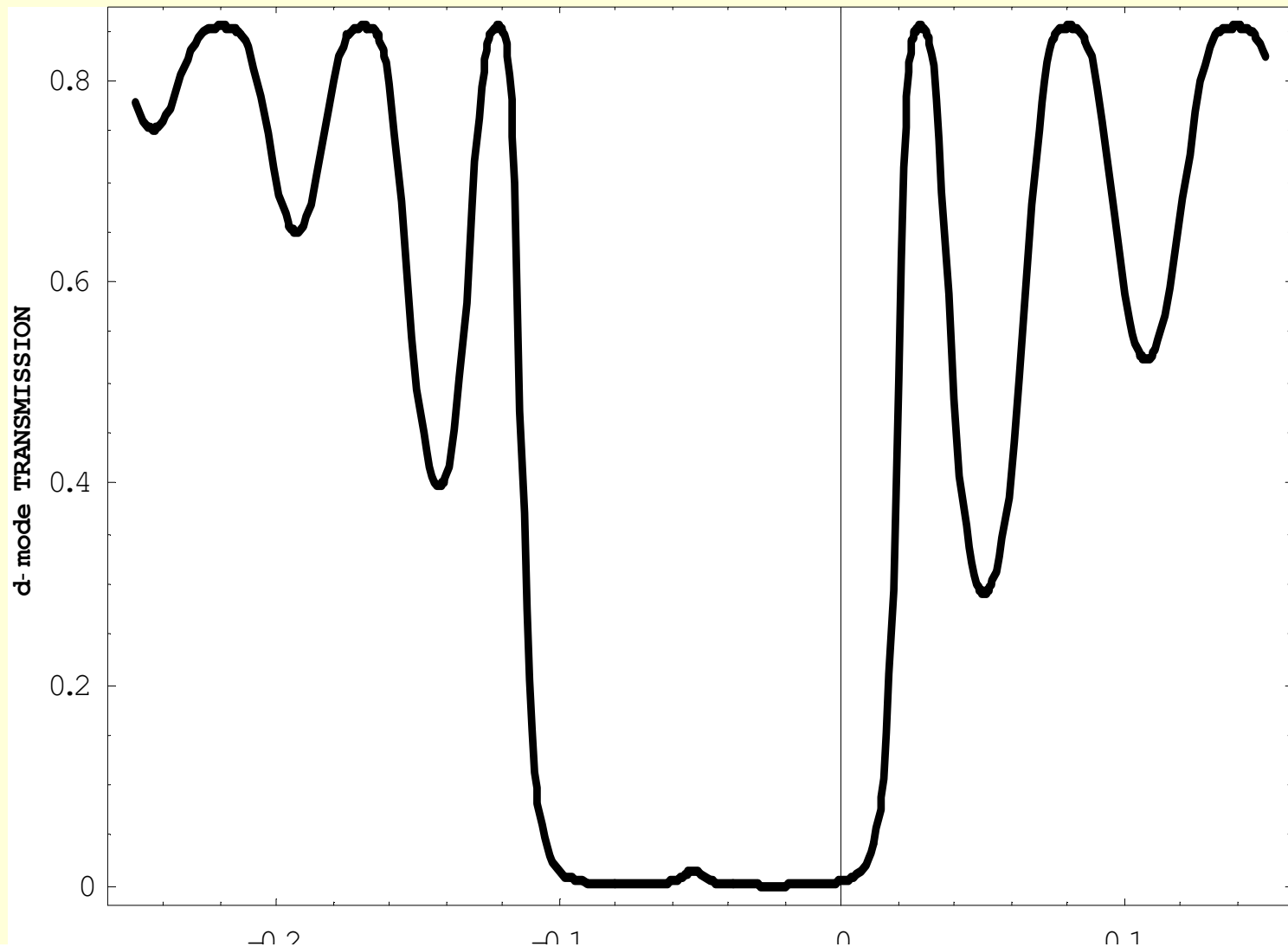


Fig.2g Transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency **for diffracting incident and exiting polarizations** at the birefringent phase shift at the defect layer thickness $\Delta\phi = \pi/2$

