APPLICATION OF MARKOV CHAIN AND ENTROPY FUNCTION FOR CYCLICITY ANALYSIS OF A LITHOSTRATIGRAPHIC SEQUENCE - A CASE HISTORY FROM THE KOLHAN BASIN, JHARKHAND, EASTERN INDIA

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SHIPRA SINHA

Department of Geology and Geophysics
Indian Institute of Technology,
Kharagpur, India

AGENDA

- Research Objective
- Study Area
- Methodology followed
- Results and Discussion
- Conclusion

OBJECTIVE

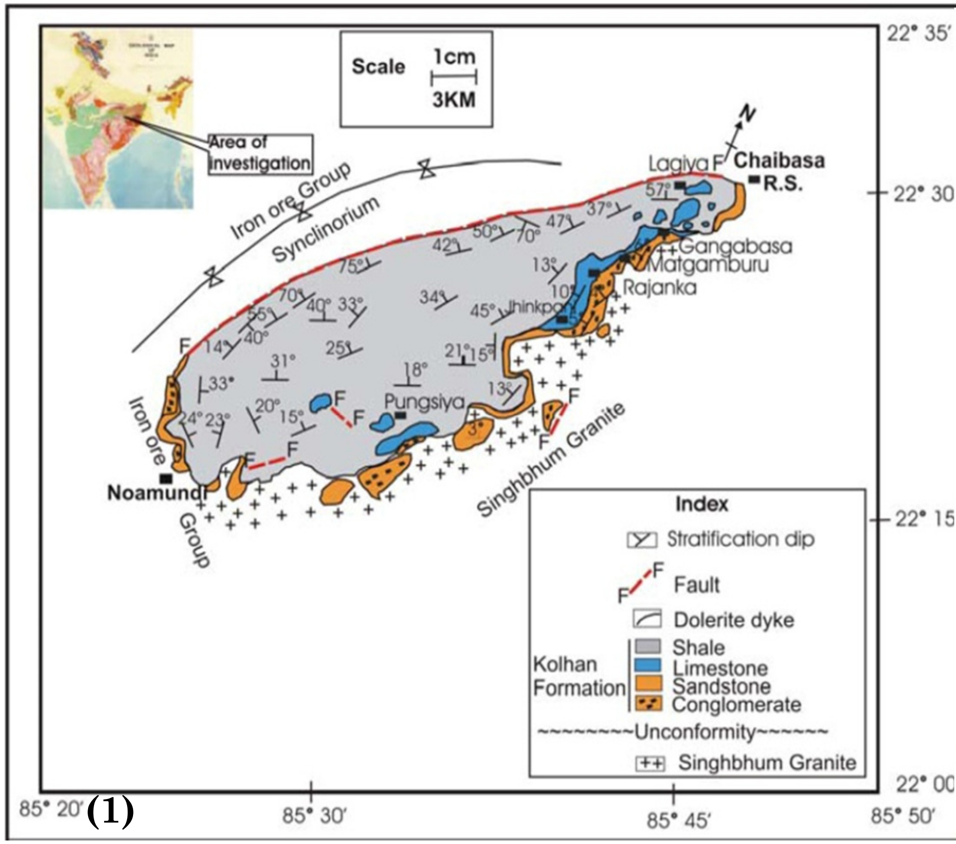
- To recognize various lithofacies present in the study area.
- To evaluate statistically cyclic character by Markov chain analysis;
- To compare the cyclicity, if present, in time and space;
- To evaluate the degree of ordering or energy regime of the facies deposition using entropy functions
- To recognize the broad depositional environment of the basin

STUDY AREA

- Length – 60 km
  Width- 10-1 2km
  Strike- NNW-SSE
  Dip- 5 to 10 degree

- Metasedimentry rocks lies unconformably over Singhbhum granite in the east and partly over folded and thrust faulted Iron-ore group to the west.

