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# **Study of Nanofluids Flow in Micro-channels Using Kinetic Theory**

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# OUTLINE

- ❖ **Background and Motivation**
- ❖ **Objectives**
- ❖ **Problem Definition**
- ❖ **Governing equations and Suggested heat transfer models**
- ❖ **Results and Analysis**
- ❖ **Concluding Remarks**

# Background

- When nanoparticles are added to liquid flow systems, some scalar transport properties can be significantly enhanced.
- Nanofluids are used to enhance heat transfer performance or to maximize drug delivery.
- Concerning micro-channel heat sinks, various designs using different cooling fluids were fabricated and tested, and the superiority of this cooling technique when using nanofluids was confirmed by experimental results.
- In general, investigators have found:
  - a base fluid of high Prandtl number together with
  - nanoparticles of high thermal conductivity in
  - a channel of high aspect ratio,form a desirable combination for optimal performance.

## Background (Cont')

- According to the literature, two approaches are commonly used to study numerically the nanofluid heat and fluid flow in micro-channels; namely, the homogenous (single-phase) approach and the two-phase approach.
- In the two-phase approach, the nanoparticles and the base liquid are considered as separate phases. Then, the heat and momentum transfers between these phases are modeled using different models.
- In most of the two-phase flow studies in micro-channels, either thermal equilibrium conditions (the same temperature) are considered for the phases which is not usually the case, or, the conjugate heat transfer problem is neglected.
- Therefore, accurate predictions of fluid flow and heat transfer in both the solid and cooling fluid are essential for an effective micro-channel heat sink design.

# Motivation

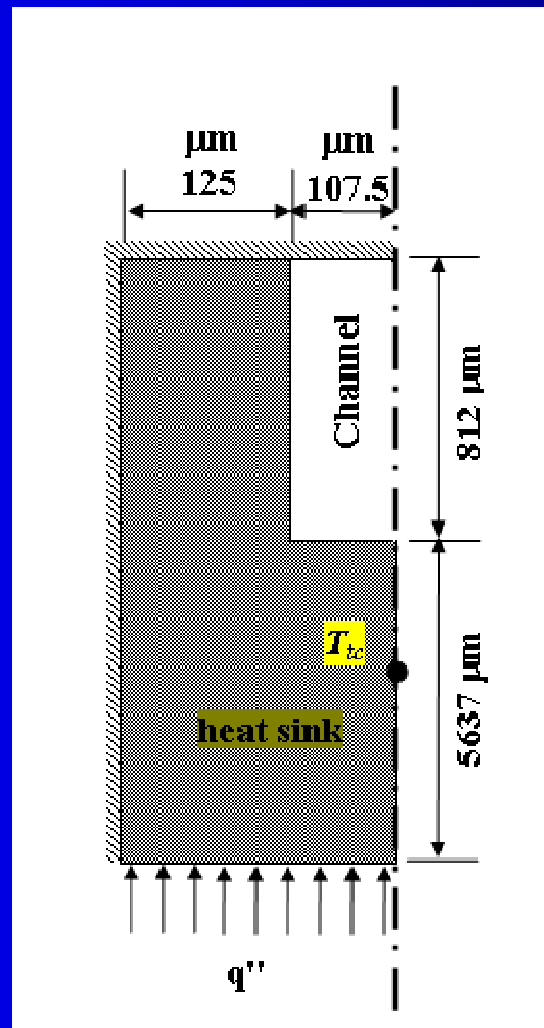
- Focusing on micro-scale heat transfer, it was found that the particle Brownian motion and the induced surrounding liquid motion are key mechanisms for the experimentally observed high increase in the effective thermal conductivity of nanofluids.
- Therefore, there is a need in the two-phase approach for an accurate modeling of the phases scalar transport properties in such a way that the interaction between the phases as well as the dielectric constant of the base liquid are taking into consideration.

# Objectives

- ❑ Propose a model, based on kinetic theory, to determine both the effective thermal conductivity and dynamic viscosity of the phases of  $\text{Al}_2\text{O}_3$ /water nanofluid.
- ❑ Assess numerically the accuracy of using the Eulerian-Eulerian two-phase approach along with the proposed models in predicting the dynamics and heat transfer performance of  $\text{Al}_2\text{O}_3$ /water nanofluid in rectangular micro-channels heat sinks.
- ❑ Investigate the effects of flow Reynolds number and nanoparticles concentration on the heat transfer process.