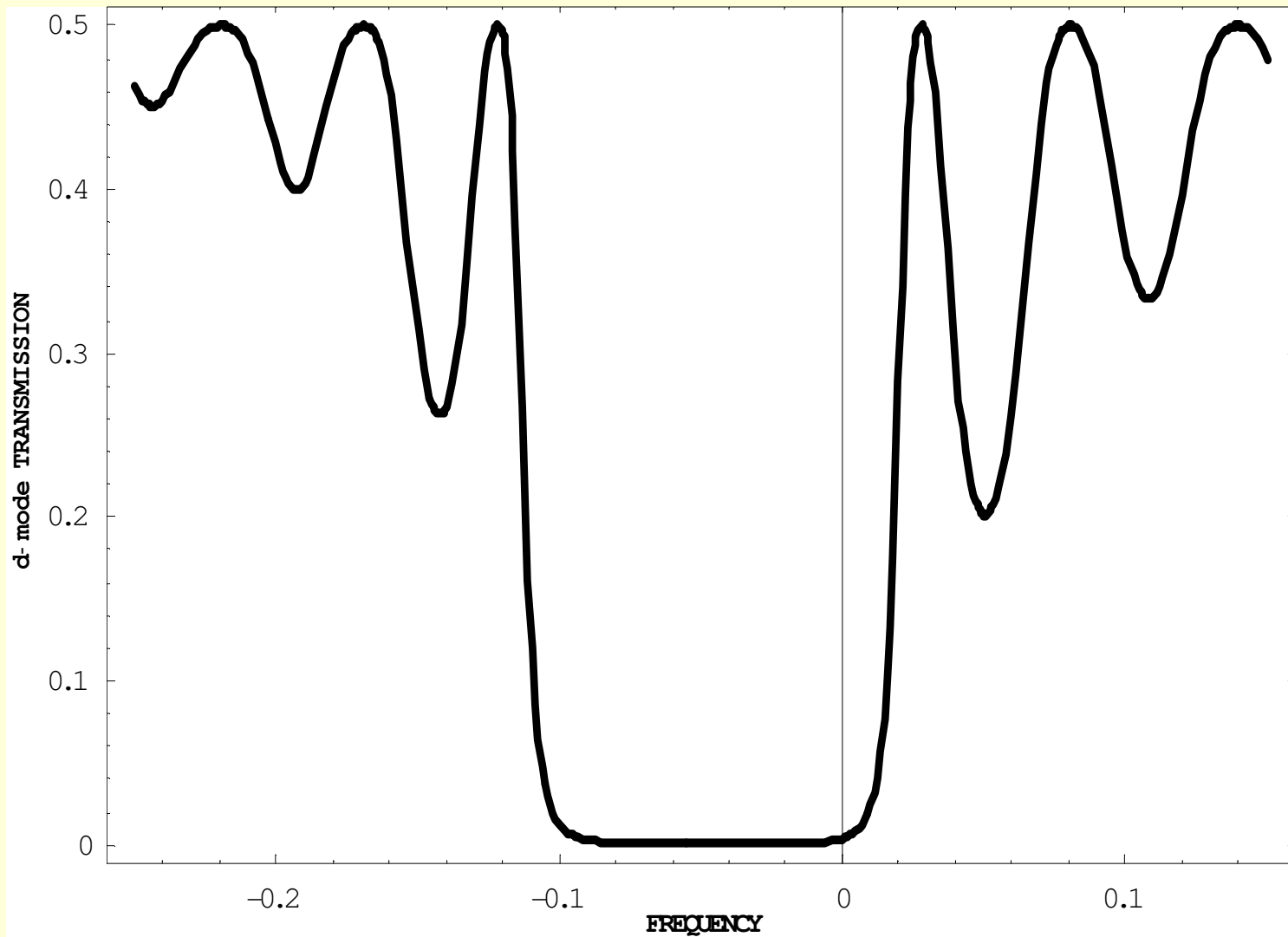
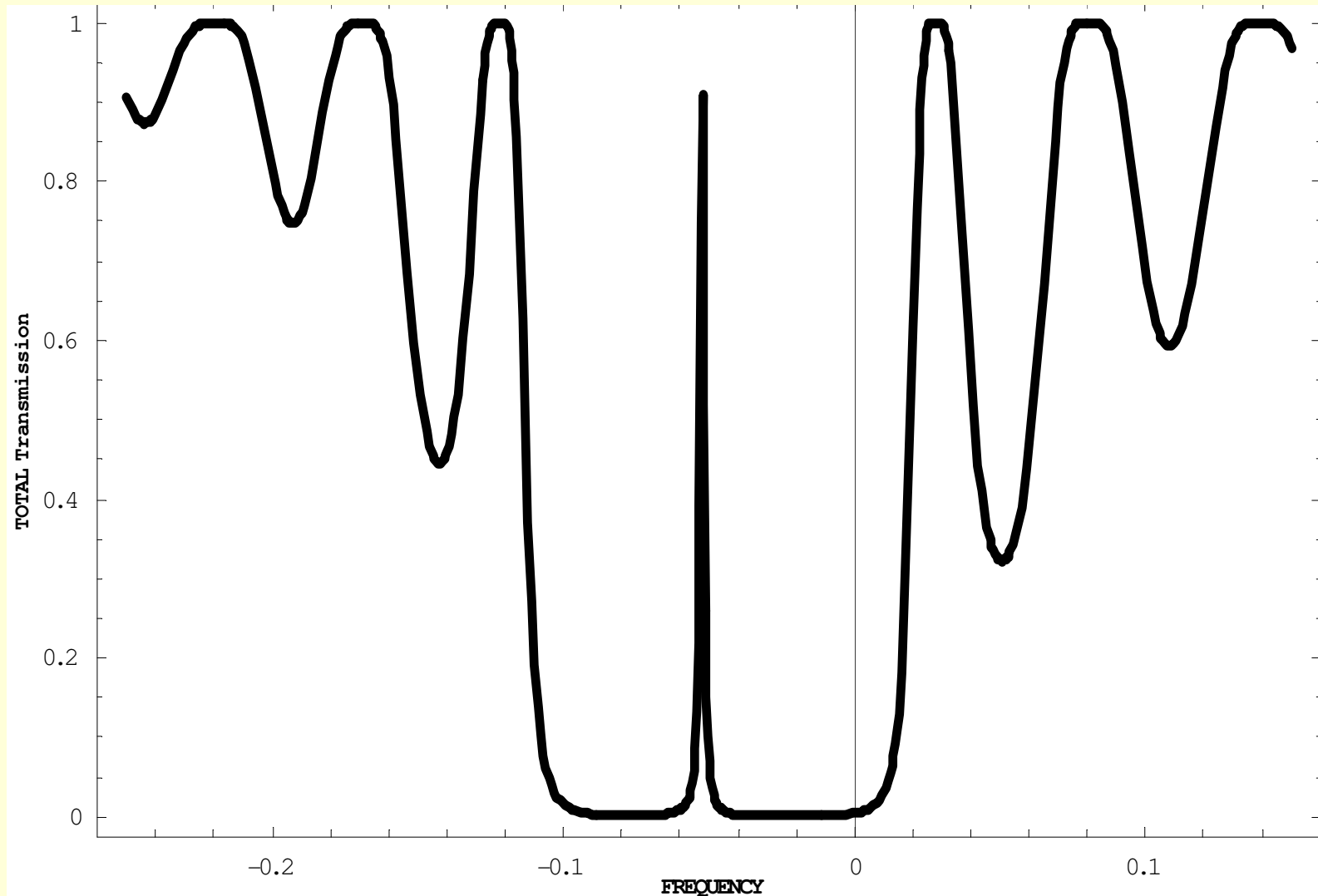
