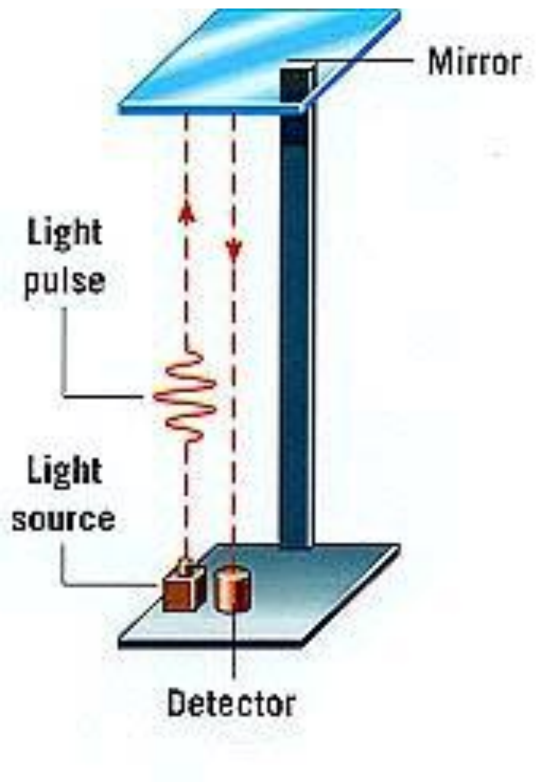
Talking about general relativity

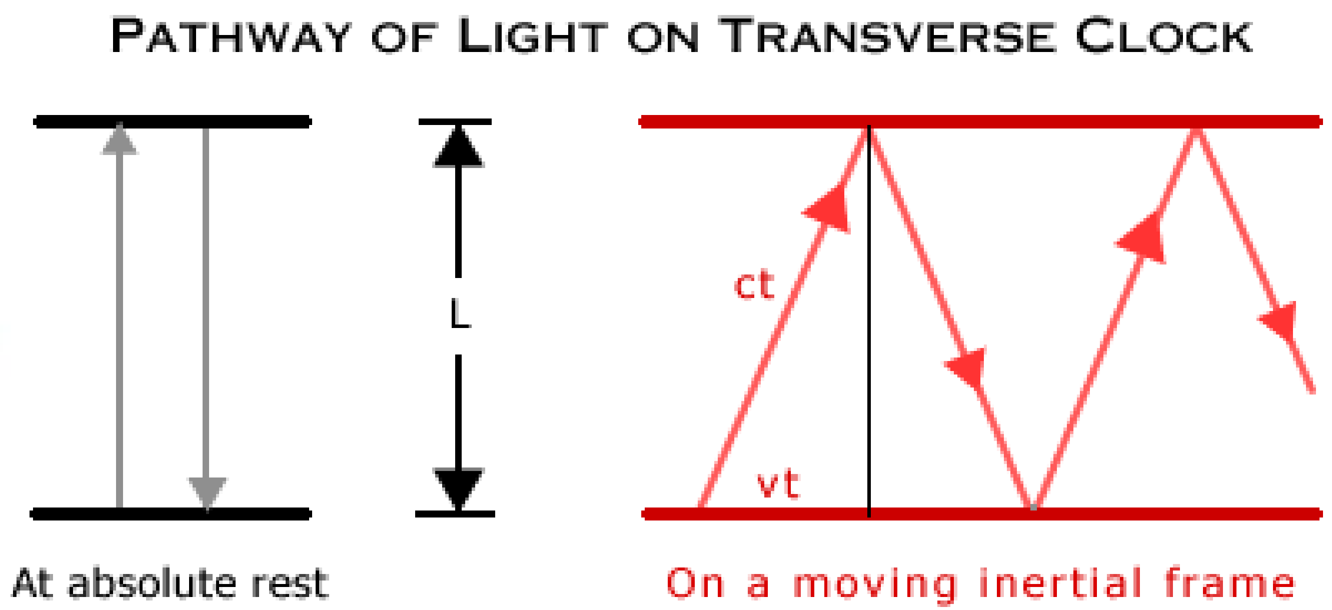
Important concepts of Einstein's general theory of relativity

Øyvind Grøn

Berlin July 21, 2016

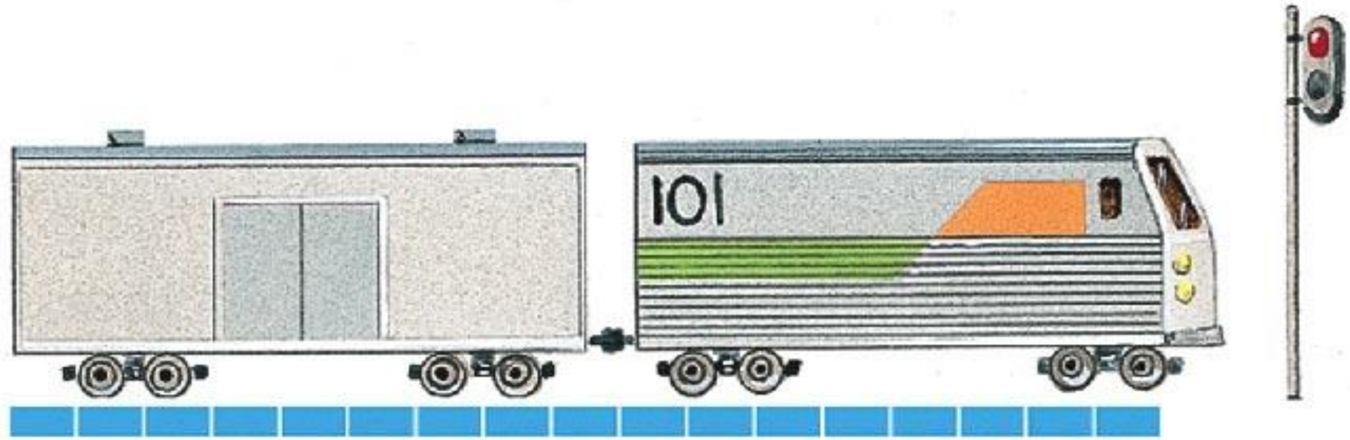
A consequence of the special theory of relativity is that the rate of a clock is less the faster it moves.



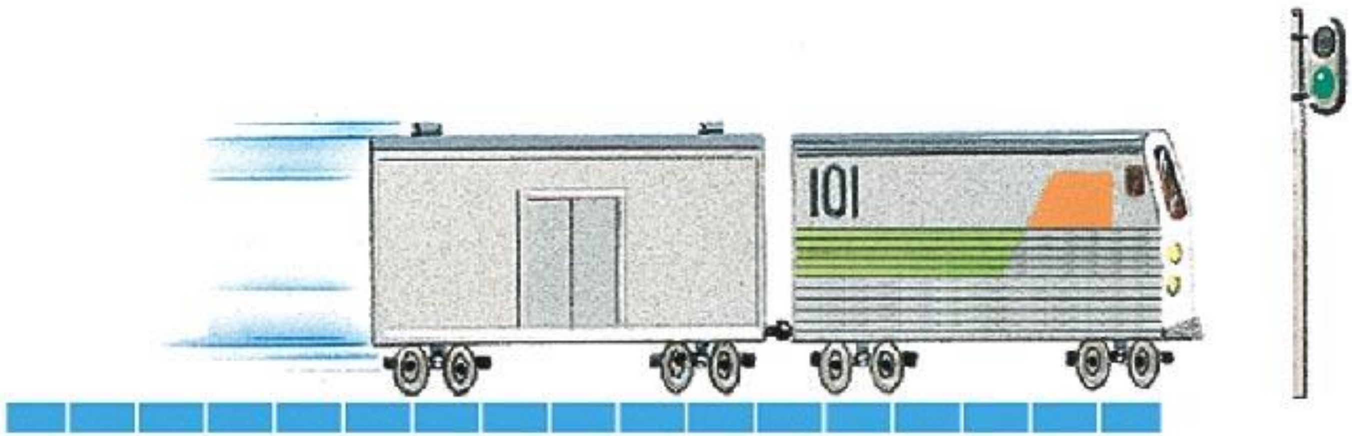


photon clock in a train **observed from the train** **observed from the station**