- Very gentle tectonic deformation and low grade of metamorphism



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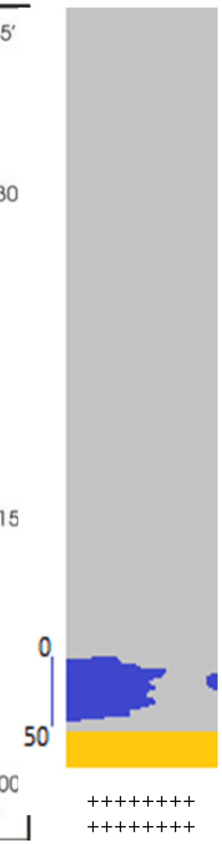


(2)

Shale

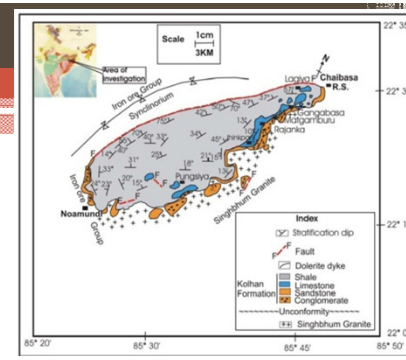
Limestone

sand

1. Geological map of Chaibasa- Noamundi basin (Chatterjee and Bhattacharya, 1969)
2. Stratigraphic succession of Kolhan Basin

INTRODUCTION

- Statistical techniques like Markov chain and entropy analyses can reveal the cyclic properties and degree of ordering of lithofacies explicitly. (Hota (1994))
- Due to absence of fossils assemblages, land vegetation and paucity of exposure, it is difficult to interpret depositional environment of Proterozoic Kolhan sequence. In the Chaibasa-noamundi basin it is observed that there is gross lithological asymmetricity present between various lithofacies.
- In the field it has been observed that there is the marked lithological asymmetricity present between the different sedimentary sequence
- Large difference in the sandstone and shale thickness: It is difficult to prove in the field, time independent depositional relational, if any, between these two lithofacies as there is absence of unconformity.
- No analytical work has been done so far in this area.
- The comples pattern in the lithogical succession are produced as a result of physical process and random events occurring simultaneously in an given depositional environment.
- To prove cyclic arrangement in the lithofacies in the study area, the Markov analysis and entropy analysis was done to test the presence of order in the sequence.



Geological map of Chaibasa- Noamundi basin (Chatterjee and Bhattacharya, 1969.

Stratigraphic succession

Reference: BANDOPADHYAY, P.C. and SENGUPTA, S. (2004)

METHODOLOGY

- Cyclicity in a sedimentary succession is defined as a series of lithologic units or lithofacies repeated through a succession in a cyclic or rhythmic pattern to some extent.

- Two types of observable cyclicity may be noteworthy: one in which there exist an order of sequence only; and another in which there is a certain order of repetition along the vertical scale of the sedimentary succession.

- In this study each "bed" provides a logical unit, therefore, examining cyclicity of a sequence is appropriate (Vistelius, 1965) [9], hence it safer to ignore thickness.

STRUCTURING DATA FOR MARKOV CHAIN

Identification of Lithofacies

- The lithofacies analysis based on the field descriptions, petrographic investigation, and their vertical packaging has been done for assessing the sediment depositional framework and the environment of deposition.

- Vertical Sequence Profile

  Seventeen lithological sections were considered for studying the vertical and areal distributions of the lithofacies within the Kolhan basin.

- The six lithofacies are

  A- Granular lag facies (GLA),
  B- Granular sandstone facies (GSD),
  C- Sheet sandstone facies (SSD),
  D- Plane laminated sandstone facies (PLSD),
  E- Rippled sandstone facies (RSD),
  F- Thin laminated sandstone facies (TLSD).

Structuring Data For Markov Chain

Lithofacies Description

GLA - Characterize by the massive, ungraded and fine matrix supported conglomerate, which is mostly mature to sub-mature (A).