# Micro-channel configuration



# Governing equations

*Conservation of mass*

*Liquid phase*

$$\frac{\partial(\phi_l \rho_l u_{i,l})}{\partial x_i} = 0 \quad (1)$$

*Particle phase*

$$\frac{\partial(\phi_p \rho_p u_{i,p})}{\partial x_i} = 0 \quad (2)$$

*Volume concentration equation*

$$\phi_l + \phi_p = 1 \quad (3)$$

*Conservation of momentum:*

*Liquid phase*

$$\frac{\partial(\phi_l \rho_l u_{i,l} u_{j,l})}{\partial x_j} = -\phi_l \frac{\partial P}{\partial x_i} + \phi_l \rho_l g_i + \frac{\partial}{\partial x_j} \left[ \phi_l \mu_l \left( \frac{\partial u_{i,l}}{\partial x_j} + \frac{\partial u_{j,l}}{\partial x_i} \right) \right] + K_{pl} (u_{i,p} - u_{i,l}) \quad (4)$$

*Particle phase*

$$\begin{aligned} \frac{\partial(\phi_p \rho_p u_{i,p} u_{j,p})}{\partial x_j} = & -\phi_p \frac{\partial P}{\partial x_i} - \frac{\partial \rho_p}{\partial x_i} + \phi_p \rho_p g_i \\ & + \frac{\partial}{\partial x_j} \left[ \phi_p \mu_p \left( \frac{\partial u_{i,p}}{\partial x_j} + \frac{\partial u_{j,p}}{\partial x_i} \right) + \phi_p \left( \lambda_p - \frac{2}{3} \mu_p \right) \left( \delta_{ij} \frac{\partial u_{k,p}}{\partial x_k} \right) \right] - K_{pl} (u_{i,p} - u_{i,l}) \end{aligned} \quad (5)$$

# Governing equations

*Conservation of energy (incompressible fluids and neglecting the radiation)*

*Liquid phase*

$$\frac{\partial(\phi_P \rho_P u_{i,P} C_{P,T_P})}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \phi_P k_{eff,j} \frac{\partial T_P}{\partial x_i} \right) + \phi_P \mu_P \left( \frac{\partial u_{i,P}}{\partial x_j} + \frac{\partial u_{j,P}}{\partial x_i} \right) \frac{\partial u_{i,P}}{\partial x_j} + \phi_P \left( \lambda_P - \frac{2}{3} \mu_P \right) \left( \frac{\partial u_{k,P}}{\partial x_k} \right)^2 + h_c (T_P - T_l) \quad (6)$$

*Particle phase*

$$\frac{\partial(\phi_l \rho_l u_{i,l} C_{P_l} T_l)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \phi_l k_{eff,j} \frac{\partial T_l}{\partial x_i} \right) + \phi_l \mu_l \left( \frac{\partial u_{i,l}}{\partial x_j} + \frac{\partial u_{j,l}}{\partial x_i} \right) \frac{\partial u_{i,l}}{\partial x_j} - h_c (T_P - T_l) \quad (7)$$

*Energy equation for solid:*

$$\frac{\partial^2 T_s}{\partial x_i^2} = 0 \quad (8)$$

# Constitutive equations

Based on Wen and Yu analysis, the Inter-phase drag force coefficient

$$K_{pl} = \frac{3}{4} C_d \frac{\phi_p \phi_l \rho_l |\vec{V}_p - \vec{V}_l|}{d_p} \phi_l^{-2.65}$$

$$C_d = \begin{cases} \frac{24}{\phi_l Re_p} \left[ 1 + 0.15 (\phi_l Re_p)^{0.687} \right] & Re_p < 1000 \\ 0.44 & Re_p \geq 1000 \end{cases}$$

Particle Reynolds number  $Re_p = \frac{\phi_l \rho_l |\vec{V}_p - \vec{V}_l|}{\mu_l} d_p$

Liquid shear viscosity,  $\mu_l = 2.414 \times 10^{-5} \times 10^{\left( \frac{247.8}{T_l - 140} \right)}$

Particle shear viscosity,  $\mu_p = \mu_{p,col} + \mu_{p,kin}$

## Constitutive equations (cont')

Particle shear viscosity,  $\mu_p = \mu_{p,col} + \mu_{p,kin}$

The collision viscosity is determined based on Gidaspow et al. relation as follows:

$$\mu_{p,col} = 0.8 \phi_p \rho_p d_p g_{0,p} (1+e) \left( \frac{\Theta_p}{\pi} \right)^{\frac{1}{2}} \quad (14)$$

The kinetic viscosity is determined based on Syamlal et al. relation as follows:

$$\mu_{p,kin} = \frac{\phi_p \rho_p d_p \sqrt{\Theta_p \pi}}{6(3-e)} \left[ 1 + 0.4 (1+e)(3e-1) \phi_p g_{0,p} \right] \quad (15)$$

The particle bulk viscosity,  $\lambda_p$ , is determined based on the Lun et al. relation:

$$\lambda_p = \frac{4}{3} \phi_p \rho_p d_p g_{0,p} (1+e) \left( \frac{\Theta_p}{\pi} \right)^{\frac{1}{2}} \quad (16)$$

## Constitutive equations (cont')

Particle pressure,  $P_p$

$$P_p = \phi_p \rho_p \Theta_p + \left[ 2 \phi_p^2 (1+e) g_{0,p} \right] \Theta_p \quad g_{0,p} = \left( 1 - \left( \frac{\phi_p}{0.63} \right)^{1/2} \right)^{-1}$$

