FAST LOW THRUST TRAJECTORIES FOR THE EXPLORATION OF THE SOLAR SYSTEM

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Introduction

- The optimization of low-thrust trajectories is a well know subject, and is performed by using numerical procedures.
- The two main approaches for optimizing a trajectory, direct and indirect methods, have been dealt with in many studies
- The indirect approach, here followed, leads to a boundary value problem in which the trajectory and the thrust profile are obtained by integrating a set of ordinary differential equations (12 first order differential equations in the general tri-dimensional case)

Introduction

- The solution of this problem requires the generation of a starting solution which is close enough to the optimized solution to allow the numerical procedure to converge toward the optimized solution
- The optimization of the trajectory and of the thrust profile is strictly linked with the optimization of the spacecraft. In 2002 an interesting optimization approach was developed to separate the optimization of the spacecraft from that of the thrust program
- The two approaches are linked by a single parameter, the specific mass of the power generator α.

Introduction

- The power generator is assumed to works always at full power, with a constant specific mass, and the thrust is regulated by suitably changing the specific impulse of the thruster I_s
- This can be done only in an approximated way, since the specific mass is defined with respect to the power of the propellant jet and the efficiency of the thruster varies with the specific impulse
- In the case solar electric propulsion the power decreases with distance from the Sun
- α is here assumed to be a known function of the distance from the Sun:

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$$\alpha = \alpha_0 \left(\frac{|\mathbf{r}|}{r_E} \right)^2$$

Equation of motion and optimization

$$\ddot{\mathbf{r}} + \nabla U = \mathbf{a}$$

α constant (NEP)

$$\frac{m_p}{m_i} = \sqrt{\frac{1}{2}\int_{T_1}^{T_2} \alpha \left|\mathbf{a}\right|^2 dt}$$

α non constant (SEP)

$$\frac{m_p}{m_i} = \sqrt{\frac{1}{2}\int_{T_1}^{T_2} \alpha_0 \left|\mathbf{a}\right|^2 \left(\frac{\left|\mathbf{r}\right|}{r_E}\right)^2 dt}$$

cost function

$$J = \frac{1}{2}\int_{T_1}^{T_2} \left|\mathbf{a}\right|^2 \left(\frac{\left|\mathbf{r}\right|}{r_E}\right)^2 dt = \frac{1}{2}\int_{T_1}^{T_2} \left|\mathbf{q}\right|^2 dt$$

$$\frac{m_p}{m_i} = \gamma = \sqrt{\alpha_0 J}$$

Final equations

$$\begin{cases} \ddot{\mathbf{r}} + \nabla U = \mathbf{q}\left(\dfrac{r_E}{|\mathbf{r}|}\right) \\ \ddot{\mathbf{q}} + \mathbf{q}\nabla\left(\nabla U\right) = 0 \end{cases}$$

2-D case
Circular, co-planar orbits

8 first order ODEs

$$\begin{cases} \dot{v}_x = -\dfrac{\mu x}{\sqrt{\left(x^2+y^2\right)^3}} + \dfrac{q_x r_E}{\sqrt{x^2+y^2}} \\ \dot{v}_y = -\dfrac{\mu y}{\sqrt{\left(x^2+y^2\right)^3}} + \dfrac{q_y r_E}{\sqrt{x^2+y^2}} \\ \dot{v}_{qx} = -\mu \dfrac{q_x\left(-2x^2+y^2\right) - 3q_y xy}{\sqrt{\left(x^2+y^2\right)^5}} \\ \dot{v}_{qy} = -\mu \dfrac{q_y\left(x^2-2y^2\right) - 3q_x xy}{\sqrt{\left(x^2+y^2\right)^5}} \end{cases}$$

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{q}_x = v_{qx} \\ \dot{q}_y = v_{qy} \end{cases}$$