Length (or Lorentz) contraction

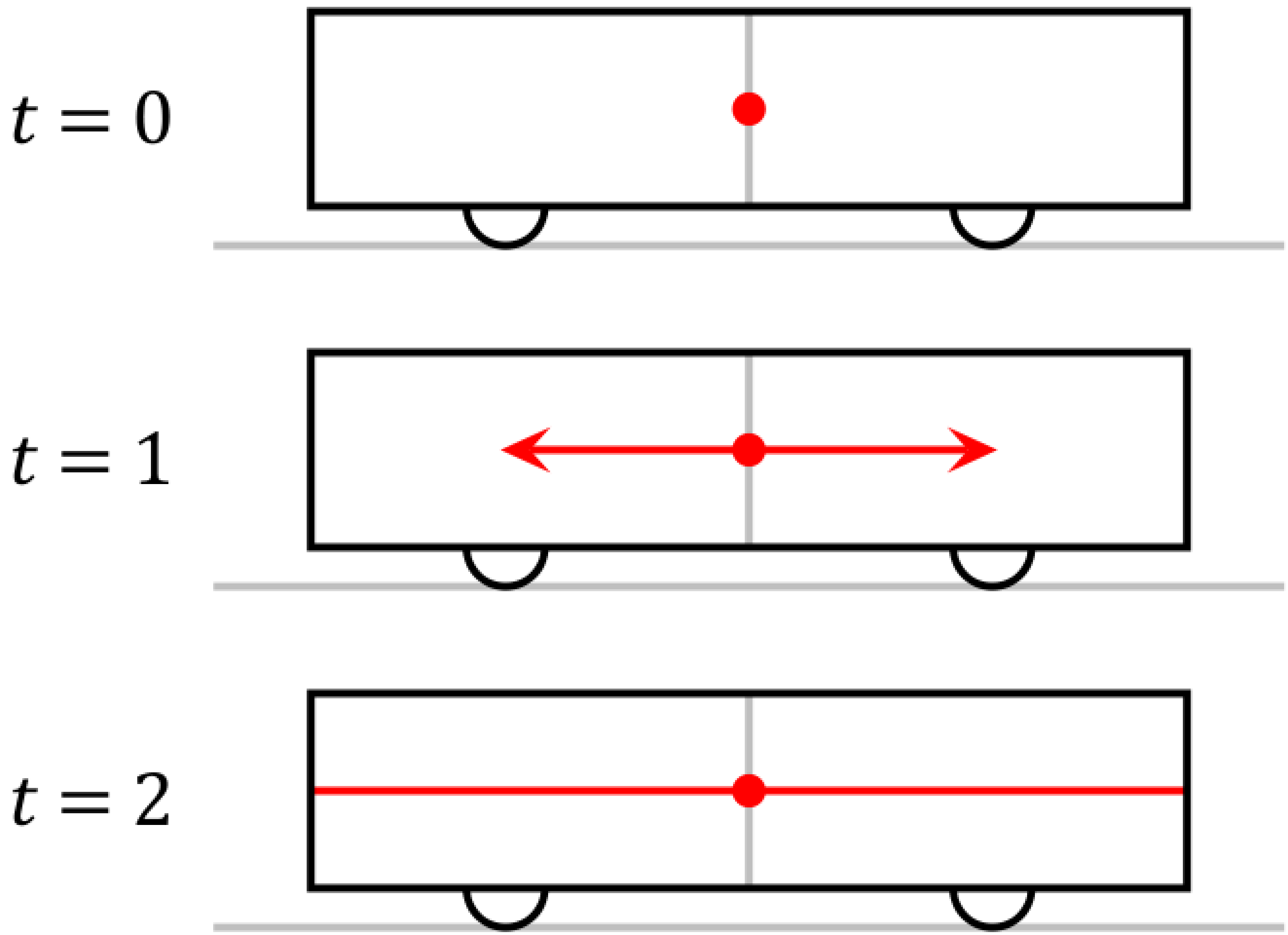


At rest



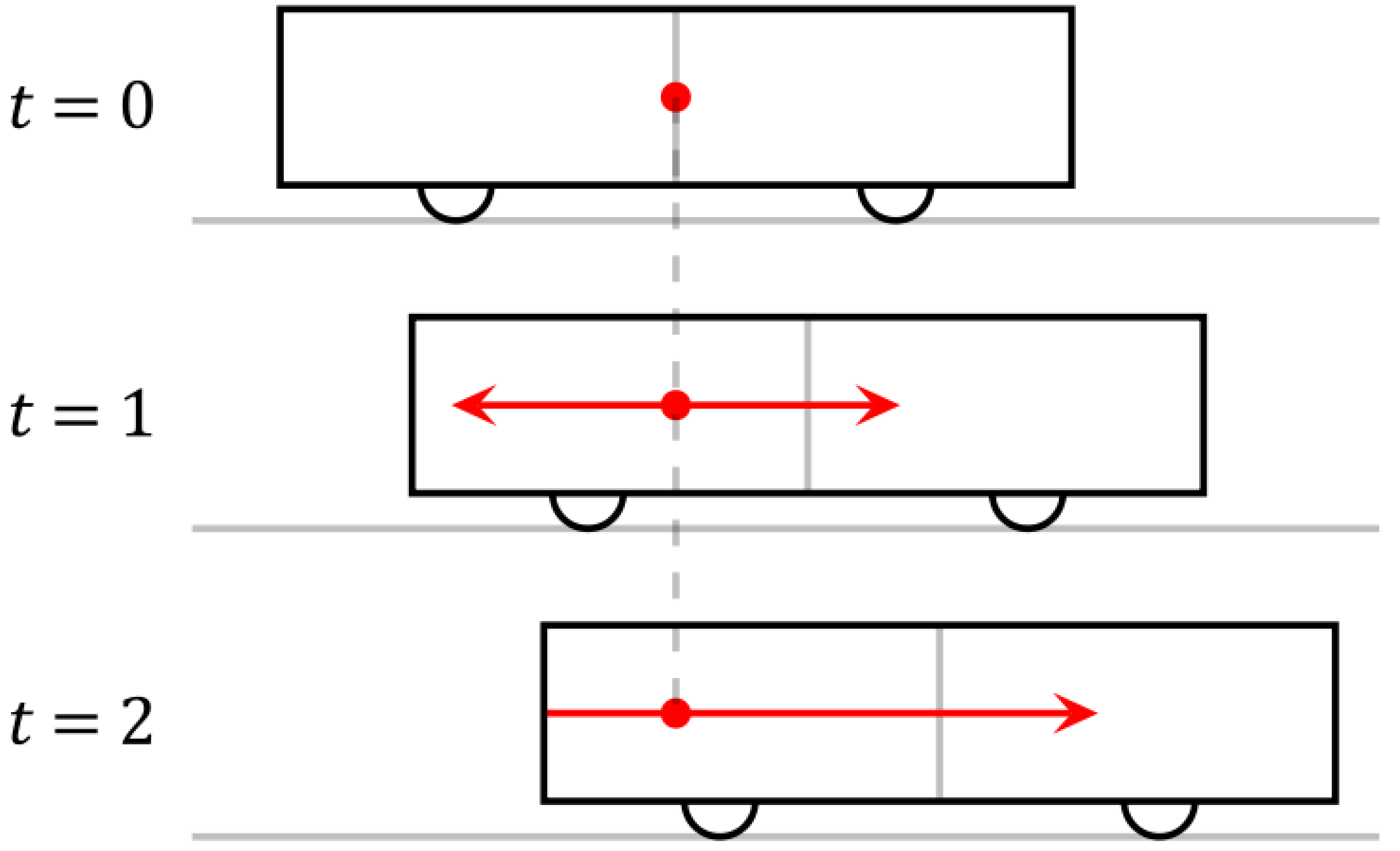
In motion

Measured by simultaneity the difference between the front position of a body and its rear position is less the faster the body moves.



Einstein synchronization of clocks:

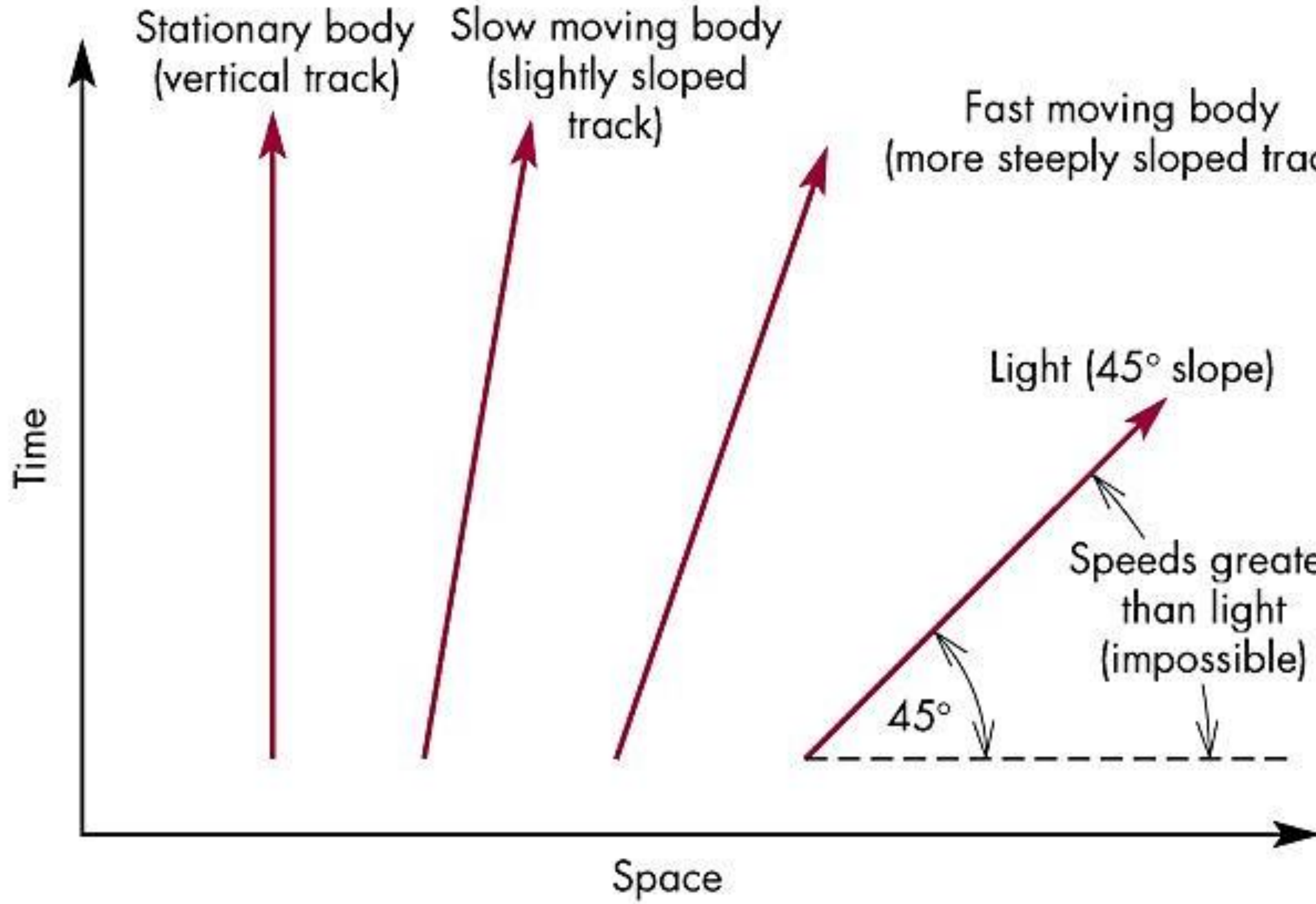
A train passes a station. As observed by a person in the train it is at rest.
She synchronizes two clocks – one to the left and one to the right in the wagon – by emitting a light signal towards the clocks from a position at the center of the wagon.
The clocks are synchronized to show the same time when they are hit by the signals.
The signals hit the clocks simultaneously as observed from the train.