Based on Kinetic theory, the transport equation for the granular temperature  $\Theta_p$ , *Schmidt and Renz* (2000) is given by:

$$\frac{3}{2} \left[ \frac{\partial(\phi_p \rho_p \Theta_p)}{\partial t_i} + \frac{\partial(\phi_p \rho_p \Theta_p \mu_{i,p})}{\partial x_i} \right] = -P_p \frac{\partial u_{k,p}}{\partial x_k} + \mu_p \left( \frac{\partial u_{i,p}}{\partial x_j} + \frac{\partial u_{j,p}}{\partial x_i} \right) \frac{\partial u_{i,p}}{\partial x_j} + \left( \lambda_p - \frac{2}{3} \mu_p \right) \left( \frac{\partial u_{k,p}}{\partial x_k} \right)^2 + \frac{\partial}{\partial x_i} \left( k_{\Theta,p} \frac{\partial \Theta_p}{\partial x_i} \right) - \chi_{\Theta_p} + \psi_{ip}$$

Detailed discussion about  $\chi_{\Theta_p}$  &  $\psi_{ip}$  the collisional dissipation of energy & the exchange of fluctuating energy between liquid and solid phase can be found in *Boemer et al. (1997)* and *Schmidt and Renz (2000)*.

- *A. Schmidt and U. Renz., "Numerical prediction of heat transfer in fluidized beds by a kinetic theory of granular flows," Int. J. Therm. Sci. 39 (2000) 871–885.*
- *Boemer A., Qi H., Renz U., "Eulerian simulation of bubble formation at a jet in a two-dimensional fluidized beds," Int. J. Multiphase Flow 23 (5) (1997) 927–944.*

## Suggested nanoparticle thermal conductivity model

- Suggested model for the particle effective thermal conductivity,  $k_{eff,p}$

$$k_{eff,p} = \underbrace{\sqrt{\phi_{pe}k_l} + \frac{3(k_{pe} - k_l)\phi_{pe}}{(k_{pe} + 2k_l) - (k_{pe} - k_l)\phi_{pe}}}_{k_{p,se}} + \underbrace{57.7(\phi_p)^{0.9159} \rho_l C_{pl} \sqrt{\frac{\beta T_p}{\rho_p d_p}} f(T, \phi_p)}_{k_{p,Brownian}}$$

$$f(T, \phi_p) = (-6.04\phi_p + 0.4705)T_p + (1722.3\phi_p - 134.63)$$

Boltzmann constant,  $\beta = 1.3807 \times 10^{-23} \text{ J/K}$

- The increased volume concentration resulting from assuming the presence of a liquid nanolayer around each particle [Yu and Choi]

$$\phi_{pe} = (1 + \gamma)^3 \phi_p \quad k_{pe} = \frac{2(k_p - k_{lay}) + (1 + \gamma)^3(k_p + 2k_{lay})}{-(k_p - k_{lay}) + (1 + \gamma)^3(k_p + 2k_{lay})} k_{lay}$$

- $\gamma = (\text{nanolayer thickness, } h) / (\text{original particle radius, } d_p/2)$ .

*In the present study, for  $Al_2O_3/\text{water}$  nanofluid,  $\gamma$  is taken as 0.167.*

- The liquid nanolayer thermal conductivity  $k_{lay}$  is equal to 2.1 W/m.k for  $Al_2O_3/\text{water}$  nanofluid [Xue].

- Liquid effective thermal conductivity,  $k_{eff,l}$ , [Schmidt and Renz (2000)]

$$k_{eff,l} = \left( \frac{1 - \sqrt{\phi_{pe}}}{1 - \phi_{pe}} \right) k_l$$

# Suggested nanoparticle Nusselt number correlation

Volumetric inter-phase heat transfer coefficient,  $h_v$ ,

$$h = \frac{\delta(1 - \phi_{pe})}{d_{pe}} h_{ip} \qquad h_{ip} = \frac{Nu_p k_{pe}}{d_{pe}}$$

In this study, the Zerradi *et al.* (2014) empirical correlation for the Nusselt number,  $Nu_p$ , has been utilized to calculate the fluid-particle heat transfer coefficient  $h_{ip}$

$$Nu_p = Pr^{0.1039} \left( 1.0257\phi_{pe} + 11397 Re_m^{0.205} + 0.7884\phi_{pe} Re_m^{0.205} + 1.2069 \right)$$

$$1 < Pr < 10 \text{ and } 10^{-4} < Re_m < 10^{-1} \qquad Pr = \frac{C_{p,l} \mu_l}{k_{eff,l}}$$

Brownian Reynolds number  $Re_m = \frac{\rho_l}{\mu_{eff,l}} \sqrt{\frac{18 \beta \Gamma_p}{\pi \rho_p d_{pe}}} \qquad \mu_{eff,l} = \left( \frac{1 - \sqrt{\phi_{pe}}}{1 - \phi_{pe}} \right) \mu_l$