950 days Earth- Saturn trajectory

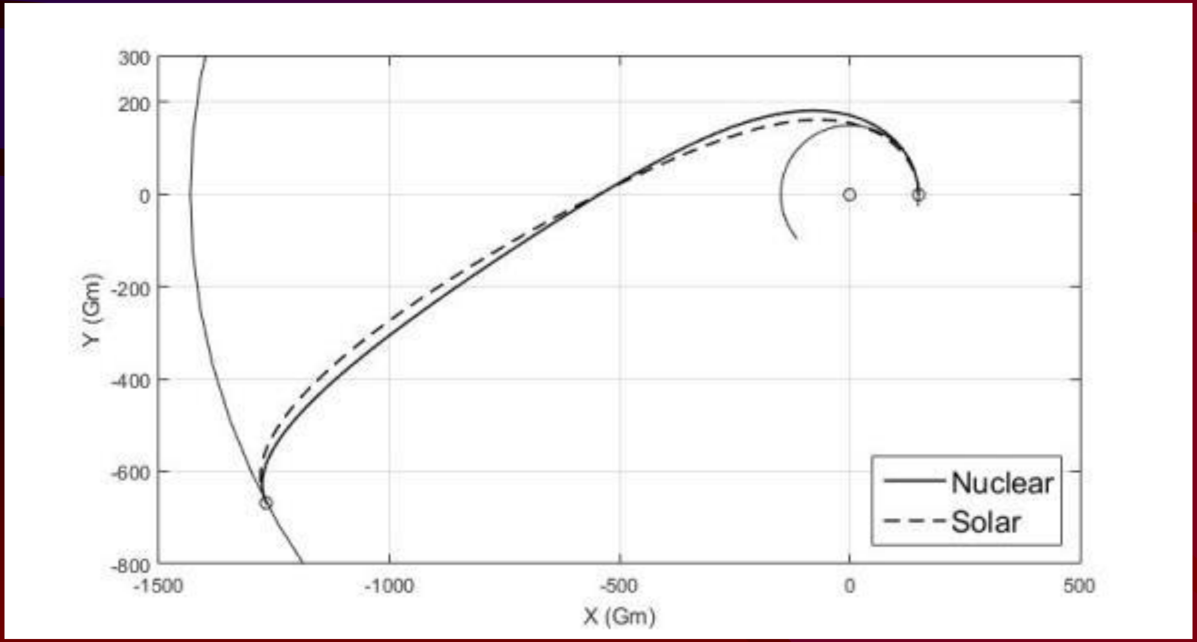
NEP: J = 13.82 m^2/s^3

SEP: J = 45.45 m^2/s^3

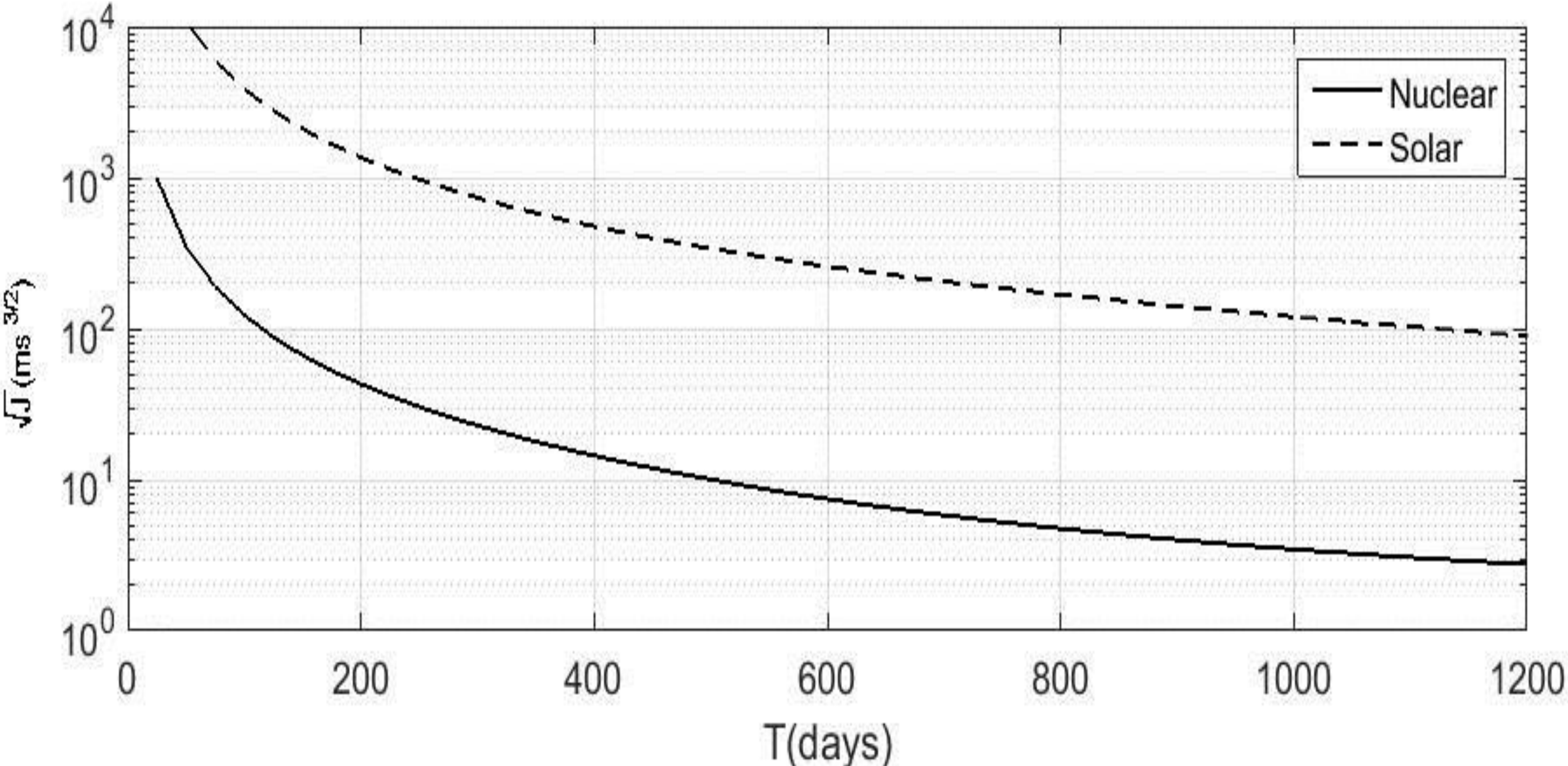
Maximum value of α_0: SEP: 72.4 kg/kW

NEP: 22 kg/kW

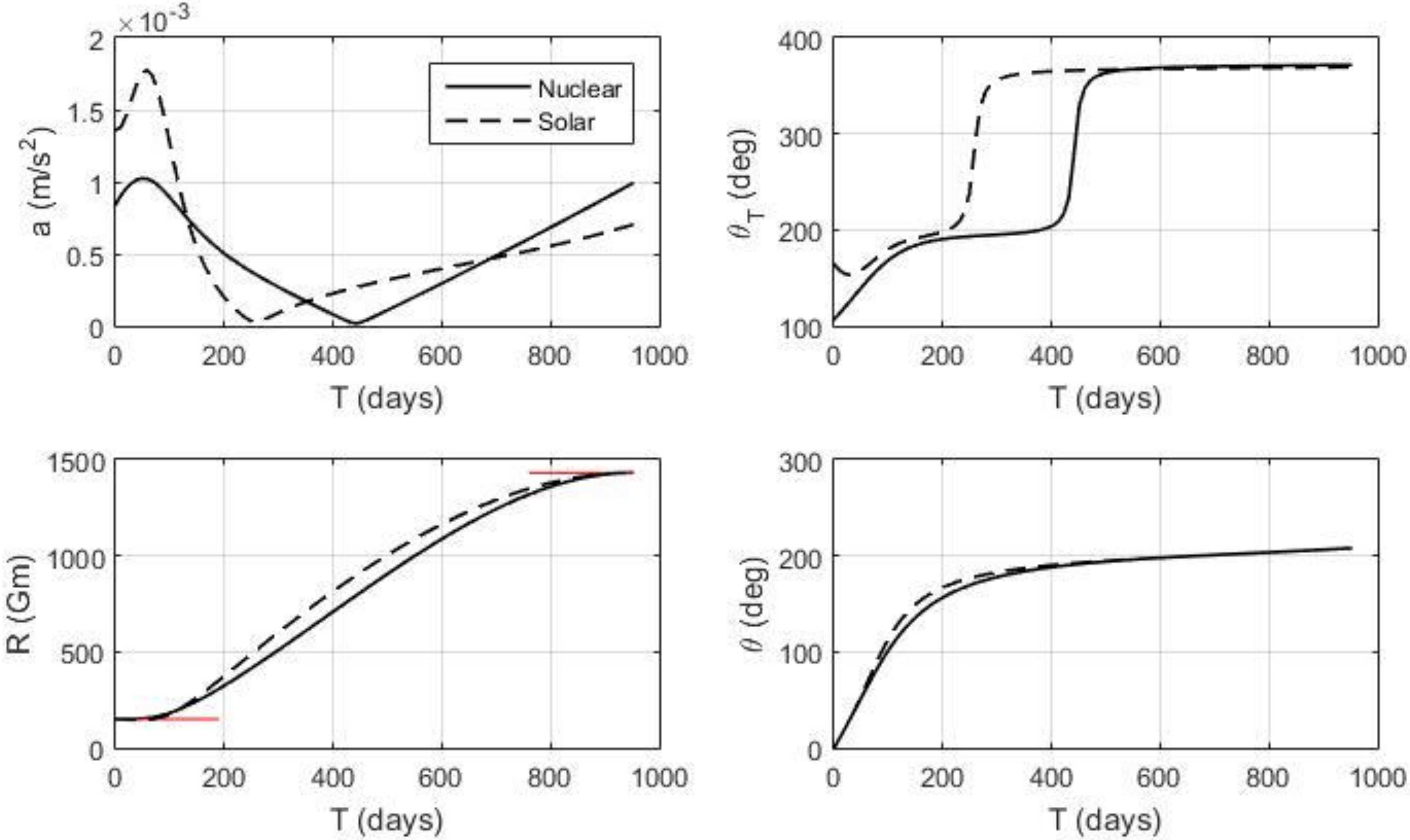
However, much lower value must be used



J as a function of *T*



950 days Earth- Saturn trajectory



Complete travel from orbit to orbit

The total mission is thus made of 3 parts, spiral trajectories about the start and destination planet and the interplanetary cruise