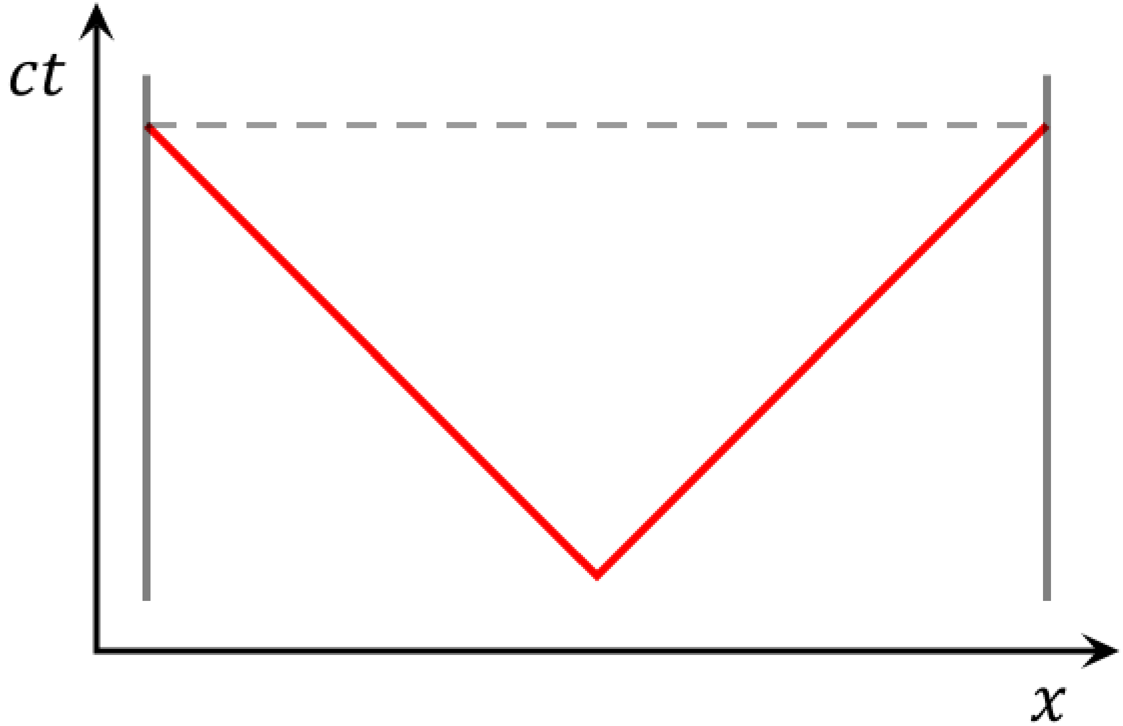


Relativity of simultaneity:

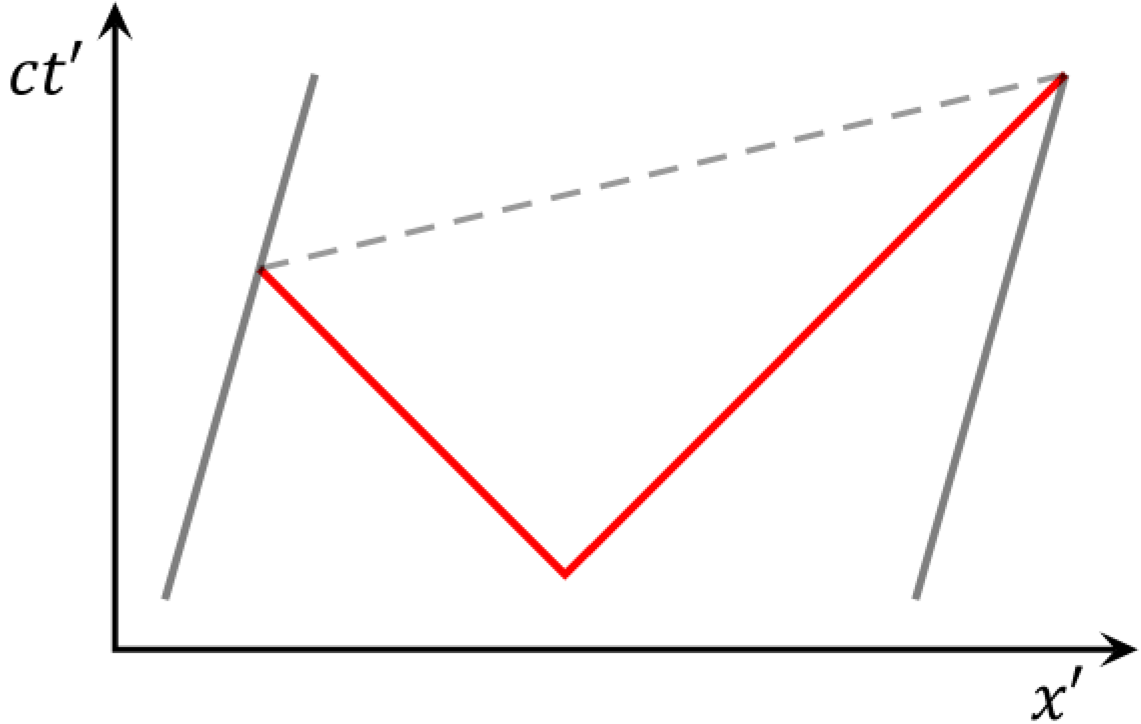
As observed from the station the train moves to the right,
and the light signals hit the rear wall of the wagon before the front wall.
Thus, these events that are simultaneous as observed from the train,
are not simultaneous as observed from the station.

Spacetime diagram also called Minkowski diagram





Minkowski diagram of the synchronization process
with reference to the train frame.
The red lines are the world lines of the light signals.
The gray lines are the world lines of the clocks.
Since the clocks are at rest in 3-space
they move only in the time direction.
The horizontal dashed line represents simultaneity in the train frame.



Minkowski diagram of the synchronization process
with reference to the station frame.
Again the red lines are the world lines of the light signals,
and the gray lines are the world lines of the clocks.
As before, the horizontal dashed line represents
simultaneity in the train frame.
It is not horizontal, showing that events that are simultaneous in the train frame are not simultaneous in the station frame.

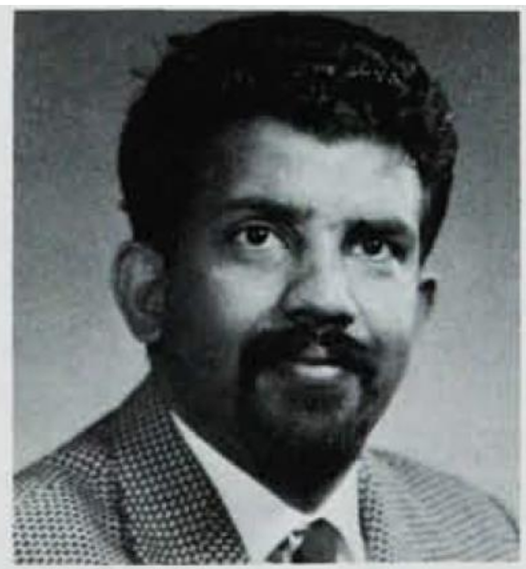
Tachyons

Are superluminal signals permitted by the theory of relativity?

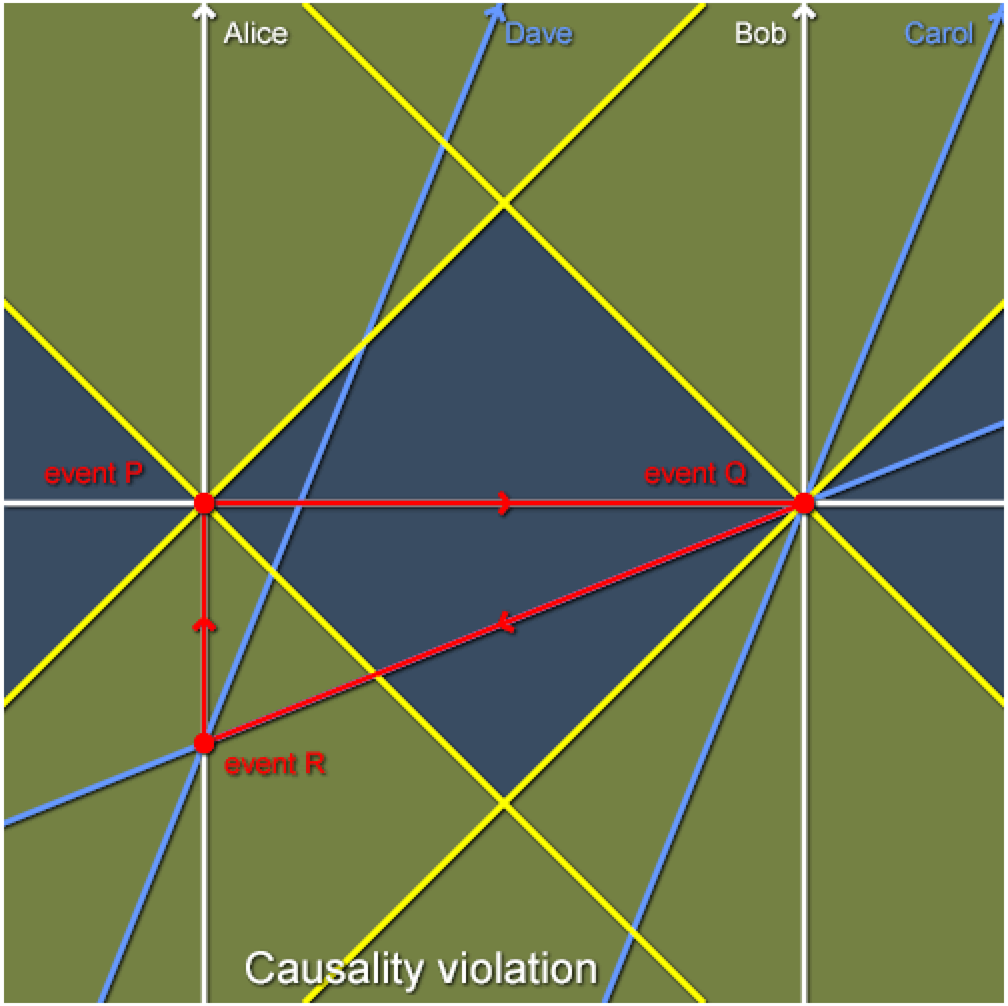
OLEXA-MYRON BILANIUK and E. C. GEORGE SUDARSHAN