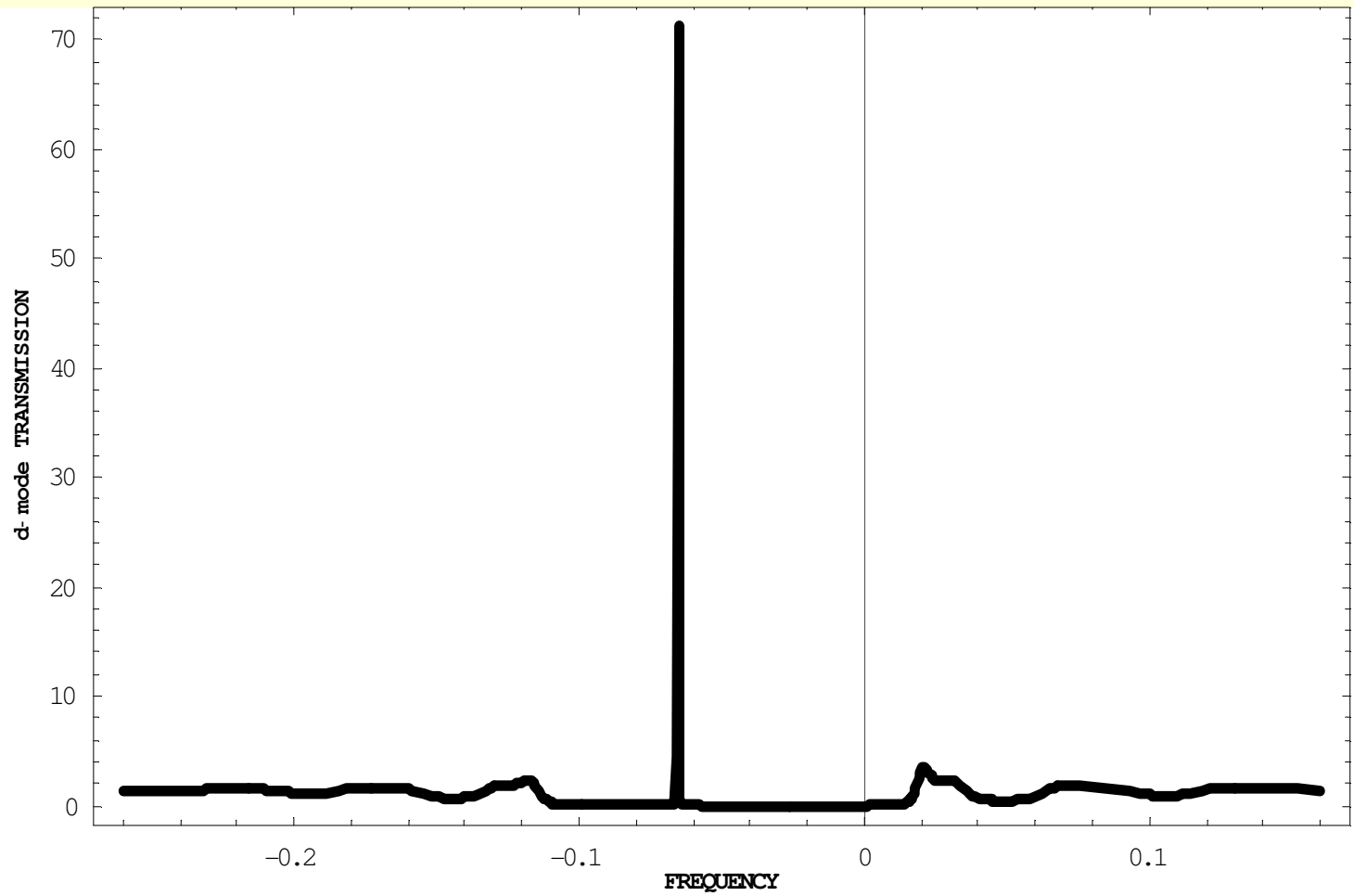