$$\frac{m_p}{m_i} = \gamma_{tot} = \sqrt{\alpha_0 J_{tot}} = \sqrt{\alpha_0 \left(J_1 + J_2 + J_3\right)}$$

To optimize the whole journey, a the cost function J_{tot} must be minimized.

To optimize the whole trajectory the value of

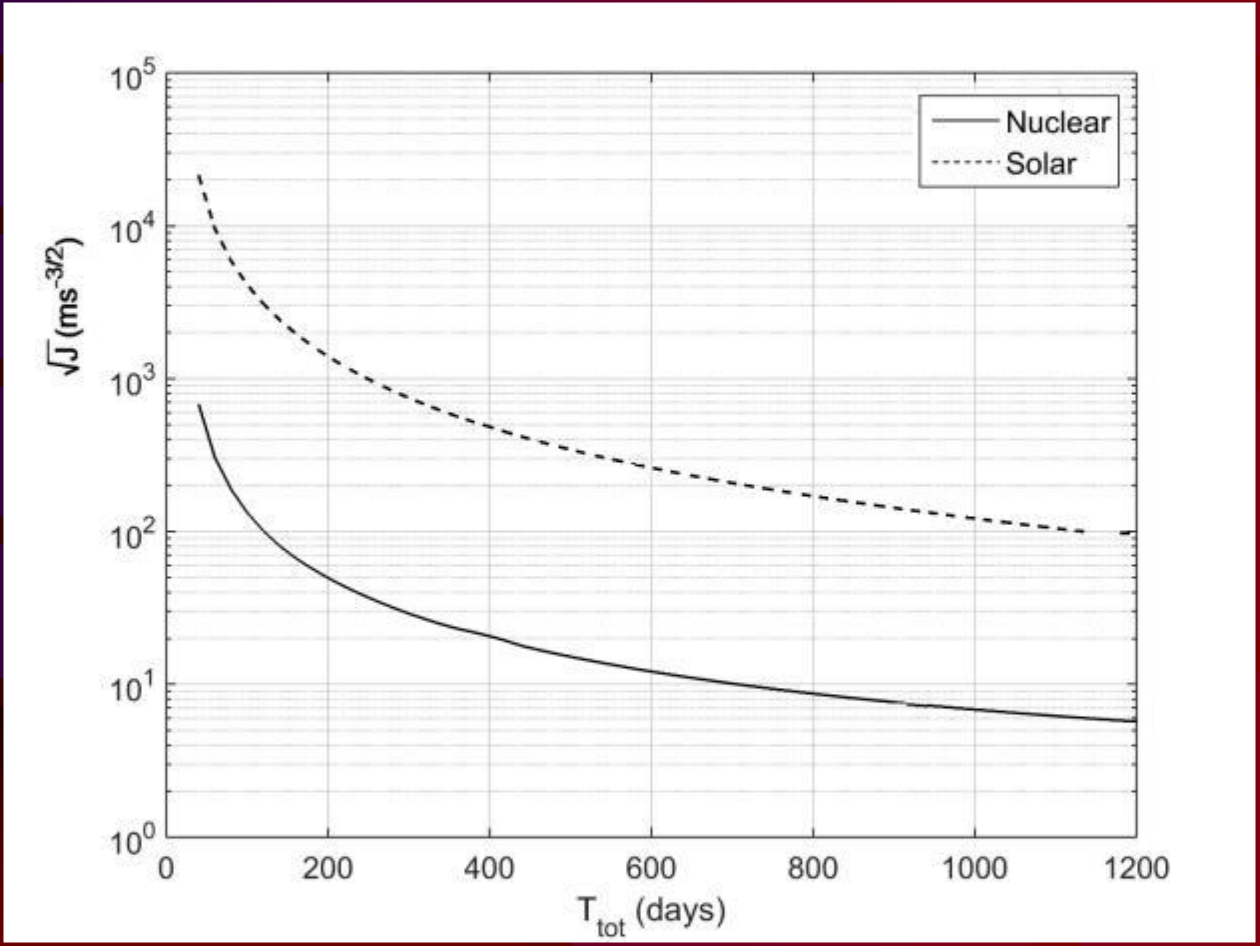
$$J_{tot} = J_1 + J_2 + J_3$$

must be minimized.

$$\frac{m_p}{m_i} = \gamma = \sqrt{\alpha_0 J_{tot}}, \qquad \frac{m_w}{m_i} = \gamma(1-\gamma), \qquad \frac{m_l + m_s}{m_i} = (1-\gamma)^2$$