## Local Nusselt number Definition

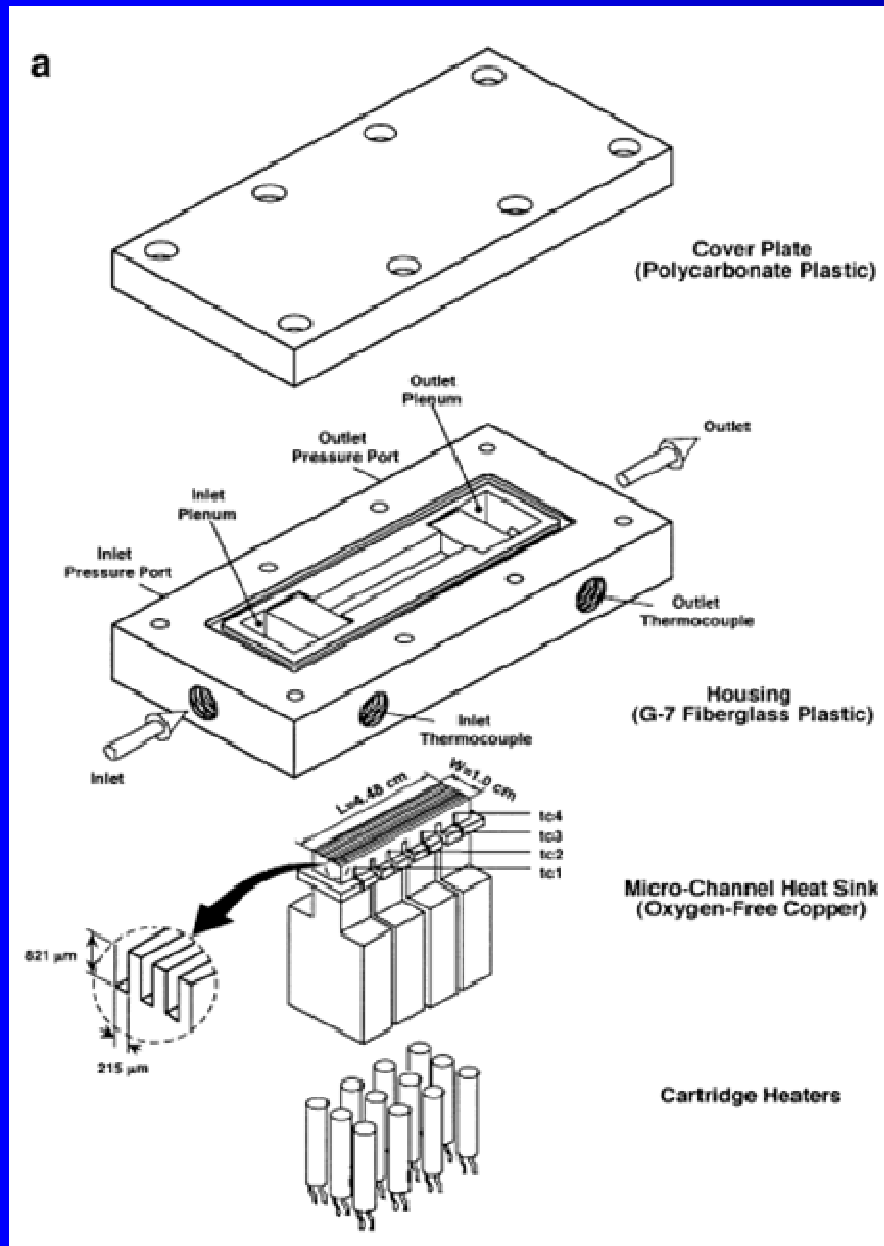
Based on the *Muzychka et. Al (2012)*, the appropriate definition of the local peripheral Nusselt number for micro-channels is based on the microchannel wall and the inlet nanofluid temperature difference and is given by:

$$Nu_h = \frac{q'' D_h}{k_{nf} (T_{w,x} - T_i)}$$

where  $q''$  and  $D_h$  are the wall convective heat transfer flux and micro-channel hydraulic diameter (341  $\mu\text{m}$  in the present study). The local wall convective heat transfer flux is determined from the following equation:

$$q'' = -k_s \left. \frac{\partial T_s}{\partial y} \right|_w - k_s \left. \frac{\partial T_s}{\partial z} \right|_w$$