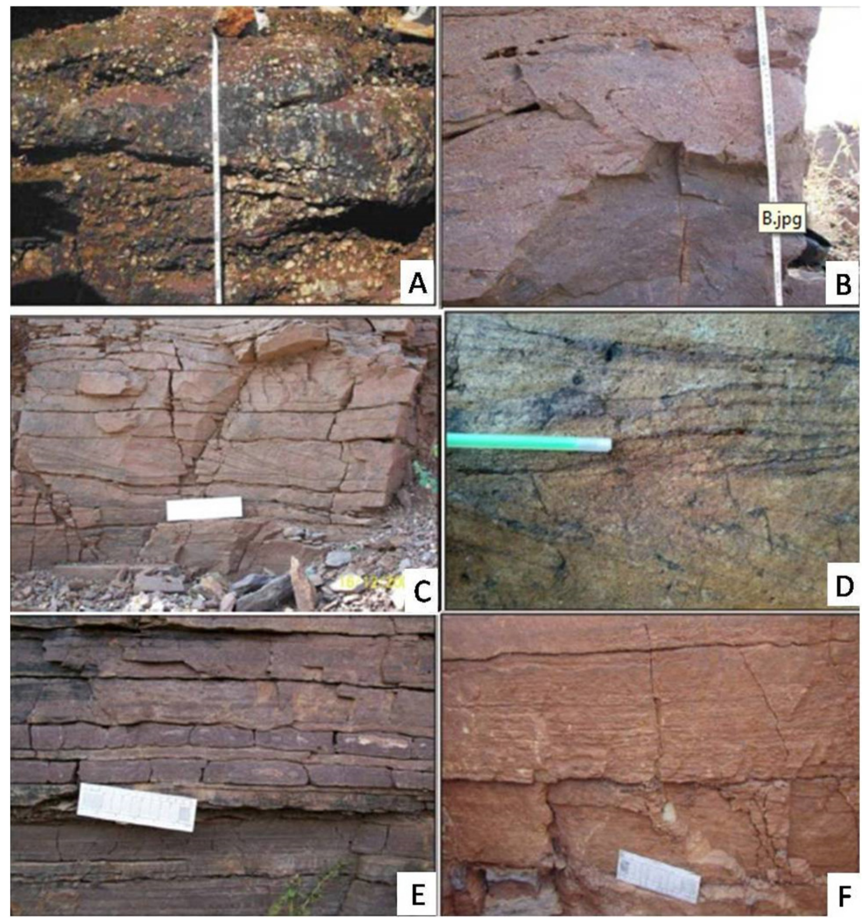
GSD – Characterized by moderately to well sorted, moderate clast/matrix ration. Planar cross stratification is common in these facies (B).

SSD- Defined by the sheets of sub-arkose to quartz arenite, sometimes intercalated with thin laminated siltstone (C).

PLSD- Defined by the amalgamated well sorted subarkose-quartz arenite, with a moderate high grain matrix ration (D).

RSD- Defined by the predominance of package of rippled sandstone with presence of symmetrical/ asymmetrical ripple (E).

TLSD- Defined by rhythmic alteration of sandstone and shale unit, sandy layer are thicker than shale. (F)



Structuring Data For Markov Chain

Transition Frequency Matrix, F:

- First order embedded chain is structured by counting transition from one facies to another, and the resulting frequency matrix will contain zeros along the principal diagonal.

- Transition are recorded where facies showing abrupt change in character.

	A	B	C	D	E	F	SRi	T-SRi
A	0	1	0	4	1	0	6	43
B	3	0	3	2	5	2	15	34
C	0	5	0	0	1	1	7	42
D	2	1	2	0	4	0	9	40
E	0	2	0	3	0	0	5	44
F	1	3	0	3	0	0	7	42
SCj	6	12	5	12	11	3	Total= 49	

Transition count matrix

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

MARKOV CHAIN METHOD

Upward transition probability matrix (P): $P_{ij} = F_{ij} / s_{ri}$

Downward Transition Probability Matrix (Q): $Q_{ji} = F_{ij} / s_{Cj}$

	A	B	C	D	E	F
A	0	0.166	0	0.666	0.166	0
B	0.2	0	0.2	0.133	0.333	0.133
C	0	0.714	0	0	0.142	0.142
D	0.222	0.111	0.222	0	0.444	0
E	0	0.4	0	0.6	0	0
F	0.142	0.428	0	0.428	0	0

Upward transition probability matrix

	A	B	C	D	E	F
A	0	0.5	0	0.333	0	0.166
B	0.0833	0	0.416	0.083	0.166	0.25
C	0	0.6	0	0.4	0	0
D	0.333	0.166	0	0	0.25	0.25
E	0.090	0.454	0.090	0.363	0	0
F	0	0.666	0.333	0	0	0

