Part-I …

Comparative Study and Improvement in Shallow Water Model

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Date: 16. 09. 2014

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- Governing Equations and projection method
- Wetting and drying treatment
- Numerical Validation
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Introduction

- Free-surface water flows occur in many real life flow situations
- Many of these flows involve irregular flow domains with moving boundaries
- These types of flow behaviours can be modelled mathematically by Shallow-Water Equations (SWE)
- The unstructured finite-volume methods (UFVMs) not only ensure local mass conservation but also the best possible fitting of computing meshes into the studied domain boundaries
- The present work extends the unstructured finite volumes method for moving boundary problems

Governing Equations and projection method

Shallow Water Equations:

Continuity Equation

$$\frac{\partial Z_s}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \qquad (1)$$

Momentam Equations

$$\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{\partial (huv)}{\partial y} = f(hv) - gh\frac{\partial Z_s}{\partial x} + \frac{\partial}{\partial x}\left[A_H \frac{\partial (hu)}{\partial x}\right] + \frac{\partial}{\partial y}\left[A_H \frac{\partial (hu)}{\partial y}\right] - \frac{\tau_{bx}}{\rho_o} \qquad (2)$$

$$\frac{\partial (hv)}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2)}{\partial y} = -f(hu) - gh\frac{\partial Z_s}{\partial y} + \frac{\partial}{\partial x}\left[A_H \frac{\partial (hv)}{\partial x}\right] + \frac{\partial}{\partial y}\left[A_H \frac{\partial (hv)}{\partial y}\right] - \frac{\tau_{by}}{\rho_o} \qquad (3)$$

Projection Method:

Convection-diffusion step

$$\begin{aligned}\frac{\partial q_x^*}{\partial t}+\frac{\partial(uq_x^*)}{\partial x}+\frac{\partial(vq_x^*)}{\partial y}&=A_H\left[\frac{\partial^2 q_x^*}{\partial x^2}+\frac{\partial^2 q_x^*}{\partial y^2}\right]\\ \frac{\partial q_y^*}{\partial t}+\frac{\partial(uq_y^*)}{\partial x}+\frac{\partial(vq_y^*)}{\partial y}&=A_H\left[\frac{\partial^2 q_y^*}{\partial x^2}+\frac{\partial^2 q_y^*}{\partial y^2}\right]\end{aligned} \qquad (4)$$

Wave propagation step

$$\left[1-\frac{\gamma^2 gh}{A}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\delta Z_s=\frac{B}{A} \qquad where \qquad \delta Z_s=Z_s^{n+1}-Z_s^n \qquad (5)$$

$$\begin{cases} B=\gamma gh\left(\frac{\partial^2 Z_s^n}{\partial x^2}+\frac{\partial^2 Z_s^n}{\partial y^2}\right)-\frac{\gamma}{dt}\left(\frac{\partial q_x^*}{\partial x}+\frac{\partial q_y^*}{\partial y}\right)-\gamma\left(\frac{\partial L_x}{\partial x}+\frac{\partial L_y}{\partial y}\right)-\frac{1-\gamma}{dt}\left(\frac{\partial q_x^n}{\partial x}+\frac{\partial q_y^n}{\partial y}\right)\\ A=dt^{-1}\left(dt^{-1}+\gamma F\right),\quad F=\frac{g}{C_h^2 h^2}\sqrt{\left(q_x^n\right)^2+\left(q_y^n\right)^2},\quad L_x=\frac{\tau_{wx}}{\rho},\quad L_y=\frac{\tau_{wy}}{\rho}\end{cases} \qquad (6)$$

- **Velocity correction step**

$$q_x^{n+1} = \left(q_x^* - \gamma g h dt \frac{\partial Z_s^{n+1}}{\partial x} - (1-\gamma) g h dt \frac{\partial Z_s^n}{\partial x} + L_x dt - (1-\gamma) F q_x^n dt \right) (1+\gamma F dt)^{-1} \qquad (7)$$