PARTICLES BEYOND THE LIGHT BARRIER

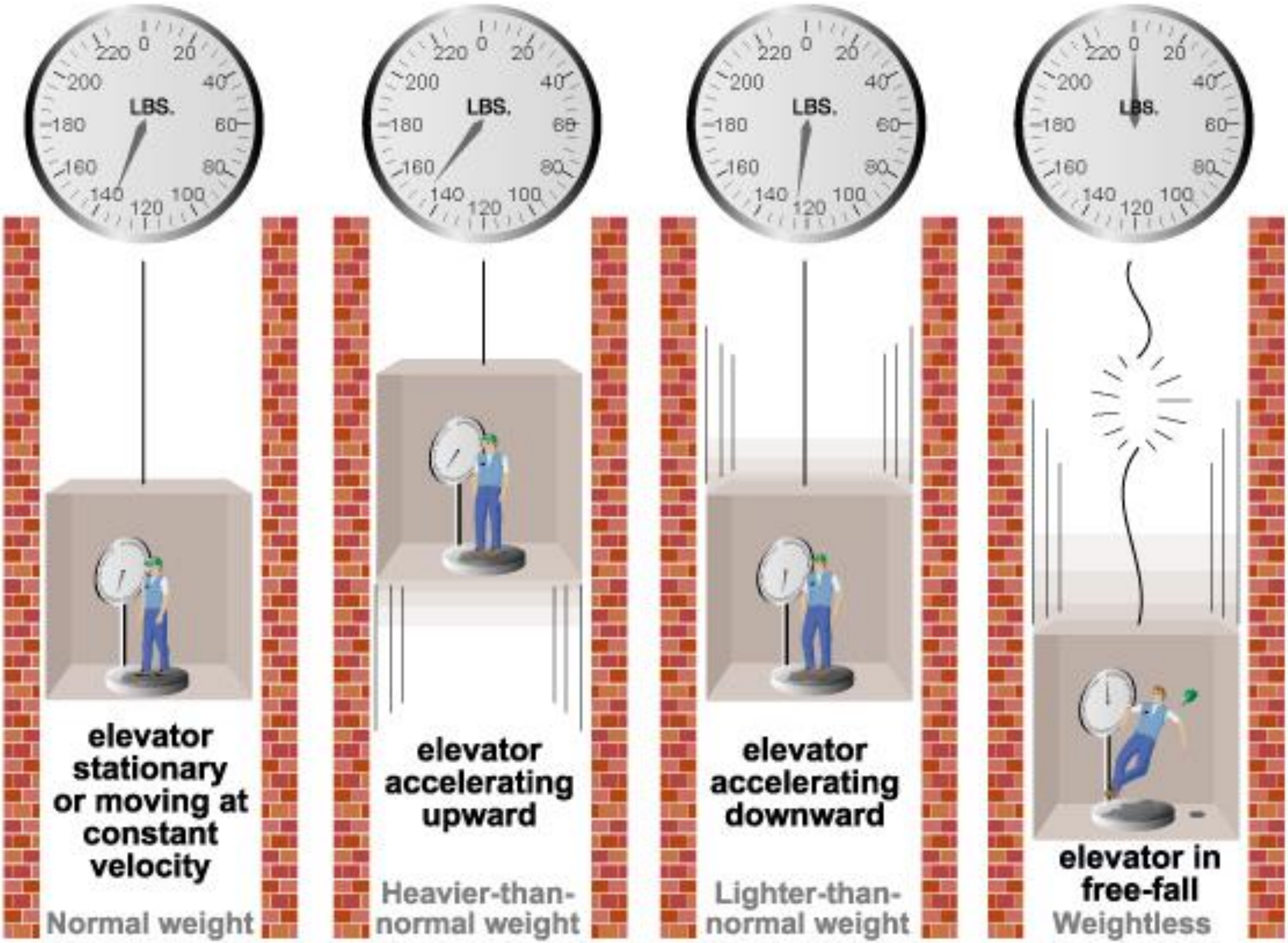
PHYSICS TODAY • MAY 1969 • 43



The tachyon telephone paradox



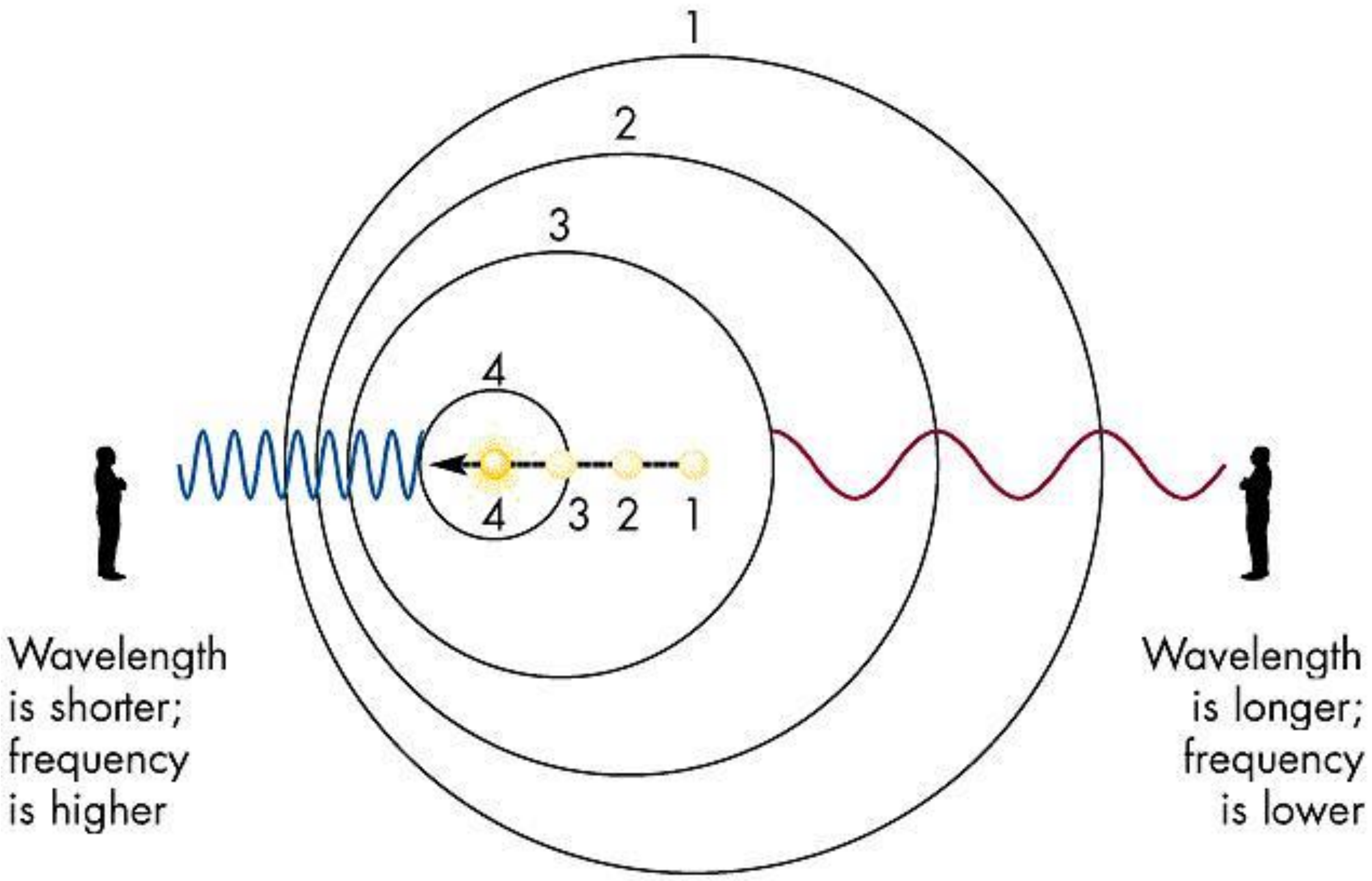
We have a tachyon telephone transferring signals instantaneously.
Event P is emission of a message from Berlin, and event Q that the message arrives at Paris.
It is immediately transferred to a tachyon telephone line on a train moving from Berlin to Paris.
The line Q-R represents simultaneity on the train.
Hence the signal arrives back in Berlin before it was emitted.

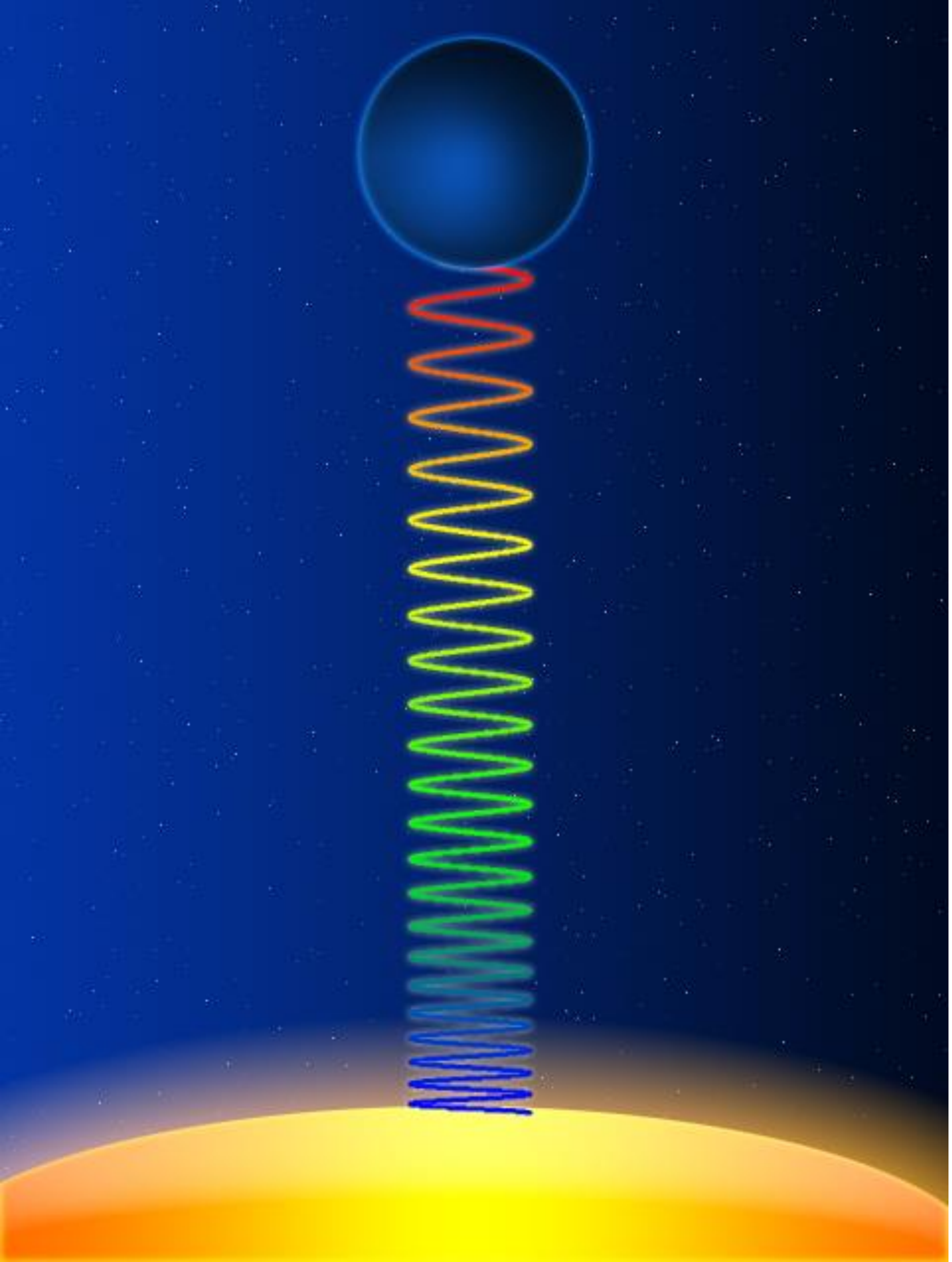


LBS.
220
0
20
40
60
80
100
120
140
160
180
200
elevator stationary or moving at constant velocity
Normal weight
elevator accelerating upward
Heavier-than-normal weight
elevator accelerating downward
Lighter-than-normal weight
elevator in free-fall
Weightless