Downward transition probability matrix

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

MARKOV CHAIN METHOD

Independent random trail matrix: $R_{ij} = S_{Cj} / (S_T - S_{Ci})$

Difference Matrix (D): $D_{ij} = P_{ij} - R_{ij}$

Expected Frequency Matrix (E): $E_{ij} = R_{ij} * S_{Ri}$

	A	B	C	D	E	F
A	0	0.279	0.116	0.279	0.255	0.069
B	0.162	0	0.135	0.324	0.297	0.081
C	0.136	0.272	0	0.272	0.25	0.068
D	0.162	0.324	0.135	0	0.297	0.081
E	0.157	0.315	0.131	0.315	0	0.078
F	0.130	0.260	0.108	0.260	0.239	0

Independent random probability matrix

	A	B	C	D	E	F
A	0	-0.112	-0.116	0.387	-0.089	-0.069
B	0.037	0	0.064	-0.190	0.036	0.052
C	-0.136	0.441	0	-0.272	-0.107	0.074
D	0.060	-0.213	0.087	0	0.147	-0.081
E	-0.159	0.084	-0.131	0.284	0	-0.0789
F	0.012	0.167	-0.108	0.167	-0.239	0

Difference matrix

	A	B	C	D	E	F
A	0	1.674	0.697	1.674	1.534	0.418
B	2.647	0	2.205	4.994	4.852	1.323
C	1	2	0	2	1.833	0.5
D	1.35	2.7	1.125	0	2.475	0.675
E	0.681	1.363	0.568	1.363	0	0.340
F	1	2	0.833	2	1.833	0

Expected matrix

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

Entropy Analysis

Hattori (1976) applied the concept of entropy to sedimentary successions to determine the degree of random occurrence of lithologies in the succession.

Two types of entropies with respect to each lithological state;

1. One is post–depositional entropy ($E_i^{(post)}$) corresponding to matrix P.
2. Pre–depositional entropy ($E_i^{(pre)}$), corresponding to matrix Q.

$$Ei^{(post)} = -\sum_{j=0}^{n} Pij * \log(Pij) \qquad Ei^{(pre)} = -\sum_{j=0}^{n} Qij * \log(Qij)$$

CHI-SQUARE TEST

Test of Significance

$$\chi^2 = \sum_{i=0}^{n} \sum_{j=0}^{n} (\mathrm{Fij} - \mathrm{Eij})^{\wedge}2/\mathrm{Eij}$$

Test of Equation	Computed value of	Limiting Value at 0.5% significance leval	Degree of freedom
Billingslay	27.112	45.55	19

- Non-cyclic depositional pattern

DISCUSSION ON MARKOV CHAIN

	A	B	C	D	E	F
A	0	0.166	0	0.666	0.166	0
B	0.2	0	0.2	0.133	0.333	0.133
C	0	0.714	0	0	0.142	0.142
D	0.222	0.111	0.222	0	0.444	0
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Upward transition probability matrix

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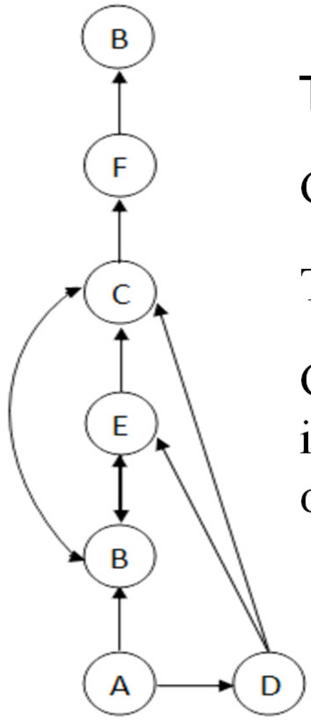
Difference matrix

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

Discussion On Markov Chain



The preferred upward transition path for the lithofacies is

GLA ➡ GSD ➡ SSD ➡ PLSD ⬌ RSD ➡ TLSD ➡ GSD

The transition between

GLA ➡ GSD, GSD ⬌ SSD, SSD ➡ PLSD and RSD ➡ TLSD is non-Markovian and the lineage is non-repetitive in nature. So cyclicity is absent or very weak.