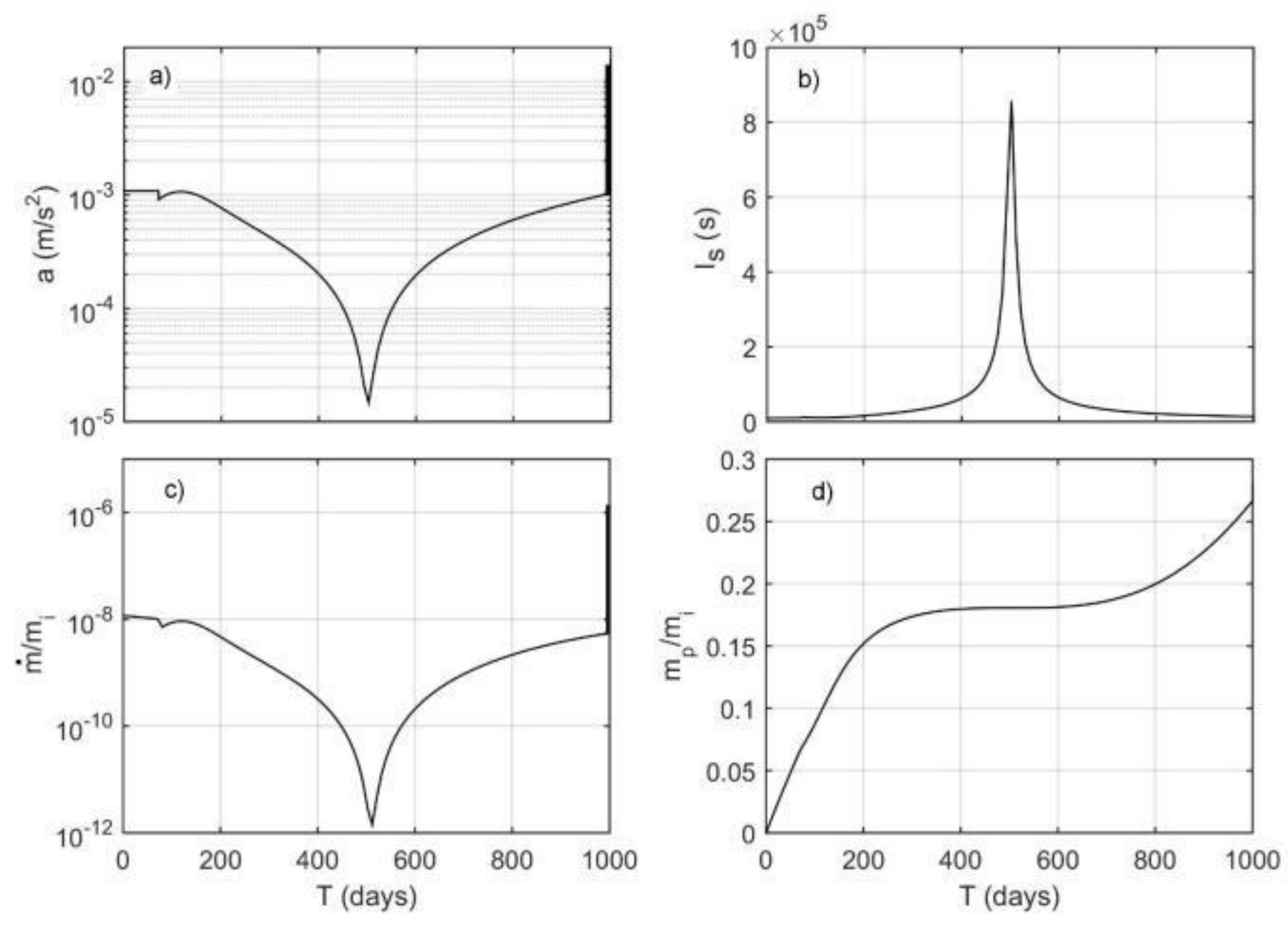
Complete travel from orbit to orbit

Complete Earth (600 km orbit) – Saturn (Titan orbit) mission



Complete travel from orbit to orbit

Complete 1000 days (2 years 9 months) Earth – Saturn mission



The space elevator

A space elevator is a gateway to the solar system

- The reduction of the cost allows
  - to accept larger IMLEO for exploration missions
  - to make it easier the development of innovative propulsion device in high orbits
- Any outbound spacecraft has already a much higher energy at its start and inbound spacecraft needs much less braking
- The release velocity may be higher than escape velocity, and the interplanetary trajectory is less expensive

The aim of this part is defining optimal low thrust interplanetary trajectories starting from a space elevator

Launching from the upper platform

- The anchor must be above GEO (radius 42,164 km)
- At larger orbits the vehicle is already injected in the interplanetary trajectory: at a radius of 63,378 km the spacecraft is on its trajectory to Mars (except for the orbital plane)
- If the spacecraft is released above 53,127 km its speed is above the escape velocity, and it has an hyperbolic excess velocity

$$V_{\infty} = \sqrt{V_t^2 - V_e^2}$$

Launching from the upper platform

- The direction of the hyperbolic excess velocity can be determined by accurately timing the instant in which the spacecraft leaves the elevator's anchor.
- Assuming that the Earth orbit is circular, the initial velocity of the interplanetary trajectory is

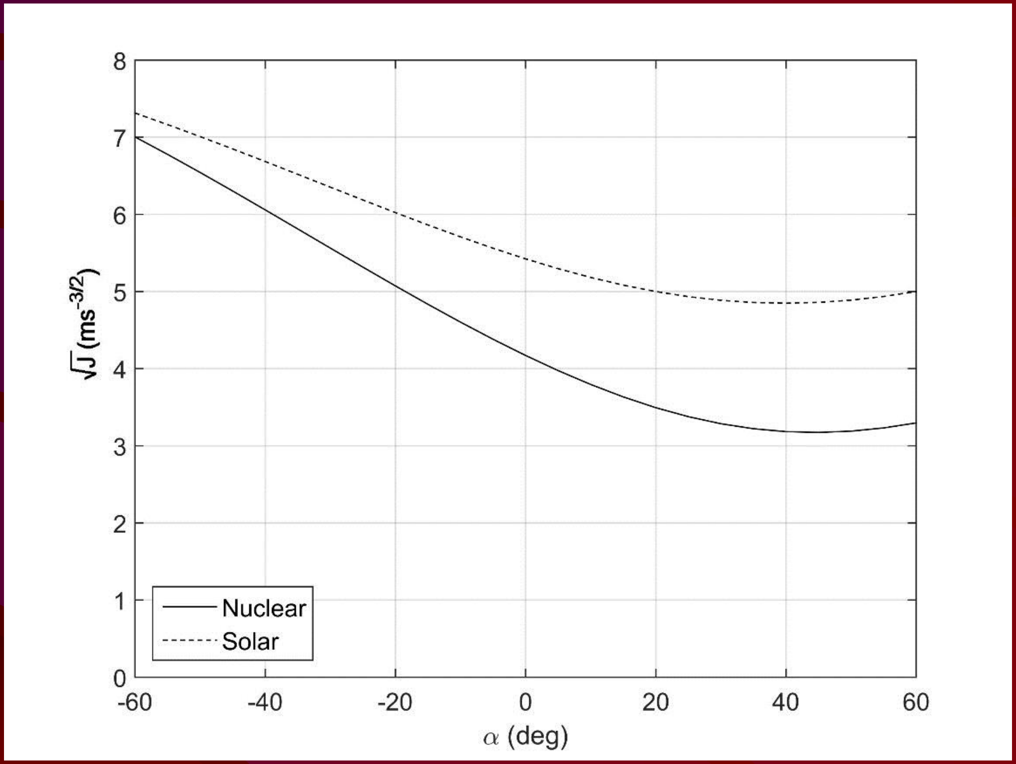
$$V_i = \begin{Bmatrix} V_{\infty\perp} \\ V_p + V_{\infty\parallel} \end{Bmatrix} = \begin{Bmatrix} V_\infty \sin(\ ) \\ V_p + V_\infty \cos(\ ) \end{Bmatrix}$$