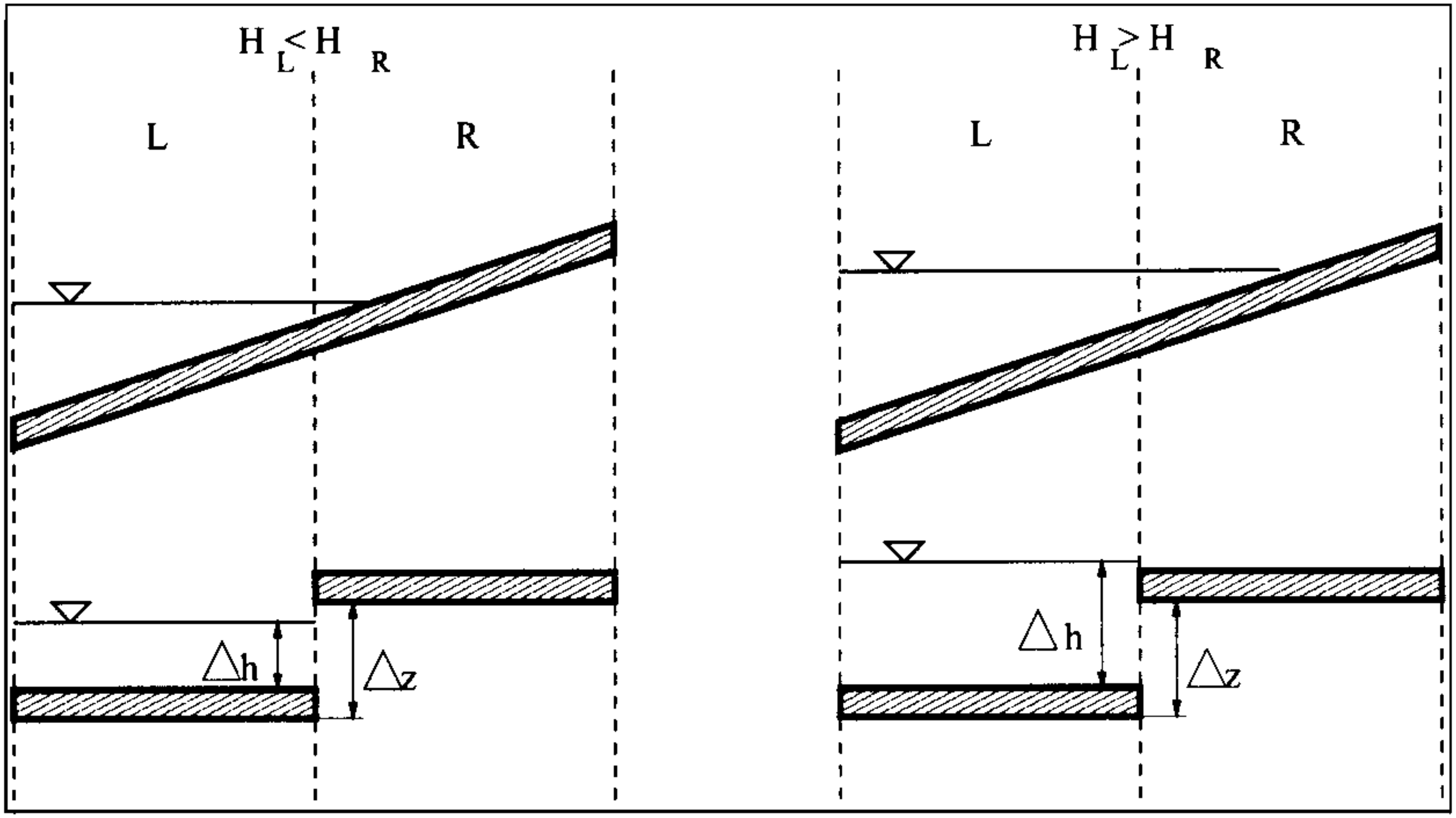
$$q_y^{n+1} = \left(q_y^* - \gamma g h dt \frac{\partial Z_s^{n+1}}{\partial y} - (1-\gamma) g h dt \frac{\partial Z_s^n}{\partial y} + L_y dt - (1-\gamma) F q_y^n dt \right) (1+\gamma F dt)^{-1} \qquad (8)$$

- **Equations (4)-(8) have been integrated by a technique based on Green's theorem and then discretised by an Unstructured Finite-Volume Method (UFVM).**