# Problem Definition



**Micro-channel configuration  
used experimentally by  
Jaeseon Lee and Issam Mudawar;**

$$Re_{Dh} = \frac{\rho_{nf} V_o}{\mu_{nf}} D_h$$

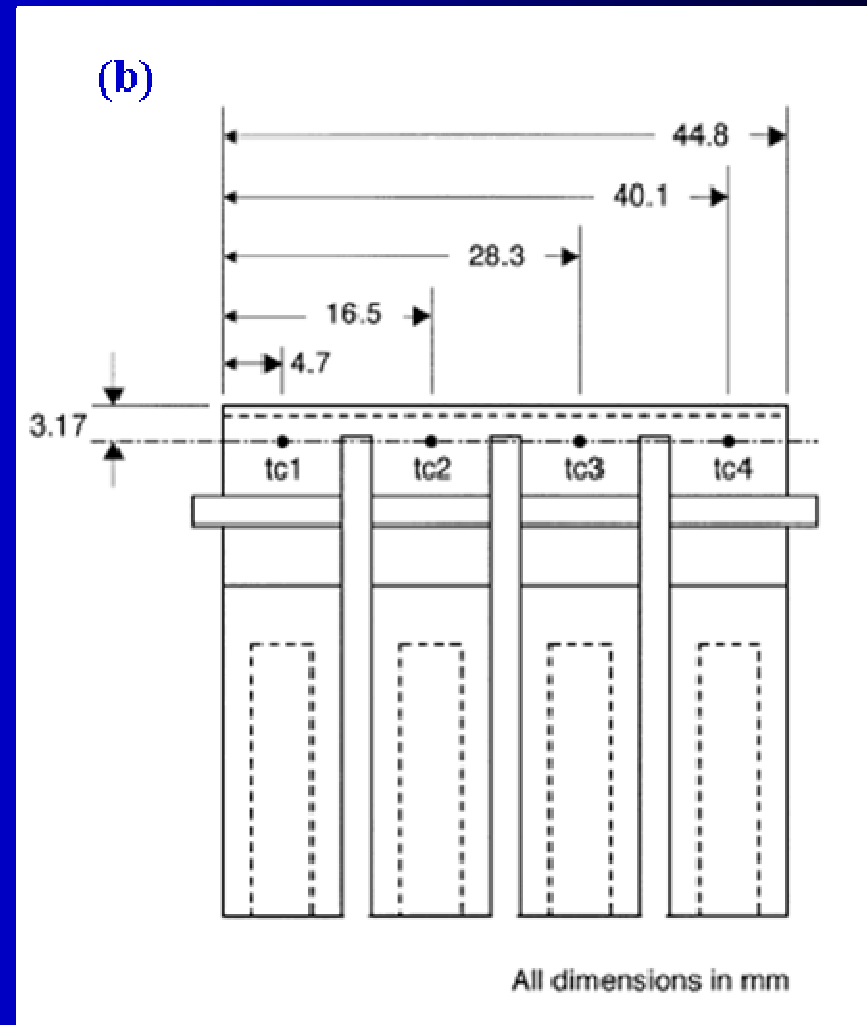
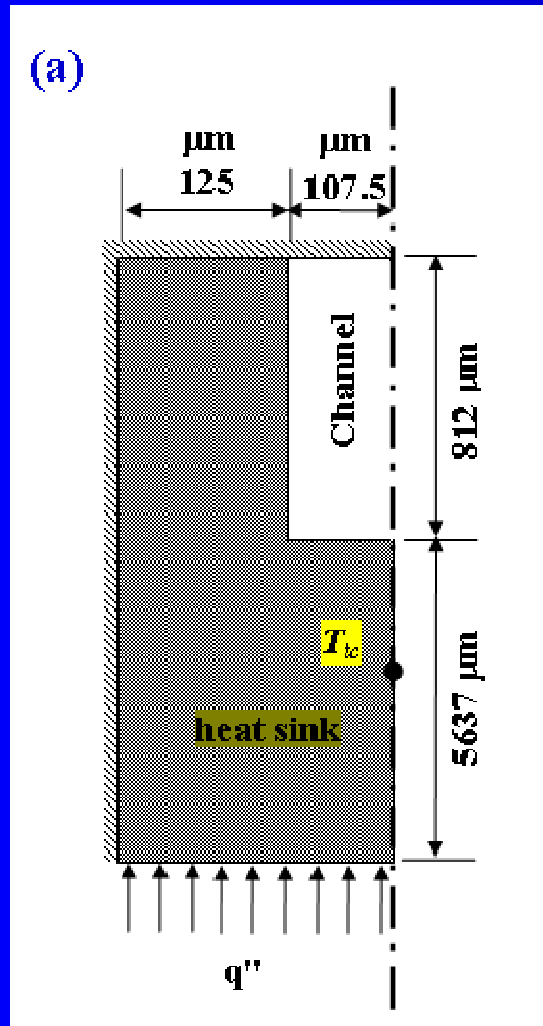
$$\rho_{nf} = (1 - \phi_p) \rho_l + \phi_p \rho_p$$

$$\mu_{nf} = \mu_l (123 \phi_p^2 + 7.3 \phi_p + 1)$$

$$k_{nf} = \left[ \frac{k_p + (n - 1)k_{bf} - (n - 1)\phi(k_{bf} - k_p)}{k_p + (n - 1)k_{bf} + \phi(k_{bf} - k_p)} \right] k_{bf}$$