Earth – Mars trajectory

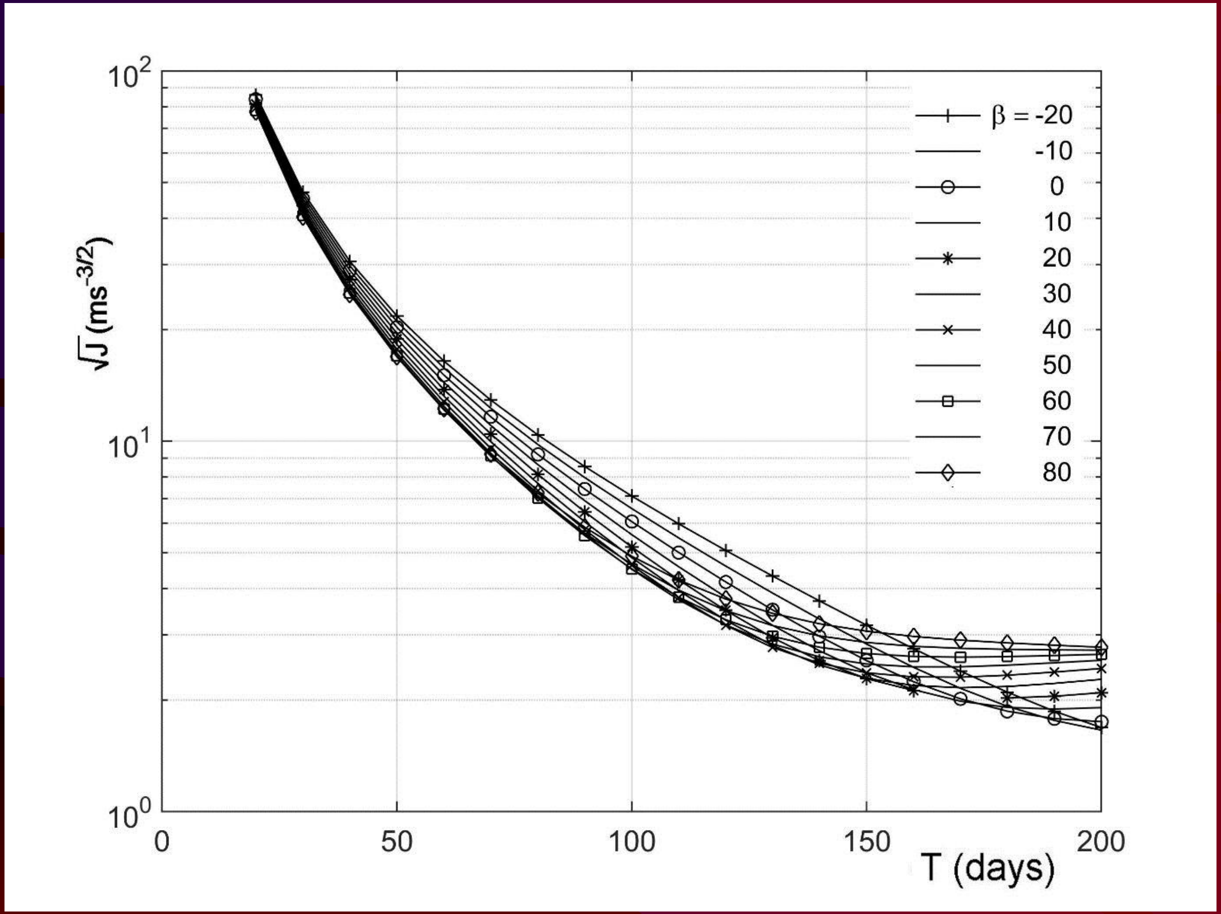
- Assumed that the radius of the anchor is 100,000 km
- A quick interplanetary trajectory is obtained, but that a larger quantity of propellant is needed for the maneuver at Mars.
- Consider for instance

a 120 days Earth-Mars trajectory and search for an optimal value of β

J is the cost function



Earth – Mars trajectories



Earth – Mars trajectory

Without a space elevator, the optimized value of J is

NEP: $J = 27.92\ \mathrm{m^2/s^3}$ ($\sqrt{J} = 5.284\ \mathrm{m/s^{3/2}}$)

SEP: $J = 40.48\ \mathrm{m^2/s^3}$ ($\sqrt{J} = 6.362\ \mathrm{m/s^{3/2}}$)

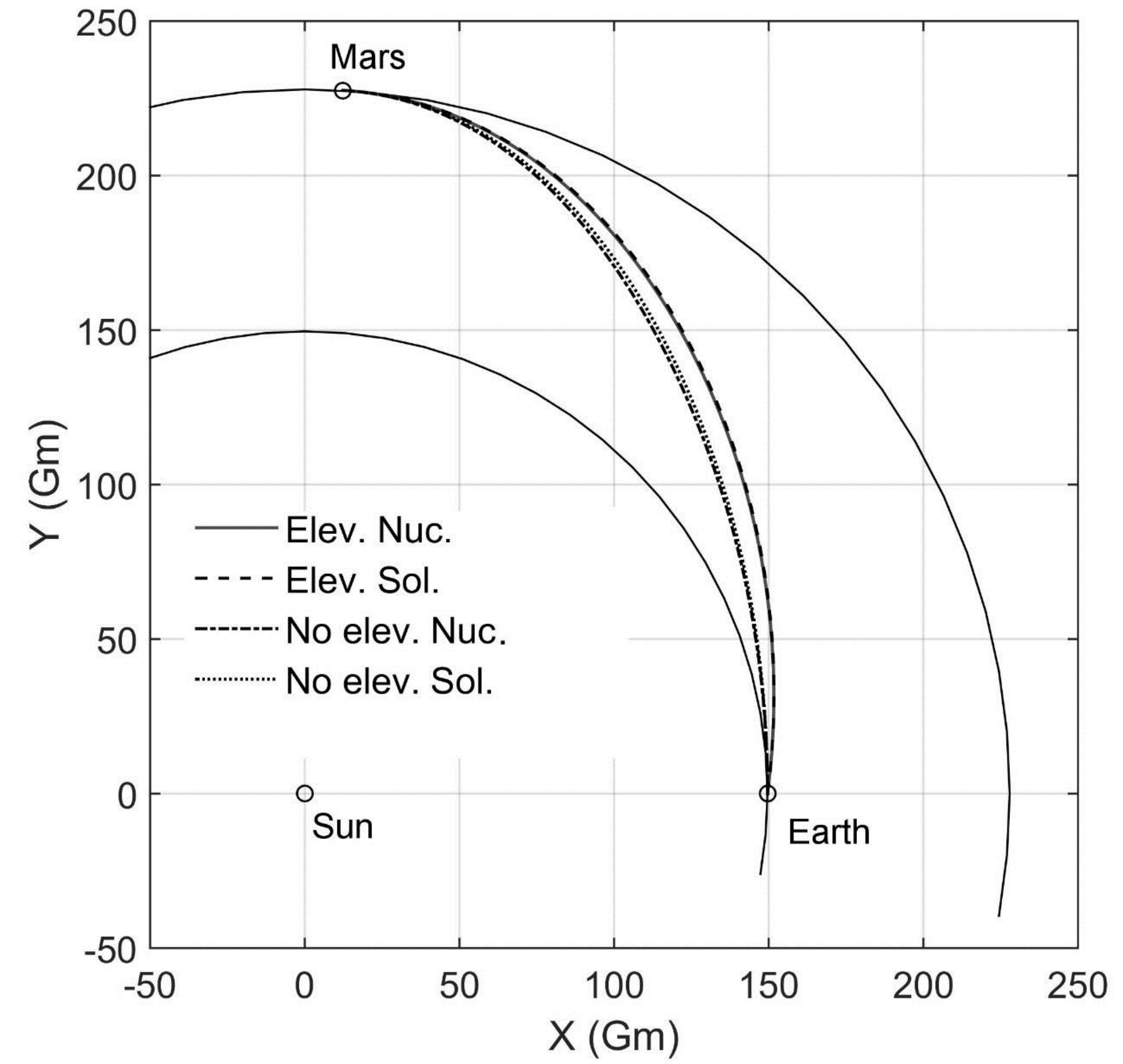
With a space elevator, and $\beta = 40^\circ$