Fig. Facies relationship diagrams showing upward transition of facies states of Chaibasa-Noamundi Basin, Kolhan group, Jharkhand.

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

DISCUSSION ON ENTROPY ANALYSIS

	E(Post)	E(Pre)	En(Post)	En (Pre)
A	0.822	0.959	0.353	0.413
B	2.232	2.054	0.961	0.885
C	1.148	0.971	0.494	0.418
D	1.836	1.959	0.791	0.846
E	0.971	1.159	0.418	0.499
F	1.448	0.918	0.624	0.395

Both E^{pre} and E^{post} are larger than 0.0 implies all six lithofacies (GLA, GSD, SSD, PLSD, RSD, TLSD) overlies and also is overlain by more than one state (Hattori, 1976).

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

DISCUSSION ON ENTROPY ANALYSIS

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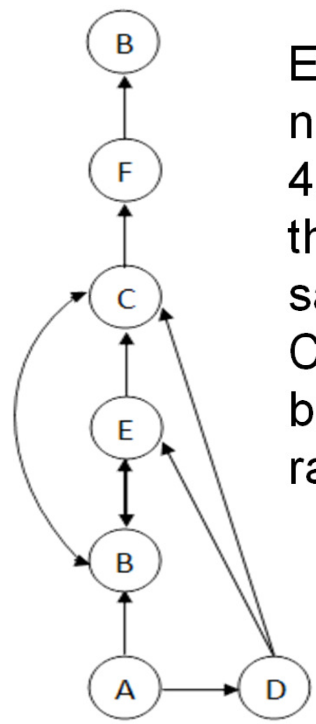

E^{pre} and E^{post} is larger in number for GSD (Table 4.2), and it is deduced that the influx of pebbly sandstone into the Chaibasa-Noamundi basin was the most random event.

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

Discussion On Entropy Analysis

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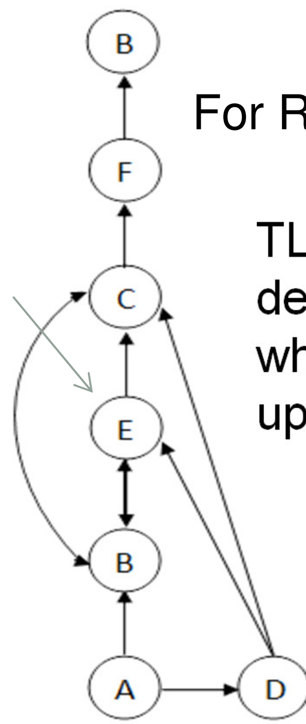
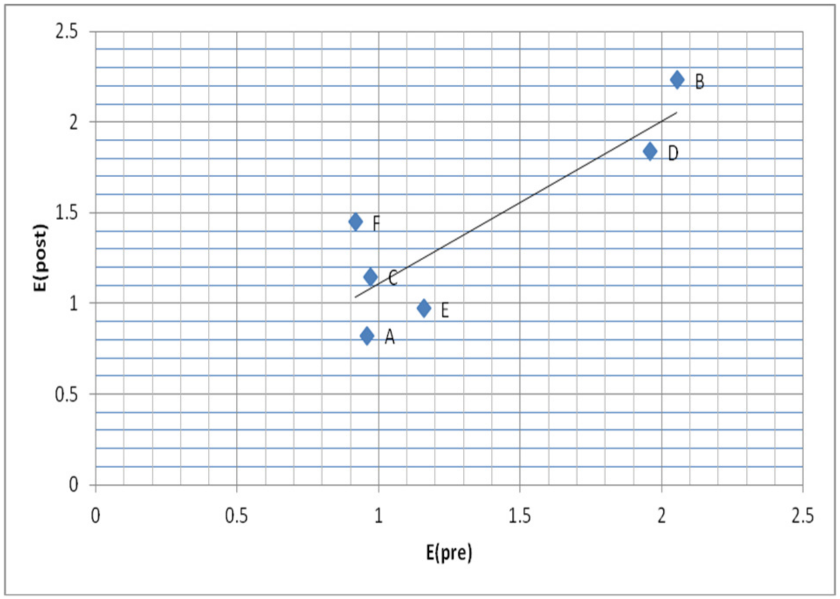


For RSD and PLSD, Epre > Epost.