J. Lee, I. Mudawar / International  
Journal of Heat and Mass Transfer  
50 (2007) 452–463

# Problem Definition

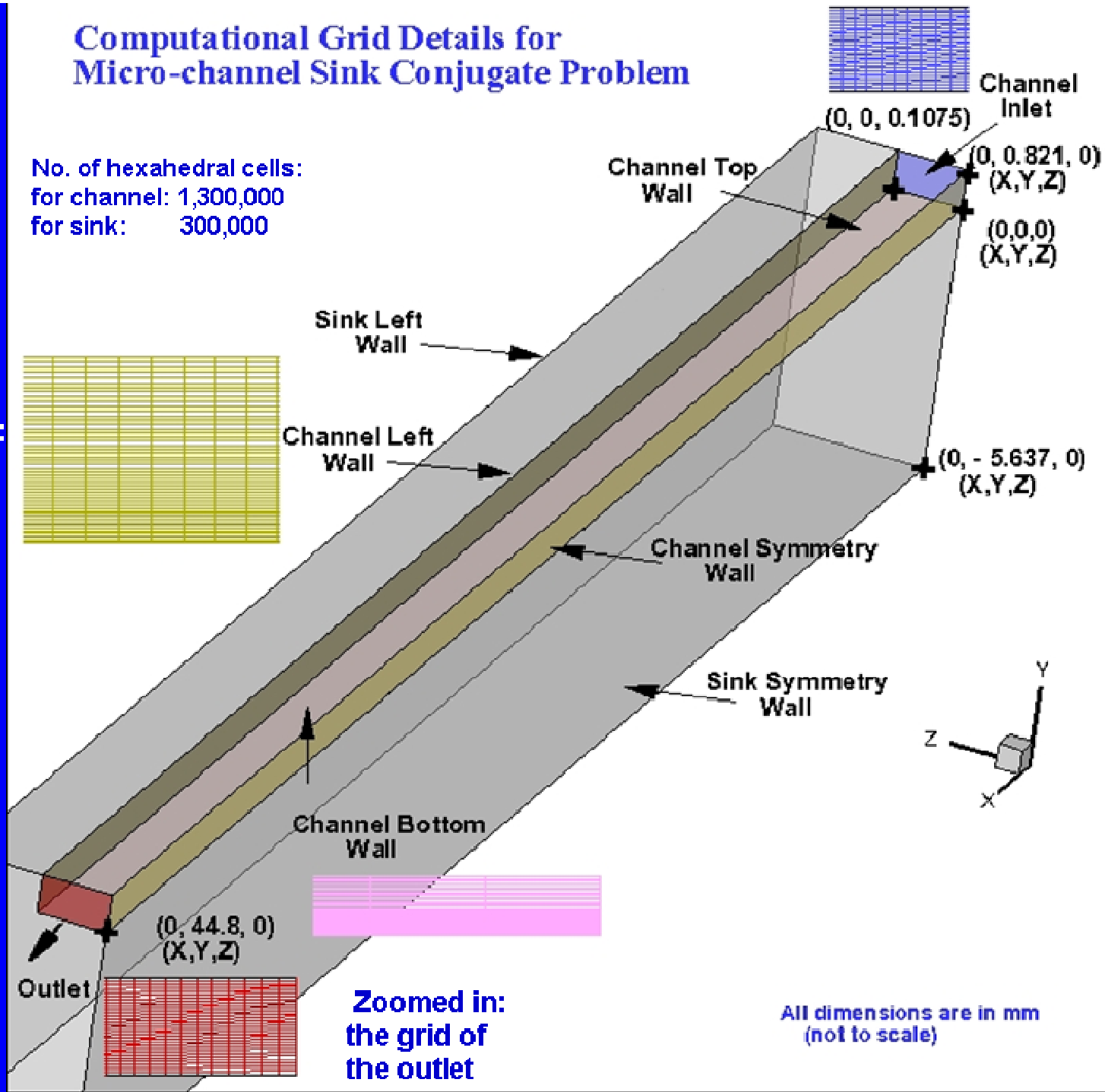
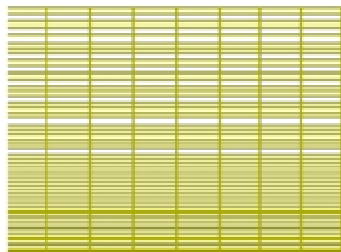


- (a) Micro-channel configuration considered in the present study;  
(b) Thermocouple locations in copper block inside the experimental module.

# Computational Grid Details for Micro-channel Sink Conjugate Problem

No. of hexahedral cells:  
for channel: 1,300,000  
for sink: 300,000

Zoomed in:  
the grid of  
the left wall



# Boundary Conditions (Channel)

➤ **At channel inlet** ( $x = 0$  plane )

*For both phases:  $u = V_o, v = w = 0$   $V_o$  is calculated from the flow Reynolds number  
 $T_i = 303.5$  K ,  $\phi_s = 0.01, 0.02, 0.05$*

➤ **At channel outlet** ( $x = 44.8$  mm plane )

*For both phases :pressure boundary condition ( $p = \text{constant}$ ) is assumed;*

$$\frac{\partial(\text{other variables})}{\partial x} = 0$$

➤ **At channel symmetry plane** ( $z = 0$  plane )

*For both phases:  $w = 0$  ,  $\frac{\partial(\text{other variables})}{\partial z} = 0$*

➤ **At channel bottom and side walls** ( $y = 0.0, z = 0.1075$  mm)

*For both phases:  $u = v = w = 0$  (no slip condition) ( $Kn < 0.001$ )The continuity of the temperature and heat flux are used as the conjugate boundary conditions to couple the energy equations for the fluid and solid phases.*

*A zero flux of the granular temperature,  $\Theta$ , is used at all walls*

➤ **At channel top walls** ( $y = 0.821$  mm)

*For both phases:  $u = v = w = 0$  (no slip condition) ( $Kn < 0.001$ )  
adiabatic boundary condition is specified.*

## Boundary Conditions (Heat sink)

- **At sink inlet** ( $x = 0$  plane );