"On the Influence of Gravitation on the Propagation of Light"
Albert Einstein (1911)
"Über den Einfluss der Schwercraft auf die Ausbreitung des Lichtes",
Annalen der Physik [35], 1911

The Doppler effect

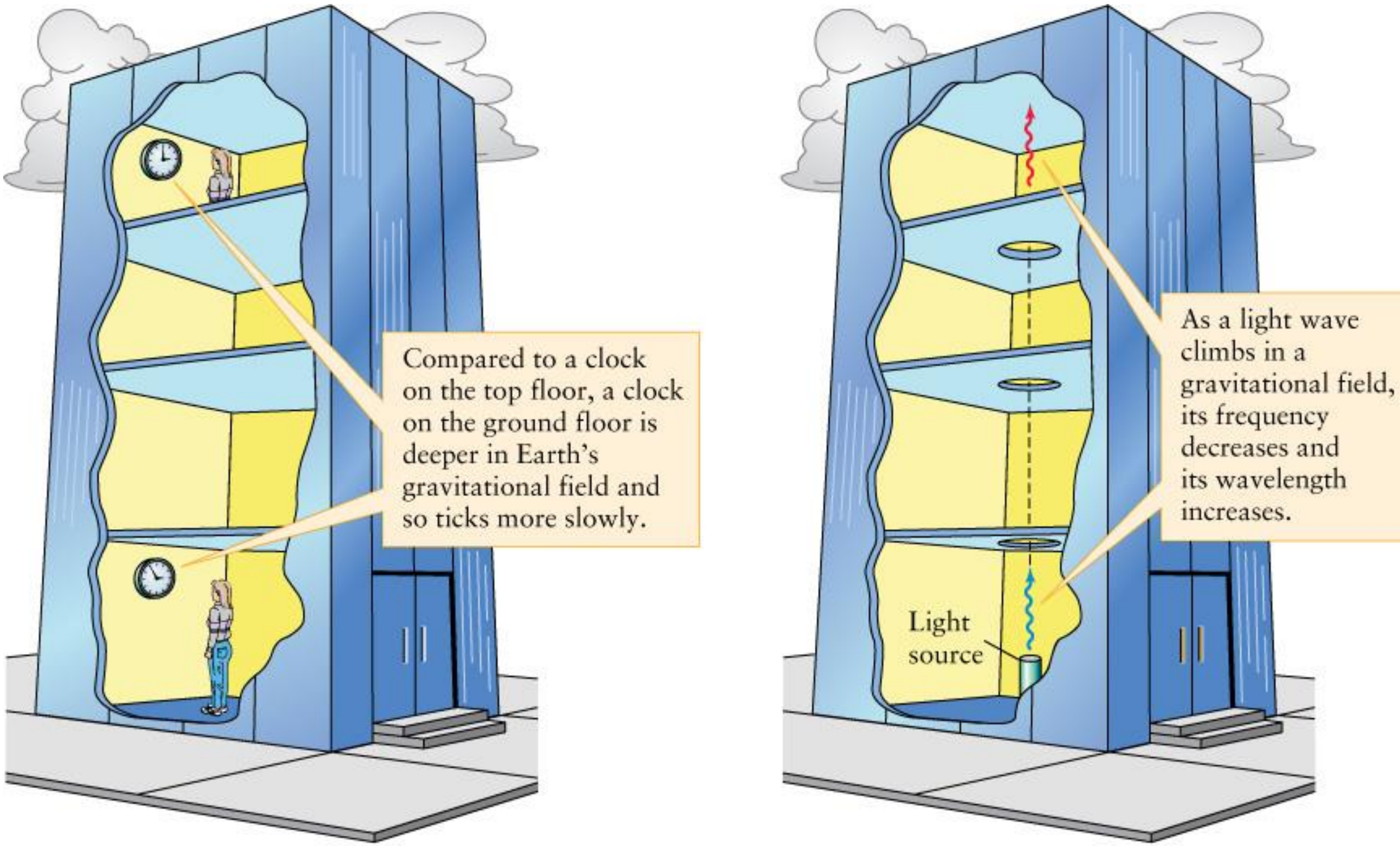




gravitational frequency shift

Light moving downwards
gets a blueshift,

and light moving upwards
gets a red shift.