NEP: $J = 10.14\ \mathrm{m^2/s^3}$ ($\sqrt{J} = 3.18\ \mathrm{m/s^{3/2}}$)

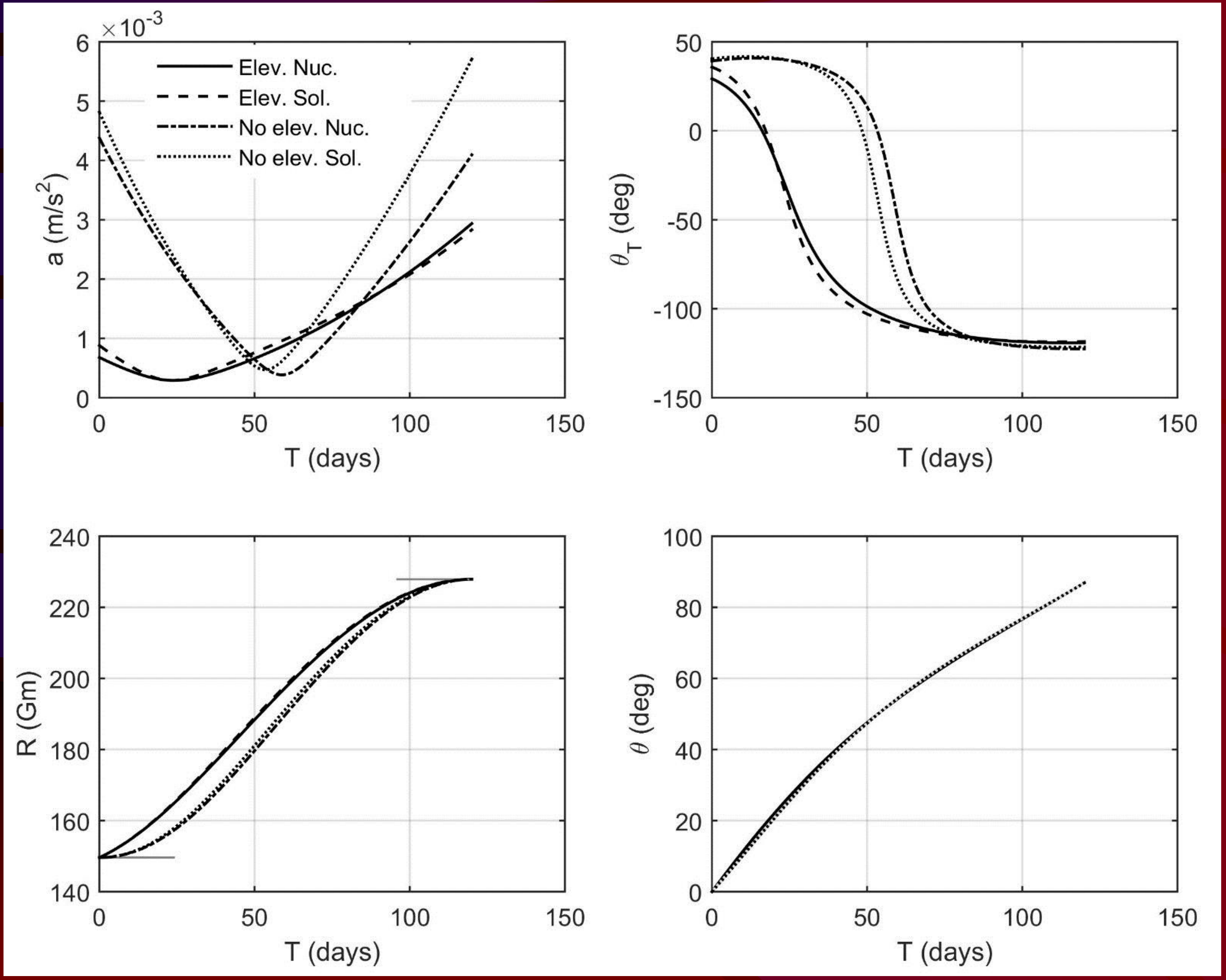
SEP: $J = 23.51\ \mathrm{m^2/s^3}$ ($\sqrt{J} = 4.85\ \mathrm{m/s^{3/2}}$)

In both cases the propellant saving with respect to the case of no elevator in quite large