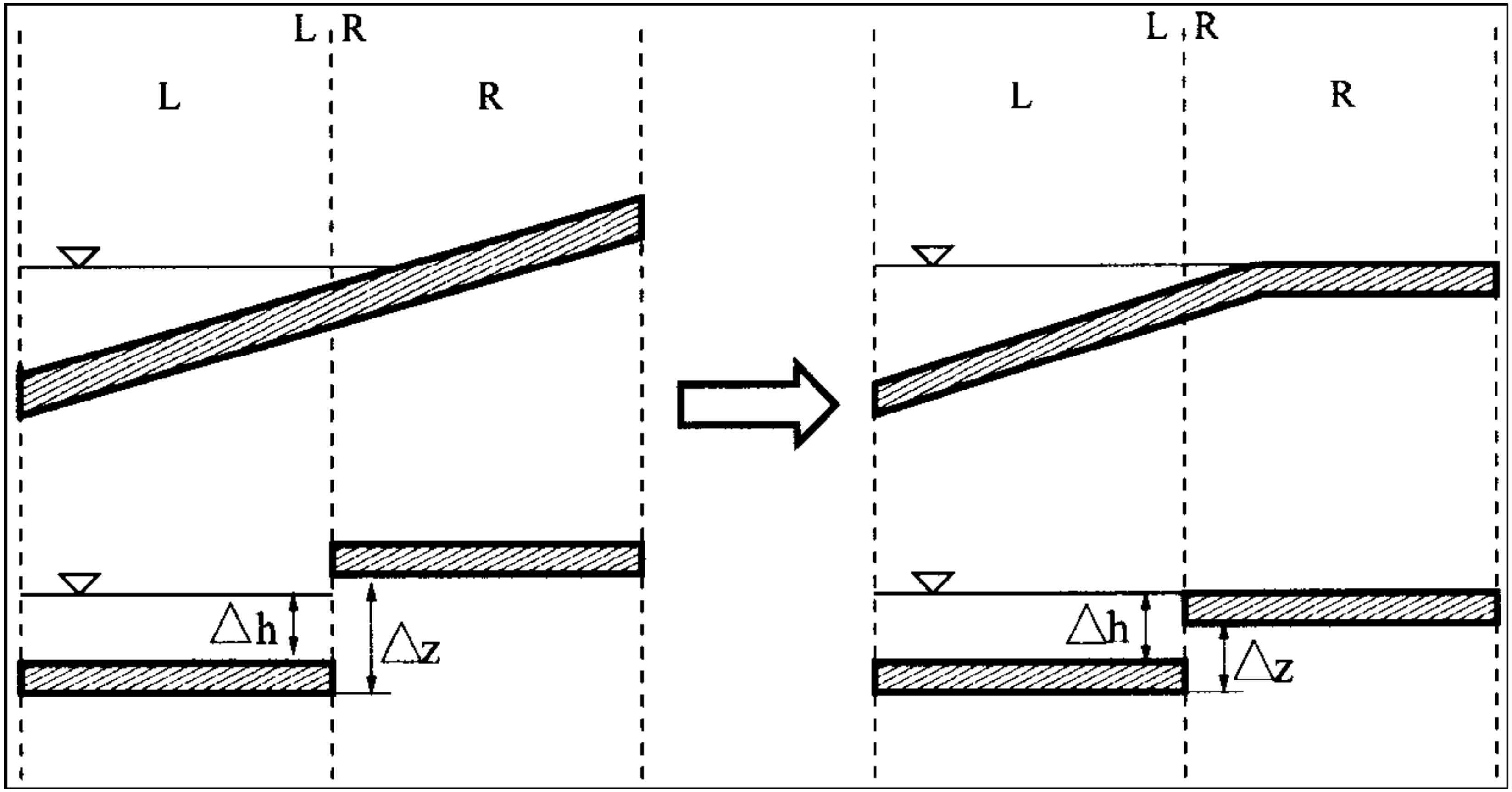
- **The convection terms are handled by a 2nd order *Upwind Least Square Scheme (ULSS)* along with the *Local Extremum Diminishing (LED) technique* to preserve the monotonicity of the scalar veriable**

- **The linear equation system issued from the wave propagation step is implicitly solved by a *Successive Over Relaxation (SOR) technique.***

Steady wetting/drying fronts over adverse steep slopes in real and discrete representations



Modification of the bed slope in steady wetting/drying fronts over adverse steep slopes in real and discrete representations



Wetting and drying treatment

- The main idea is to find out the partially drying or flooding cells in each time step and then add or subtract hypothetical fluid mass to fill the cell or to make the cell totally dry respectively, and then subtract or add the same amount of fluid mass to the neighbouring wet cells in the computational domain [*Brufau et. al. (2002)*].

- To consider a cell to be wet or dry in an particular time step, we use the threshold value $\varepsilon = O(10^{-3})$ as the minimum water depth (***h***)

- If $h \le \varepsilon,$ the cell will be considered as dry and the water depth for that cell set to be fixed as $h = \varepsilon$ for that time step

Conservative Property

Definition: **If a numerical scheme can produce the exact solution to the still water case:**

$$Z_s \equiv H, \qquad \vec{V} \equiv 0, \qquad (I)$$

then the scheme is said to satisfy the Conservative Property (*C-property*) [*Bermudez and Vázquez 1994*].