**sink outlet** ( $x = 44.8$  mm plane )

**sink top wall** ( $y = 0.821$  mm plane)

**sink side wall** ( $y = 0.232$  mm plane)

*adiabatic boundary condition is specified ( $q'' = 0$ )*

- **At sink symmetry plane** ( $z = 0$  plane )

$$\frac{\partial(T)}{\partial z} = 0$$

- **At sink bottom wall** ( $y = -5.637$  mm)

*constant heat flux is specified ( $q'' = \text{constant}$ )*

- **At channel-sink interface walls** ( $y = 0$  plane ,  $z = 0.1075$  mm plane)

*The continuity of the temperature and heat flux are used as the conjugate boundary conditions to couple the energy equations for the fluid and solid phases.*

## CFD Flow-Solver Employed

- The implicit segregated algorithm included in the flow simulation package designated ANSYS FLUENT® V14.0 is used in the present study.
- This algorithm uses a control volume approach to solve sequentially both the conservation equations of phases and the constitutive equations needed for the Eulerian-Eulerian approach.
- The diffusive and convective terms are second order upwind differenced, whereas, the viscous terms are centrally differenced.
- The pressure interpolation is obtained using the PRESTO scheme.
- The Phase Coupled SIMPLE (PCSIMPLE) algorithm is used to achieve the coupling between the velocity and the pressure.
- Due to the non-linearity and the coupling of the governing equations, many iterations have been carried out to obtain a converged solution.
- A value of  $10^{-10}$  has been set as a criterion for the convergence of the flow and transport equations.

## Test cases

Cases	Volume concentration, $\phi_p$	m, g/s	Input heat Q, W
1 (a, b, c)	0, 0.01, 0.02	2.09	300
2 (a, b, c)	0, 0.02	5.49	300

## Thermo-physical properties

### Liquid phase: Water

$\rho_l$	density	995.7 kg/m <sup>3</sup>
$C_{p,l}$	Specific heat	4.183 kJ/kg.K
$k_l$	Thermal conductivity	0.603 W/m.K

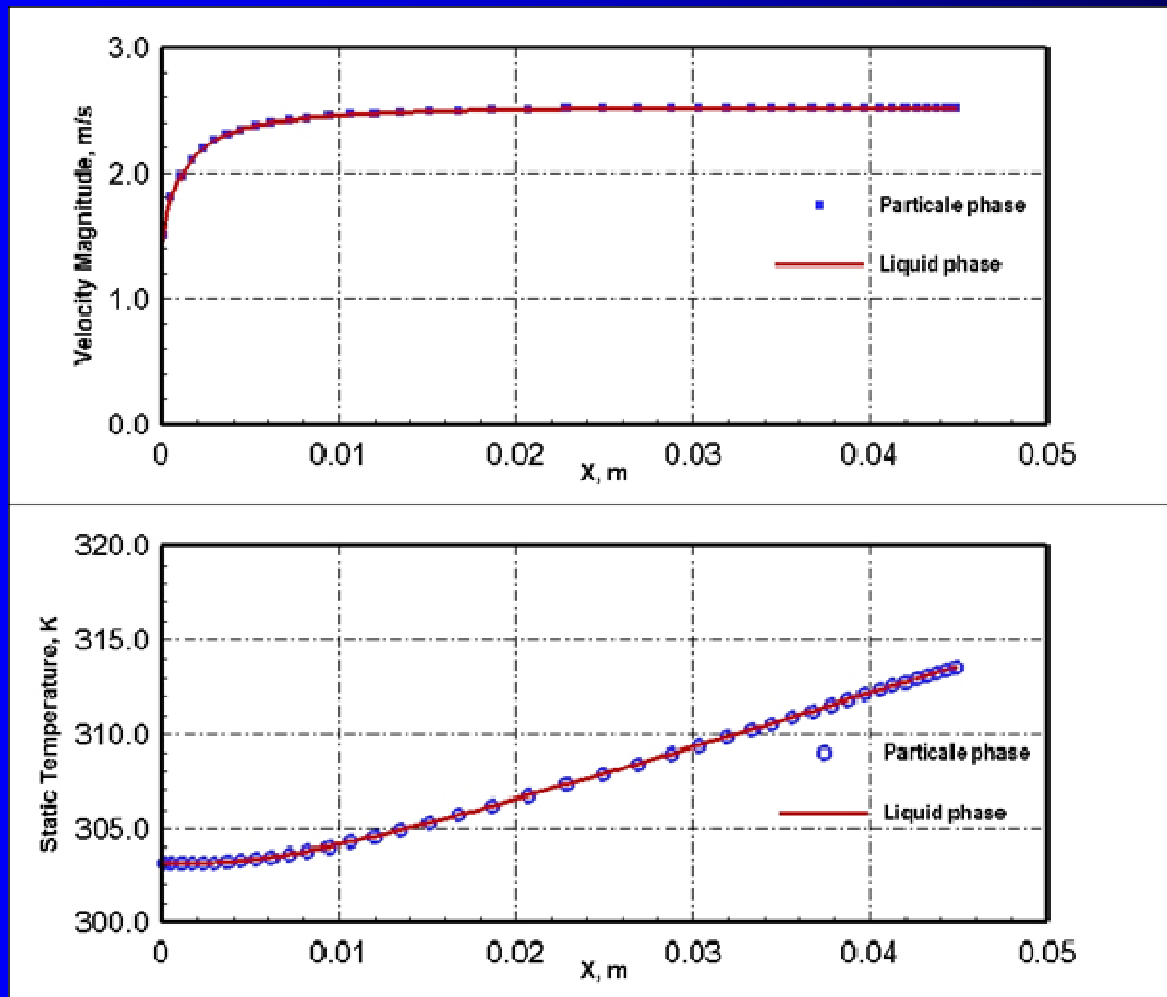
### Particle phase: Al<sub>2</sub>O<sub>3</sub>