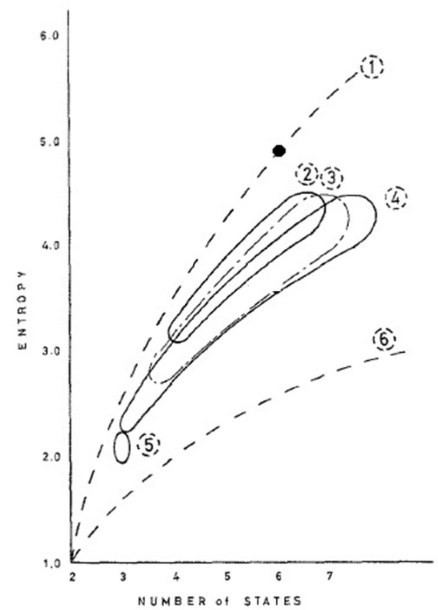
TLSD: indicates its strong dependence on its precursor which is visualized from the upward transition diagram

A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD

DISCUSSION ON ENTROPY ANALYSIS

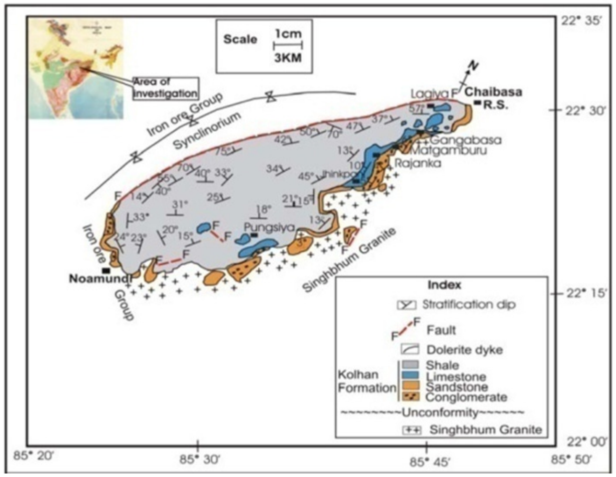


Entropy set derived from chaibasa-noamundi basin.
A-GLA; B-GSD; C-SSD; D-PLSD; E-RDS; F-TLSD



Relationship between entropy and depositional environmentof l ithlogical sequences (after Hattori, 1976). 1-maximum entropy; 2-etropies for coal measure succession; 3-entropies for fluvial-alluvial successions; &entropies for neritic successions; 5-entropies for flysch sediments; 6-minimum entropy; Black dot indicate entropy of basin under study

SEDIMENT FLOW MODEL



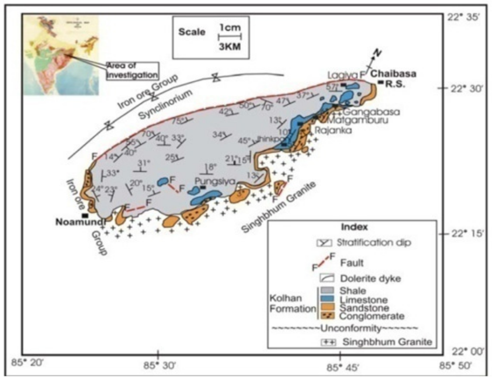
Fluctuating value of entropy suggests the changing Environment and asymmetric sequence is related to the sediment bypassing (change in energy regime).

Upward transition facies diagram of Markov Chain represent the fining upward non cyclic sequence.

Absence of marine features and facies association from field suggest that the sedimentary deposition are of fluvial deposit.

Variation in layer thickness is suggestive of deposition by unsteady flow in a fluvial regime within the channel

SEDIMENT FLOW MODEL



GLA and GSD represents high energy environment and product of rapid fluvial deltaic deposition.

Shallow depth in the basin is because the basin has risen against iron-ore thrust fault. This changes the slope of basin and hence energy of flow decreases.

Because of basin depth risen, depositional environment changes to low energy lacustrine and deposited shale away from the basin boundary.

CONCLUSION

- Chaibasa-Noamundi basin represents fining upward non-cyclic deposit.
- Environment of deposition changes from deltaic to lacustrine.

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Thank You!!