Proposition 1. **The present numerical scheme satisfies the *C-property*.**

Proof. **The details of the proof can be found in *Shi et at. 2013* (*Comp & Fluids*).**

Numerical Validation

Parabolic Bowl :

- To test the capacity of the present model in describing the wetting and drying transition
- The bed topography of the domain is defined by $\mathrm{b(x)} = \alpha r^2$, where α is a positive constant and $r^2 = x^2 + y^2$
- The water depth $h(r,t)$ is non-zero for $r < \sqrt{(X + Y\cos\omega t) / \alpha(X^2 - Y^2)}$
- The analytical solution is periodic in time with a period $\tau = 2\pi / \omega$
- The analytical solution is given within the range as

$$h(r,t) = \frac{1}{X + Y\cos\omega t} + \alpha\left(Y^2 - X^2\right)\frac{r^2}{\left(X + Y\cos\omega t\right)^2},$$

$$(u,v)(\vec{x},t) = -\frac{Y\omega\sin\omega t}{X + Y\cos\omega t}\left(x/2 , y/2 \right).$$

Numerical Validation …

Parabolic Bowl …

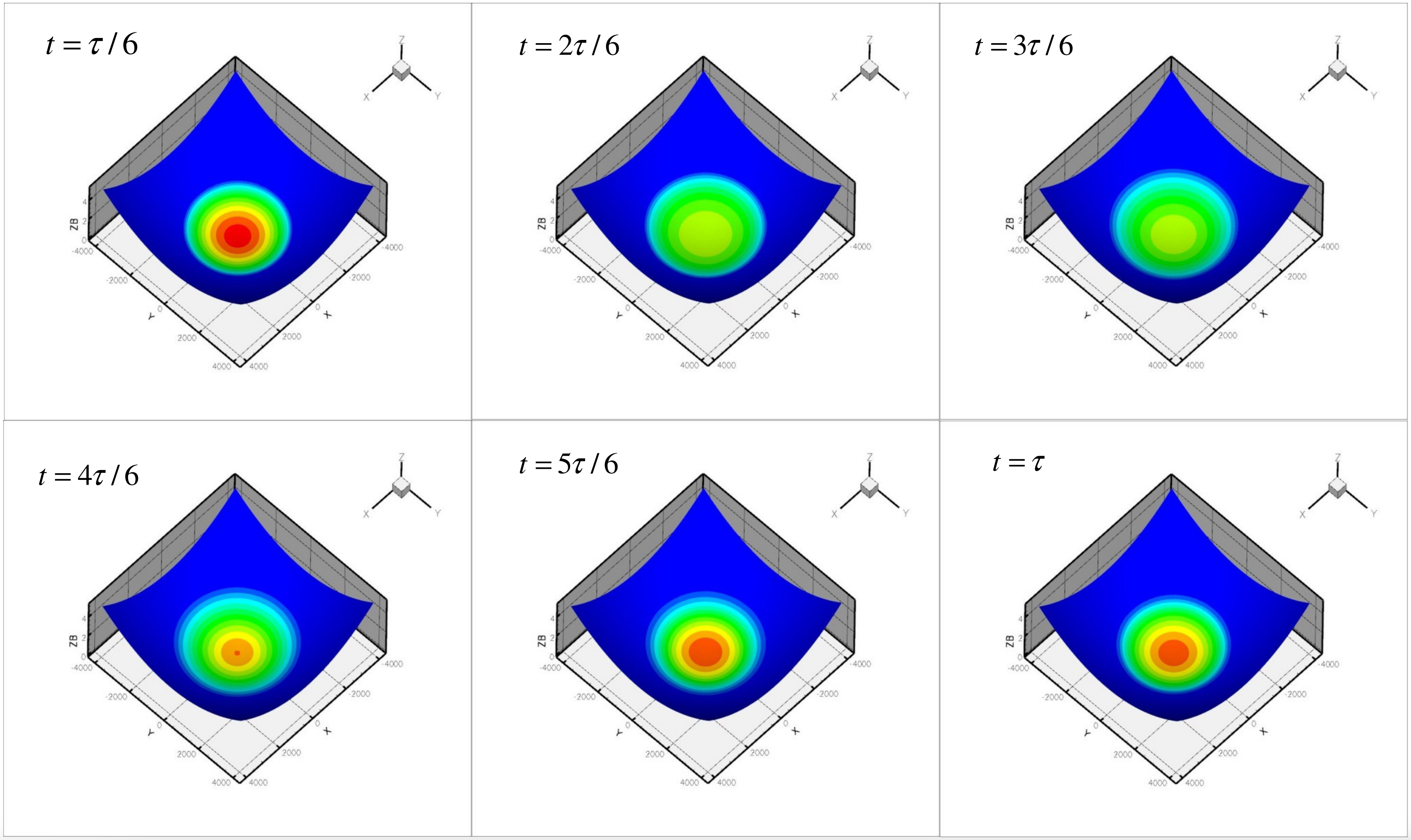
- For computation purpose, α , X and Y are fixed as $1.6\times10^{-7} m^{-1}$, $1m^{-1}$ and $-0.41884m^{-1}$ respectively