Fig.3a Calculated **total intensity transmission** coefficient versus the frequency **for diffracting incident** polarization at the birefringent phase shift at the defect layer thickness $\Delta\phi = \pi/20$ for a nonabsorbing CLC at $d/p=0.25$.



Transmission coefficient $|T(d,L)|^2$ for a low birefringent defect layer versus the frequency for diffracting incident and exiting polarizations at the birefringent phase shift at the defect layer thickness $\Delta\varphi = \pi/6$, $\gamma = -0.002355$ ($\delta=0.05$, $N=33$, $d/p=2.25$)



CONCLUSION

1. Presented approach allows to reveal clear physical picture of the localized modes
2. Predicted low lasing threshold under the conditions of anomalously strong absorption effect
3. An interconnection between the gain and other LC and defect layers parameters at the threshold pumping energy for lasing at the defect (as well at the stop band edge) mode frequency is revealed
4. Much to be done in the theory and experiment

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REFERENCES

1. Belyakov, V.A. and Dmitrienko, V.E. (1989). *Optics of Chiral Liquid Crystals*, in *Soviet Scientific reviews / Section A, Physics Reviews* (ed. I.M.Khalatnikov, Harwood Academic Publisher), v.13, p.1-203.
2. Belyakov, V.A. (1992). *Diffraction Optics of Complex Structured Periodic Media*, Springer Verlag, New York, Chapt. 4 .
3. V.A.Belyakov, *Ferroelectrics*, **344**, 163 (2006).
4. V.A.Belyakov, *MCLC*, **453**, 43 (2006).
5. V.A.Belyakov, *MCLC*, **494**, 127 (2008).
6. V.A.Belyakov, and S.V.Semenov, *MCLC*, **507**, 209 (2009).
7. V.A.Belyakov, and S.V.Semenov, *Zh.Eksp.Teor.Fiz.*, **136**, 797 (2009); (English translation *JETP*) **109**, 687 (2009).
8. V.A.Belyakov, and S.V.Semenov, *JETP*, **112**, 694 (2011).
9. V.A.Belyakov, *MCLC*, **559**, 39 (2012); *MCLC*, **559**, 50 (2012).
10. V.A.Belyakov, and S.V.Semenov, *JETP*, **118**, 798 (2014).
11. V.A.Belyakov, Localized Optical Modes in optics of Chiral Liquid Crystals in “New Developments in Liquid Crystals and Applications”, 2013 (Ed. P.K. Choudry, Nova Publishers, New York), Chp. 7, p.199.

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