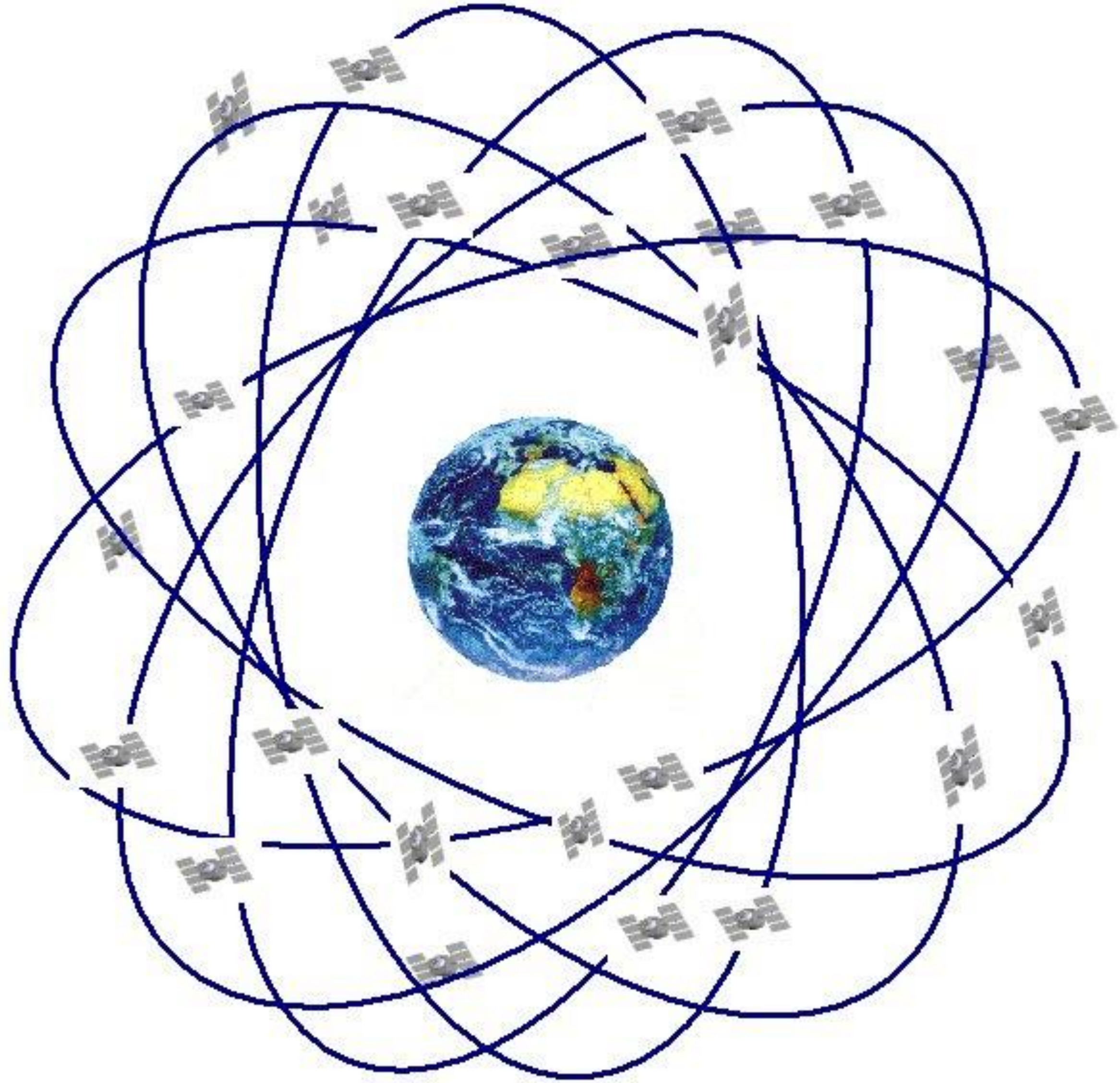


(a) The gravitational slowing of time

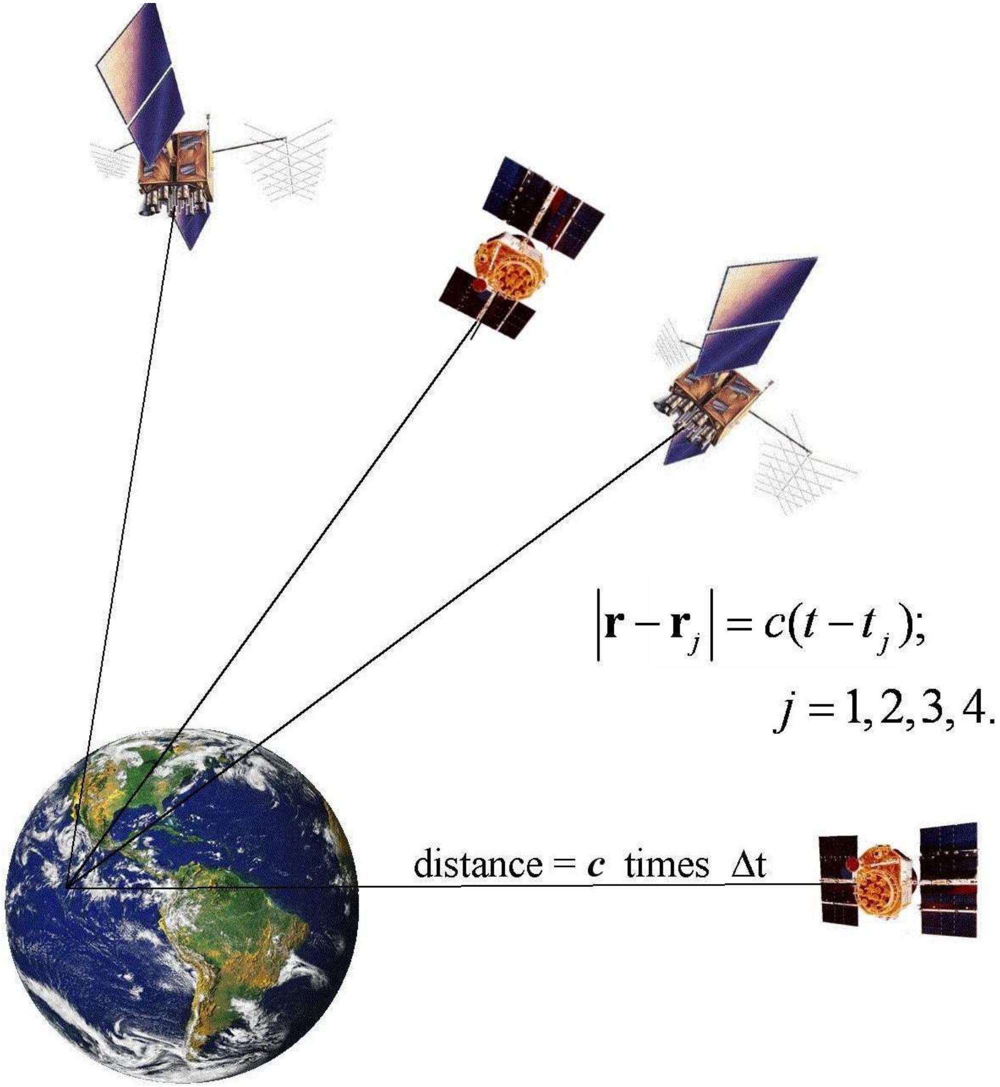
(b) The gravitational redshift

Relativistic time effects in the GPS-system

In order to obtain correct positions with an accuracy of a few meters
by means of the GPS-system
one has to take account of both
the kinematical and gravitational time effects.



24 satellittes move along approximately circular paths
20 000 km above the surface of the Earth.



From

Relativistic Effects in the Global Positioning System

Neil Ashby July 18, 2006

Kinematical time dilation

If an observer on Earth measures a time interval t_E between two events on a satellite, then the time interval t_S measured between the same events on a satellite clock is

$$t_s = \sqrt{1 - \frac{v^2}{c^2}} t_E \approx \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right) t_E$$

where v is the velocity of the satellite with respect to the surface of the. Hence to 2. order in v/c this velocity dependent time dilation makes the satellite clock slow down with

$$\Delta t_v \approx \frac{1}{2}\frac{v^2}{c^2} t_E$$

during the time t_E. With $v = 4 \bullet 10^3$ m/s, c = $3 \bullet 10^8$ m/s and t_E = 24 h = 86400 s we get

$$\Delta t_v \approx 7 \cdot 10^{-6} s$$

Gravitational time distortion

The gravitational time dilation in a spherically symmetrical gravitational field is given by

$$t_G = \sqrt{1 - \frac{2Gm}{c^2 r}}\, t_0 \approx \left(1 - \frac{Gm}{c^2 r}\right) t_0$$

where t_G is a time interval as measured on a clock
at a distance *r* from the center of a spherical body with mass *m*,
and *G* is Newton's constant of gravity.
The difference in time as measured by by a satellite clock
at a distance r_E from the center of the Earth
and a clock on the surface of the Earth at r_S is

$$\Delta t_S \approx \frac{Gm}{c^2}\left(\frac{1}{r_E} - \frac{1}{r_S}\right) t_0 = \frac{\Delta\phi}{c^2} t_0 \approx \frac{\Delta\phi}{c^2} t_E$$

where $\Delta\phi$ is the difference of gravitational potential at the two clocks.
This expression says that
the gravitational time dilation is proportional to the difference of gravitational potential at the positions of the two clocks.

The Schwarzschild radius of a mass *m* is

$$R_S = \frac{2Gm}{c^2}$$

Inserting the values of Newton's constant of gravity, the velocity of light and the mass of the Earth gives

$$R_S = 0{,}88 cm$$

In terms of the Schwarzschild radius the expression of the gravitational time dilation takes the form

$$\Delta t_G = \frac{1}{2}\frac{R_S}{r_E}\frac{\Delta r}{r_S} t_E$$

where Δr is the height difference of the clocks.

During 24 h the gravitational time distortion makes the satellite clock 20 000 km above the Earth come ahead of the clock on the surface of the Earth by

$$\Delta t_G = 45 \cdot 10^{-6} s$$

Hence *for the GPS satellite clocks the gravitational increase of the rate of time is more than 6 times greater that the velocity dependent time dilation.*

What does the time difference mean for the position determination?

Taken together the relativistic effects imply that during a day
the satellite clock proceeds by 38 microseconds more
than a clock at rest on the surface of the Earth.

This introduces an accumulating error in the position determination on the surface of the Earth.

The satellites have a velocity equal to around 4 km/s.
One may wonder whether the accumulated error per day is equal to
the displacement of a satellite during 38 microseconds, 15 cm,
or equal to the distance travelled by light during 38 microseconds, i.e. 11,4 km.

Neil Ashby writes:
"This (the relativistic distortion of the rate of the satellite clock)
is actually a huge effect.
If not accounted for, in one day it could build up to a timing error
that would translate into a navigational error of 13.7 km."

So it is the distance moved by light, not the satellite, during 38 microseconds that accounts.

The reason is the following.
The position of a point on the Earth is determined
by means of a distance measurement,
where the distance is equal to

$$c\left(t_E - t_S\right)$$

Here t_E is the point of time that the signal is received on the Earth
as measured on an Earth clock,
and t_S is the point of time that the signal is emitted from the satellite
as measured on a satellite-clock.
Hence the position error induced by an error in t_S
is equal to the velocity of light times
the error of the satellite clock relative to the earth clock.

The twin paradox

It is often claimed that special relativity is sufficient for solving the twin paradox.

I will here argue that this is not correct,
and that you cannot even formulate the twin paradox within the special theory of relativity.

I will first consider the usual version of the twin paradox in the Minkowski spacetime:
Imagine that twin A remains at rest on the Earth,
and twin B travels with velocity $v = 0{,}8c$ to the nearest star Alpha Proxima
4 light years from the Earth and back. According to A this will take

$$t_A\left(A\right) = 2L_0 / v = 10\, years$$

which means that A is 10 years older at the reunion.

A would predict that the twin B is

$$t_A(B) = \sqrt{1 - v^2 / c^2}\, t_A(A) = 6 \text{ years}$$

older at the reunion.

But according to the principle of relativity
B could consider himself as at rest and A as moving.
According to the special theory of relativity
B would then predict that
he is 10 years older and A is 6 years older at the reunion.

The contradiction between these predictions is the twin paradox.

The principle of relativity
is essential for the formulation of the twin paradox.
There would be no paradox if not both A and B
could consider them self at as rest.

In the special theory of relativity
only non-accelerated motion is relative.
This special principle of relativity is, however,
not enough to formulate the twin paradox.
In order that the twins shall be able to
travel away from each other and reunite again,
at least one of them must accelerate during the departure.

So the general principle of relativity,
including accelerated and rotating motion, is needed.

B observes twin A and the Earth and Alpha Proxima move with a velocity *v = 0,8c*
a Lorentz contracted distance

$$L = L_0 \sqrt{1 - v^2 / c^2} = 2,4 \; light \;\; years$$

According to B the time taken by A's travel to Alpha Proxima and back is

$$t_B(B) = 2L / v = 6 \, years$$

i.e. B predicts that he ages by 6 years during A's travel.
This is in agreement with A's prediction.

But due to the kinematical time dilation B would predict that A ages by

$$t_B(A)_{OUT-IN} = \sqrt{1 - v^2 / c^2} \; t_B(B) = \sqrt{1 - v^2 / c^2} \, \frac{2\sqrt{1 - v^2 / c^2} \, L_0}{v} = \left(1 - \frac{v^2}{c^2}\right) \frac{2L_0}{v} = 3,6 \, years$$

which is in conflict with A's prediction that he should age by ten years.

Let us take a closer view upon what happens with A, according to B,
when B turns at Alpha Proxima.
When B accelerates towards his brother he experiences a field of gravity away from A,
who is higher up in this gravitational field than he is.
Hence as measured by B twin A at the Earth ages faster than B during the time,

$$\Delta t_B(B) = 2v / g$$

when B accelerates.
If B has constant proper acceleration it follows from the general theory of relativity
that the relation between A's ageing and B's is

$$\Delta t_B(A) = \left(1 + \frac{gL_0}{c^2}\right)\Delta t_B(B)$$

which gives

$$\Delta t_B(A) = \left(1 + \frac{gL_0}{c^2}\right)\frac{2v}{g} = \frac{2v}{g} + \frac{2vL_0}{c^2}$$

When the Earthbound twin A calculated
his own and B's ageing during B's travel,
he neglected the time taken by A
at Alpha Proxima to reverse his velocity.
This means that his calculation is correct
only in the limit of an infinitely large acceleration.
The expression for the ageing of A as calculated by B
during the time B experiences a gravitational field, then reduces to

$$\Delta t_B\left(A\right)=\frac{2vL_0}{c^2}=2\cdot 0{,}8\cdot 4\,years=6{,}4\,years$$

Hence, the total ageing of A as correctly predicted by B is

$$t_B\left(A\right)=t_B\left(A\right)_{OUT-IN}+\Delta t_B\left(A\right)=\left(1-\frac{v^2}{c^2}\right)\frac{2L_0}{v}+\frac{2vL_0}{c^2}=\frac{2L_0}{v}=10\,years$$

in agreement with A's own prediction.