- The computational domain (Ω) is considered as a square region $[-4000,4000]\times[-4000,4000]m^2$ with the origin at the domain centre

- The threshold value ε is set as 3×10^{-3}

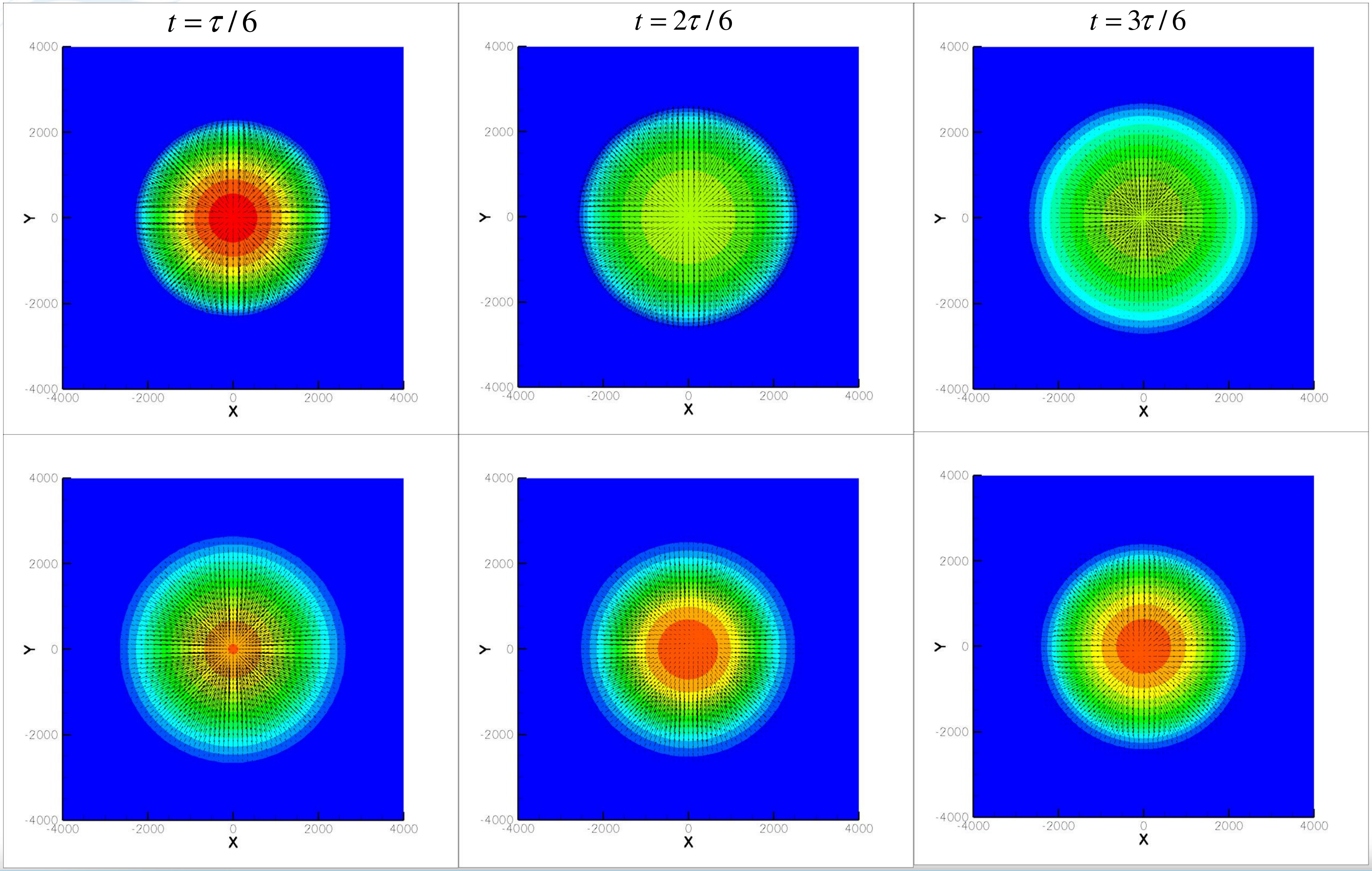
Numerical Validation …

Parabolic Bowl …



Numerical Validation …

Parabolic Bowl …



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