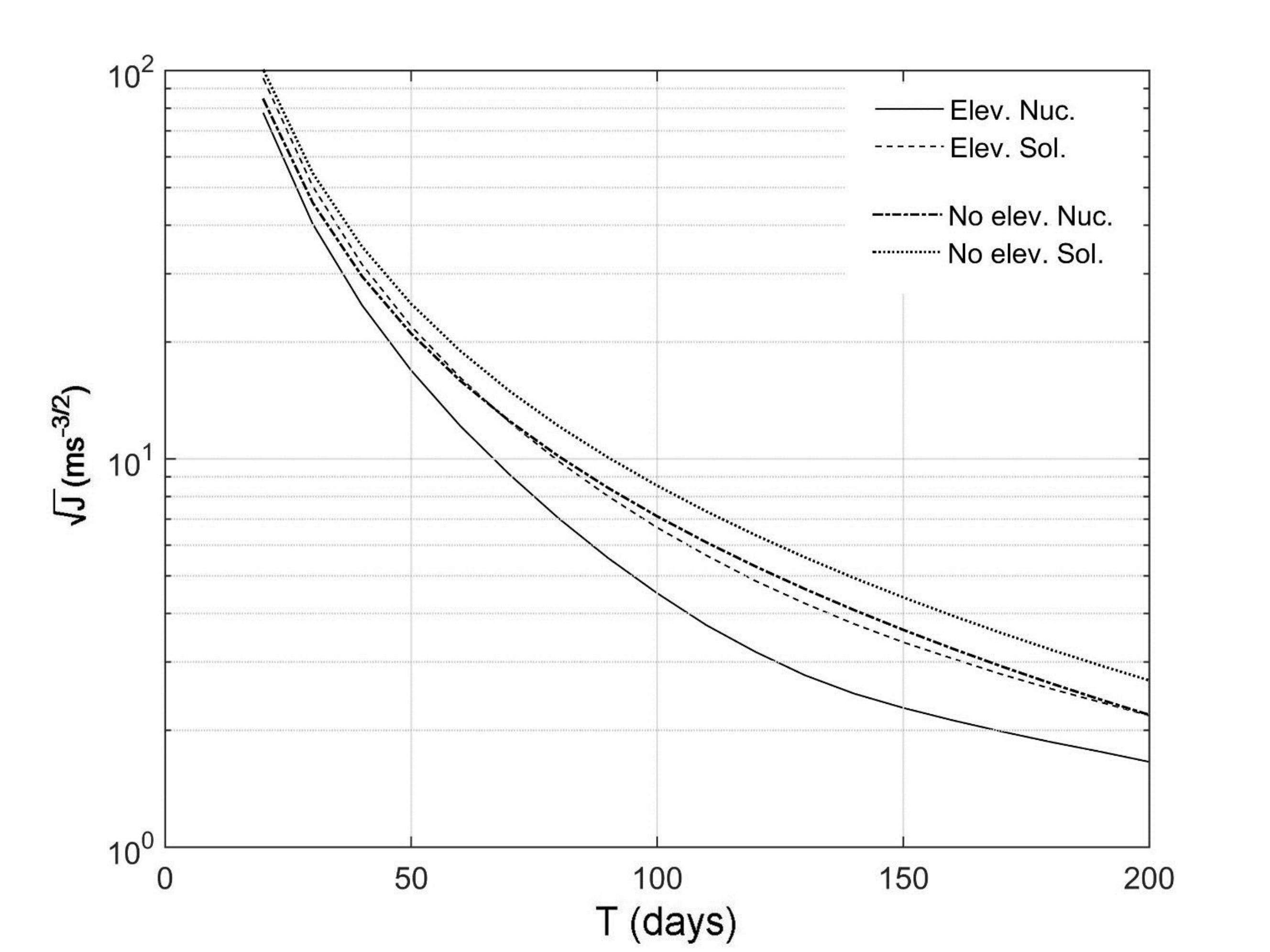
120 days Earth – Mars trajectory



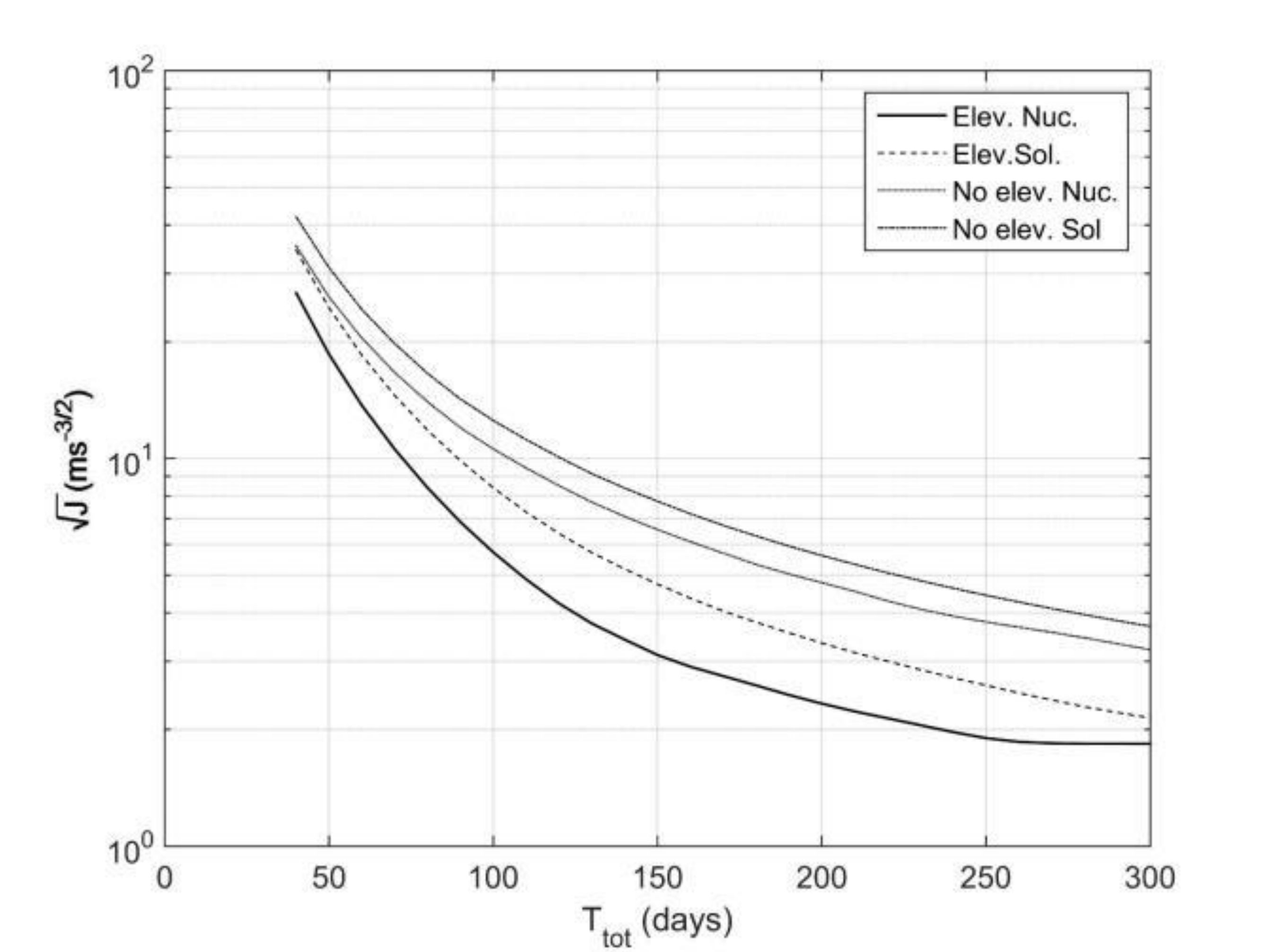
120 days Earth – Mars trajectory



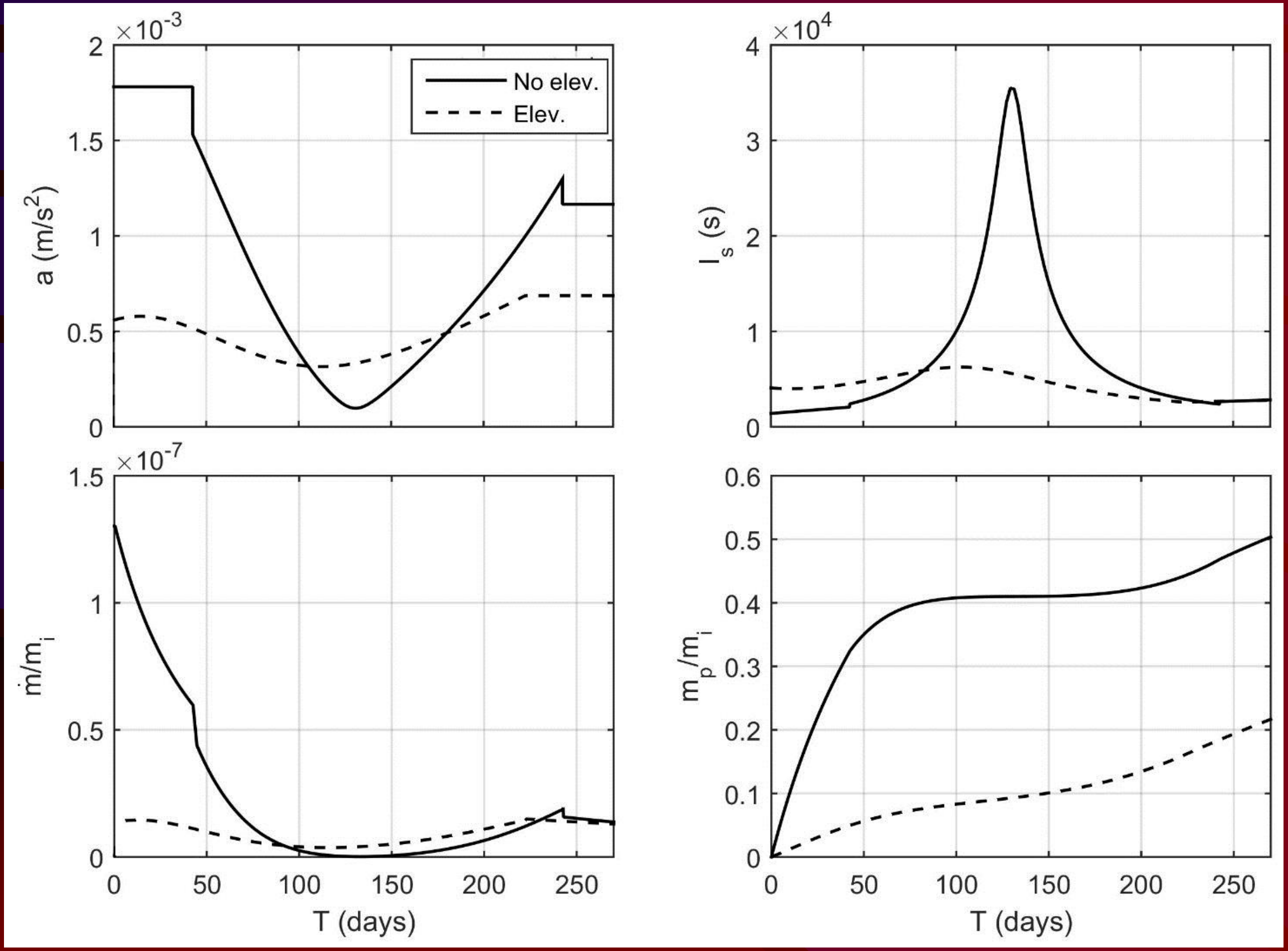
Earth – Mars trajectories



Complete Earth – Mars trajectories



Complete 270 days Earth – Mars trajectory



Examples

Example	1 (NEP, 120 days)			2 (SEP, 270 days)	
α_0 (kg/kW)	10			20	
Elevator	No	Yes		No	Yes
T_1 (days)	12.1	–		41.6	–
T_2 (days)	103.1	110		202.2	225.5
T_3 (days)	4.8	10		26.2	47.5
$\sqrt{J_{tot}}$ ($ms^{-3/2}$)	8.51	4.23		4.10	2.38
$m_l + m_s$ (t)	30	30		100	100
$\gamma = m_p/m_i$	0.851	0.423		0.580	0.337
m_w/m_i	0.127	0.244		0.244	0.223
$(m_l + m_s)/m_i$	0.022	0.333		0.176	0.439
m_i (t)	1350	90		567	227.5
m_p (t)	1150	38		329	76.7
m_w (t)	170	22		139	50.8
P_E (MW)	170	2.20		6.9	2.54

Conclusions

The exploration of the solar system is underway since almost half a century and has completely changed planetary astronomy

Most of these probes were propelled by chemical rockets and some used Solar Electric Propulsion (SEP).

Conclusions

Solar electric propulsion has a severe drawback in missions to the outer solar system: the available power close to the destination planet is just a small fraction of the power available when starting from Earth

To manoeuvre in those conditions an energy source which still has a reasonable mass/power ratio close to the destination planet is required, and at present this is characteristic only of nuclear propulsion

Conclusions

The space elevator is a true gateway to the solar system

Apart from reducing substantially the cost of entering space, and particularly high orbits, it can release outbound spacecraft (and receive inbound ones) with remarkable savings in the interplanetary cruise

It is possible to optimize the trajectory of an interplanetary low thrust spacecraft, both nuclear-electric and solar-electric, without difficulties

Conclusions

Apart from optimizing the thrust along the trajectory, an additional optimization parameter can be identified: the angle between the velocity at the escape from the Earth influence sphere and the tangent to Earth orbit

Some examples showed that in case of Earth-Mars journeys, using a space elevator allows to drastically reduce the IMLEO or the travel time, both in case of SEP and NEP.