The concept acceleration in the theory of relativity

There are two quantities called “acceleration”:
three-acceleration and four-acceleration.
Three-acceleration is defined as the derivative of the coordinate velocity with respect to coordinate time.
It *is a relative acceleration* which can be transformed away.
In spacetime three-acceleration is not a vector.

Four-acceleration is defined as the derivative of the four-velocity with respect to proper time.
It *is an absolute acceleration* which cannot be transformed away.
It is a vector in spacetime.
Four-acceleration represents deviation from free fall.
Particles falling freely have vanishing four-acceleration.
A non-vanishing four-acceleration is due to non-gravitational forces.

Acceleration of gravity

One sometimes hear that gravity is due to spacetime curvature.
This is not always true.
It depends upon what you are talking about.
The acceleration of gravity represents a *local* experience:
That a free particle falls – or that the weight is not showing zero.
This is not due to spacetime curvature.
Acceleration of gravity is something you experience
in a room which is not freely falling,
whether spacetime is flat or curved – that does not matter.

On the other hand:
Tidal forces are due to spacetime curvature.
In a *global* gravitational field,
say the gravitational field of the Earth,
there are tidal forces, and spacetime is curved.

Inertial dragging inside a rotating shell of matter

The weak field result

Inertial dragging inside a rotating shell of matter was described already in 1918 by Hans Thirring.
He calculated the angular velocity of a an object with zero angular momentum
inside a shell with Schwarzschild radius $R_S = 2GM / c^2$ and radius r_0
rotating slowly with angular velocity □
in the weak field approximation,
and found the inertial dragging angular velocity

$$\Omega_Z = \frac{8R_S}{3r_0}\omega$$

Perfect inertial dragging

In 1966 D. R. Brill and J. M. Cohen presented a calculation of the inertial dragging angular velocity inside a rotating shell *valid for arbitrarily strong gravitational fields*, but still restricted to slow rotation, giving

$$\Omega_Z = \frac{4R_S\left(2r_0 - R_S\right)}{\left(r_0 + R_S\right)\left(3r_0 - R_S\right)}\omega$$

If the shell has a radius equal to its own Schwarzschild radius, $r_0 = R_S$,
The expression above gives $\Omega_z = \omega$.
Then there is perfect inertial dragging.
This means that for example the swinging plane of a Fouceault pendulum rotates together with the shell.
Then the motion of a free object inside the shell is determined by the shell.
This is in agreement with Mach's principle.

A cosmic consequence of assuming that rotational motion is relative

The distance that light and the effect of gravity have moved since the Big Bang
is called the lookback distance, $R_0 = ct_0$,
where t_0 is the age of the universe.
We shall now *assume that rotational motion is relative*.
This means that an observer for example at the North pole of the earth
with a Foucault pendulum and a telescope,
may consider the Earth as at rest and the
stars on the sky as rotating.
The observer sees that the swinging plane of the pendulum rotates together with the stars.
If rotational motion is relative, he must be able to explain this as a gravitational effect.
Then there must be *perfect gravitational dragging* in our universe.
Since we have assumed the validity of the principle of relativity for rotational motion,
we then have to *assume that there is perfect inertial dragging in our universe*.
We shall deduce two interesting consequences of this assumption.

We shall interpret Brill & Cohen's result to mean that
in order to have perfect inertial dragging in our universe,
the Schwarzshild radius of the mass inside the look back distance
integrated along the past light cone of an observer,
must be equal to the look back distance.

The mass of the dust inside the look back distance integrated along the past light cone of the observer is given by

$$M = 4\pi \int_0^{r_L} \rho_M r^2 a^3(t) dr$$

where $r_{LH} = d_H / a(t_{EH})$ is the lookback radius of the Hubble horizon at the emission time, and $a(t)$ is the scale factor. The density of the dust at an arbitrary point of time is

$$\rho_M(t) = \rho_{M0} / a^3$$

where ρ_{M0} is the present density of the matter. Hence

$$M = 4\pi\rho_{M0} \int_0^{r_L} r^2 dr = \frac{4\pi}{3} \rho_{M0} r_L^3$$

The Schwarzschild radius of this mass is

$$R_S = \frac{2GM}{c^2} = \frac{8\pi G\rho_0}{3c^2} R_0^3$$

where ρ_0 is the present density of all mass and energy contained in the universe.

Requiring that the Schwarzschild radius of the mass inside the look back distance is equal to the look back distance, $R_S = R_0$, gives for the age of the universe

$$t_0^2 = \frac{3}{8\pi G\rho_0}$$

The present value of the critical density,
corresponding to a universe with Euclidean spatial geometry, is

$$\rho_{cr0} = \frac{3H_0^2}{8\pi G}$$

where H_0 is the present value of the Hubble parameter. Hence

$$t_0^2 = \frac{\rho_{cr0}}{\rho_0}\frac{1}{H_0^2}$$

The present value of the density parameter and the Hubble age of the universe are defined by

$$\Omega_0 = \frac{\rho_0}{\rho_{cr0}} \quad , \quad t_H = \frac{1}{H_0}$$

The assumption that rotational motion is relative thus leads to the relationship

$$t_0 = \frac{1}{\sqrt{\Omega_0}} t_H$$

A large amount of different observations indicate that the density of the universe is very close to the critical density, $\Omega_0 = 1$. The equation above then shows that for a flat universe *the assumption that rotational motion is relative, leads to the simple relationship*

$$t_0 = t_H$$

i.e. that *the age of the universe is equal to its Hubble age*.

The standard model of the universe is a flat universe with dust and Lorentz Invariant Vacuum Energy, LIVE. This type of dark energy has a constant density which may be represented by the cosmological constant, Λ. The present density parameter of the LIVE is $\Omega_{\Lambda 0}$. The age of such a universe is given in terms of the Hubble age as

$$t_0 = \frac{2}{3} t_H \frac{\arctan\left(\sqrt{\Omega_{\Lambda 0}}\right)}{\sqrt{\Omega_{\Lambda 0}}}$$

The last two equations lead to

$$\tanh\left(\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}\right) = \sqrt{\Omega_{\Lambda 0}}$$

The last two equations lead to

$$\tanh\left(\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}\right) = \sqrt{\Omega_{\Lambda 0}}$$

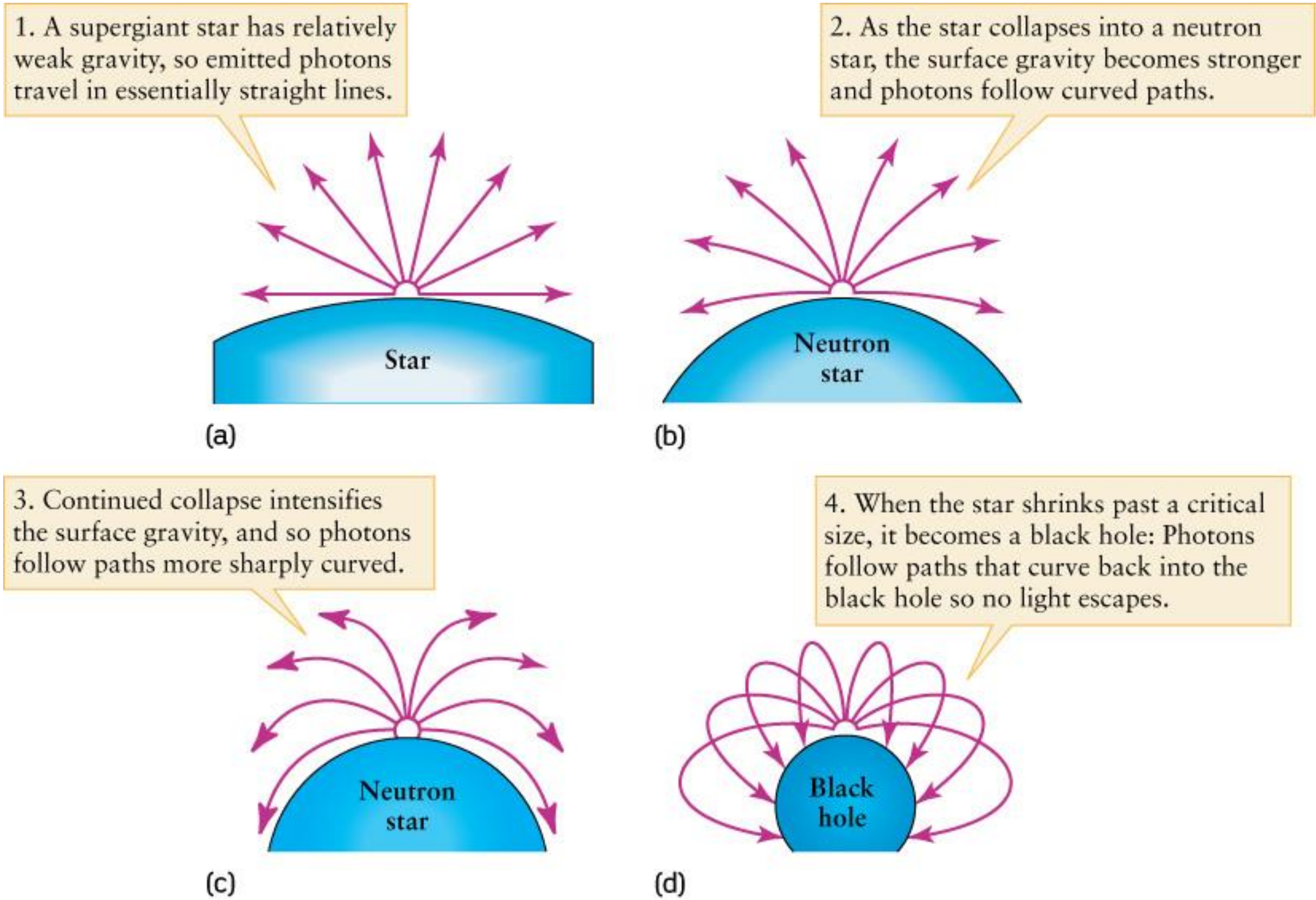
The positive, real solution of this equation gives the following present value of the density parameter of dark energy,

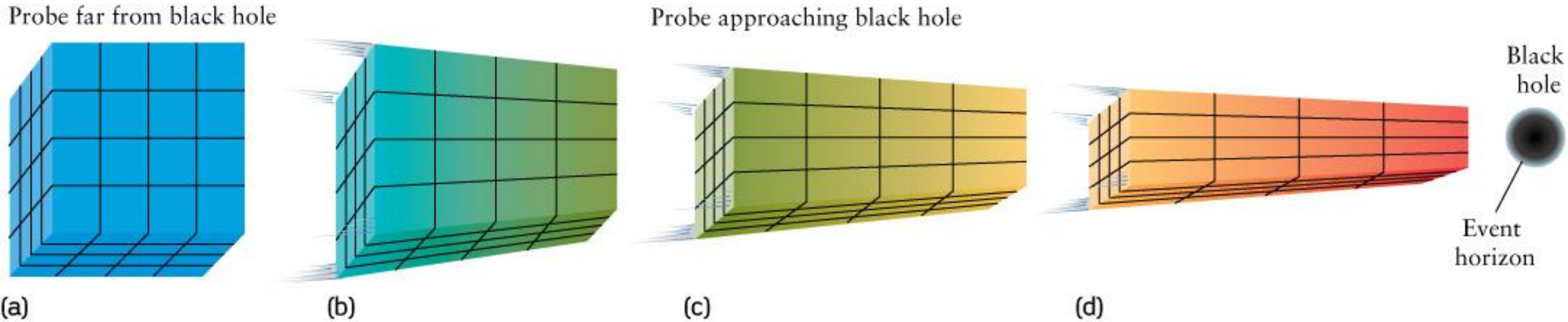
$$\Omega_{\Lambda 0} = 0.737$$

This is the prediction of the general theory of relativity
together with the assumption that rotational motion is relative in our universe
and that the universe is flat and contains dust and LIVE.
It says that 73,7 % of the contents of the universe consists of dark energy.

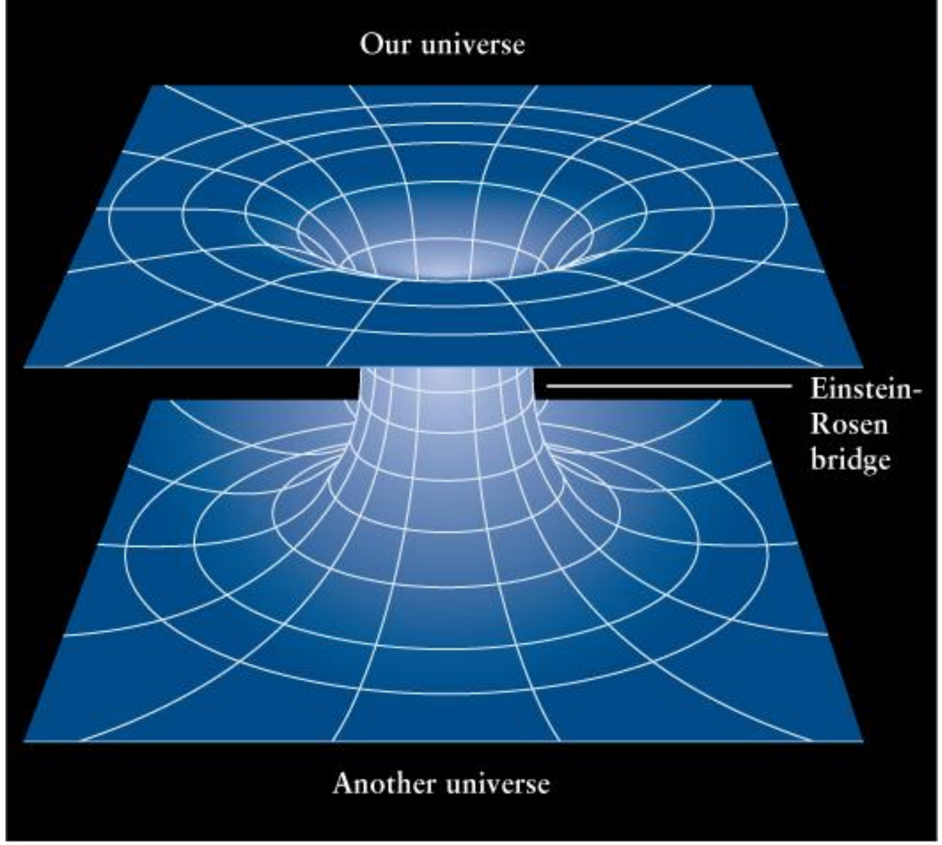
Gravitation:
Einstein’s theory contra Newton’s theory

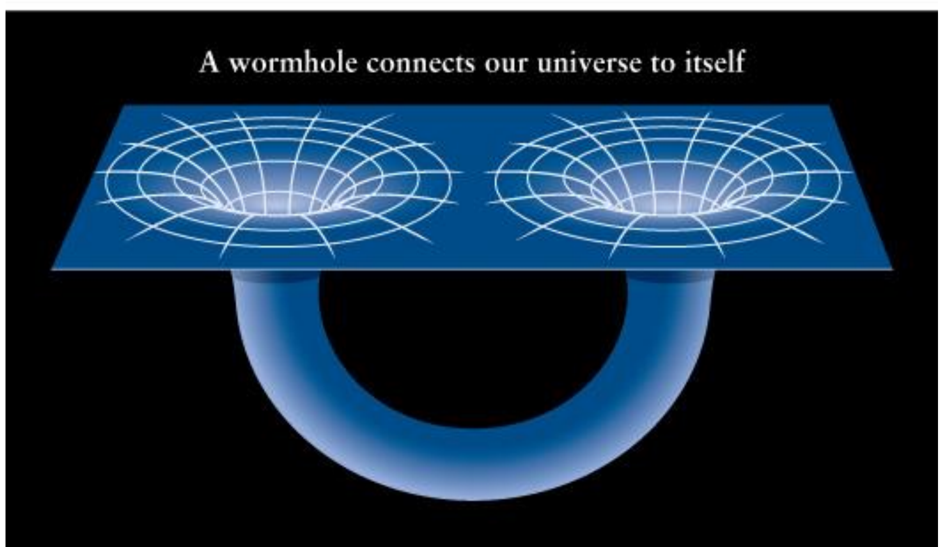
Newton	Einstein
The acceleration of gravity is due to the gravitational force produced by a body.	The acceleration of gravity is the acceleration of a free body in a room which is not freely falling.
Tidal forces are due to masses.	Geodesic deviation is due to spacetime curvature.
	Gravitational waves are emitted by asymmetrical bodies changing shape or orientation.
Tidal forces cause change of shape	Gravitational waves cause change of shape.

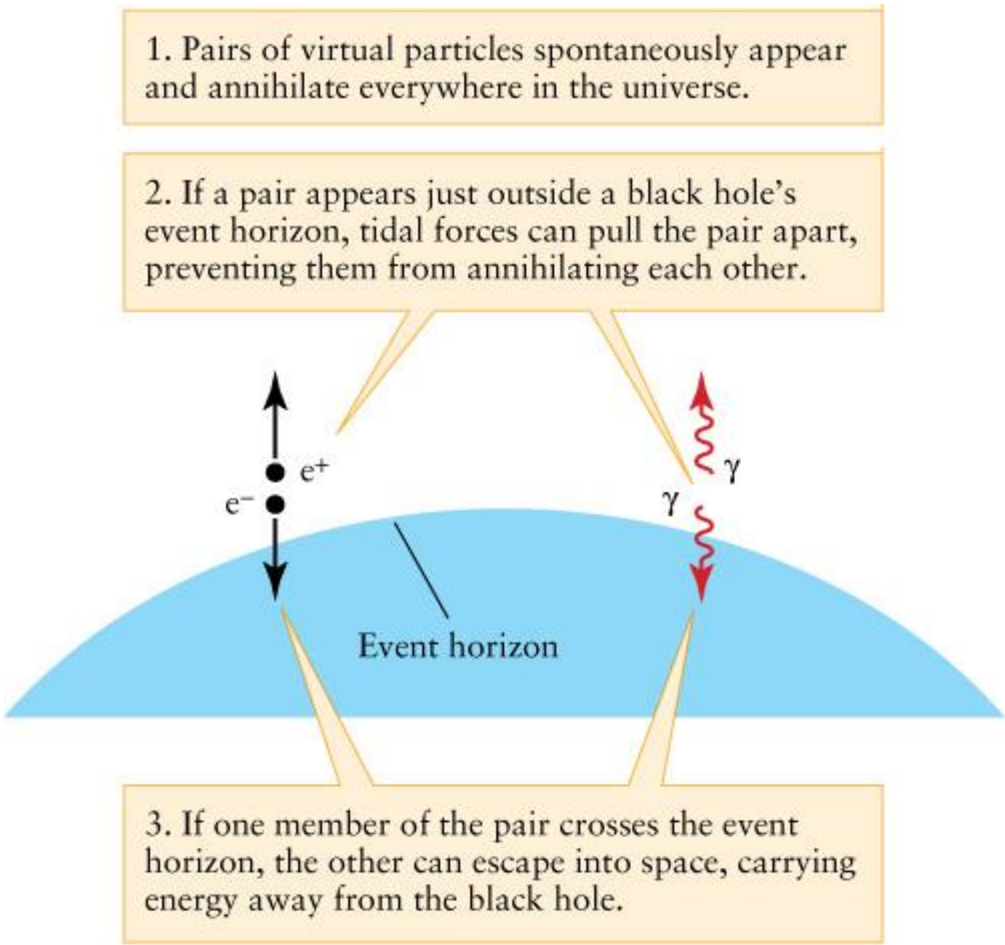

1. A supergiant star has relatively weak gravity, so emitted photons travel in essentially straight lines.
Star
(a)
2. As the star collapses into a neutron star, the surface gravity becomes stronger and photons follow curved paths.
Neutron star
(b)
3. Continued collapse intensifies the surface gravity, and so photons follow paths more sharply curved.
Neutron star
(c)
4. When the star shrinks past a critical size, it becomes a black hole: Photons follow paths that curve back into the black hole so no light escapes.
Black hole
(d)



Probe far from black hole
Probe approaching black hole
Black hole
Event horizon
(a)
(b)
(c)
(d)


Our universe
Einstein-
Rosen
bridge
Another universe


A wormhole connects our universe to itself


1. Pairs of virtual particles spontaneously appear and annihilate everywhere in the universe.
2. If a pair appears just outside a black hole's event horizon, tidal forces can pull the pair apart, preventing them from annihilating each other.
e+
e−
γ
γ
Event horizon
3. If one member of the pair crosses the event horizon, the other can escape into space, carrying energy away from the black hole.