$d_p$	diameter	40 nm
$\rho_p$	density	3600 kg/m <sup>3</sup>
$C_{p,p}$	Specific heat	0.675 kJ/kg.K
$k_p$	Thermal conductivity	36 W/m.K
$e$	Coefficient of restitution	0.95



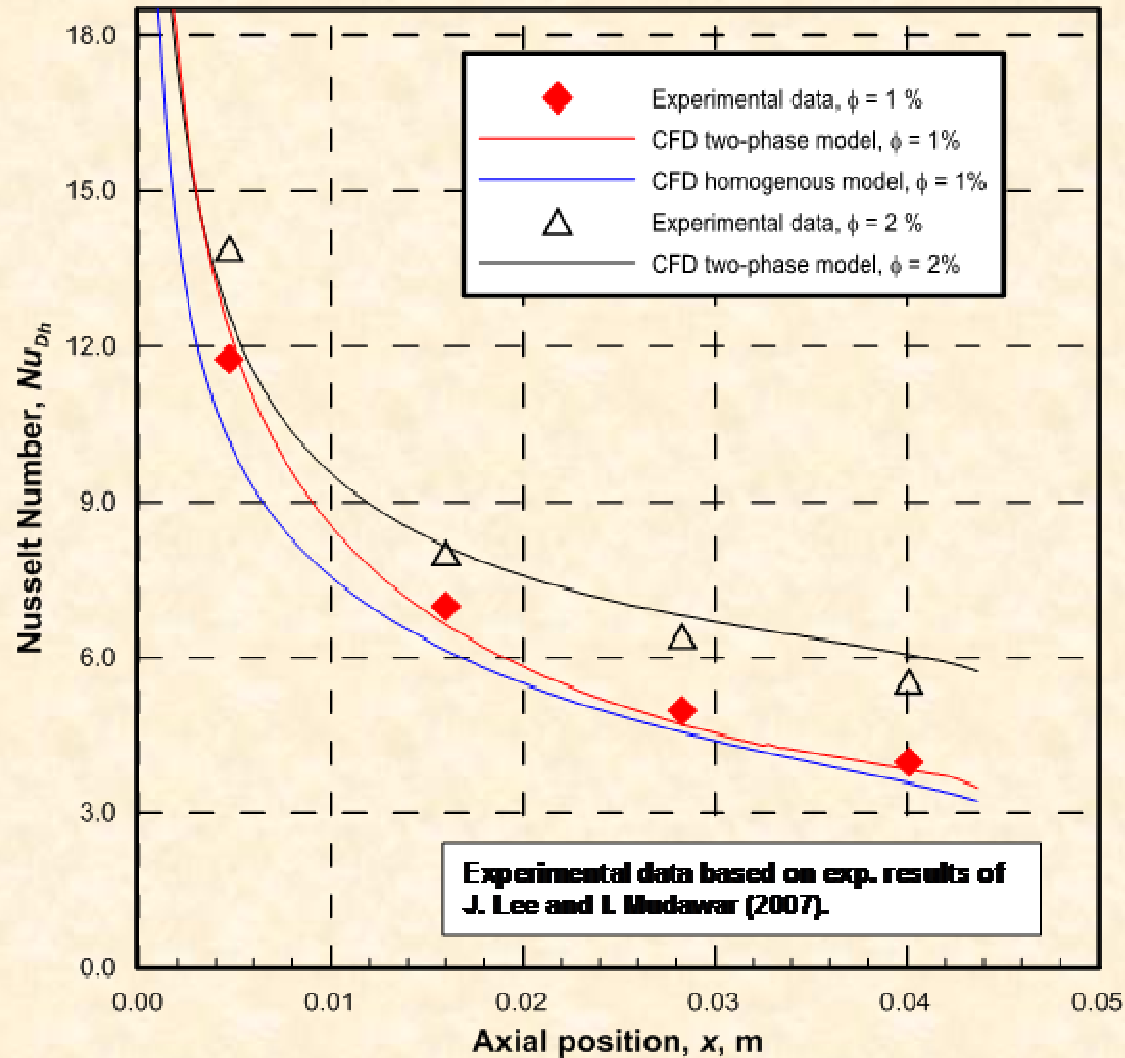
# Results and Analysis

# Velocity and Temperature along the middle of micro-channel Symmetry plane



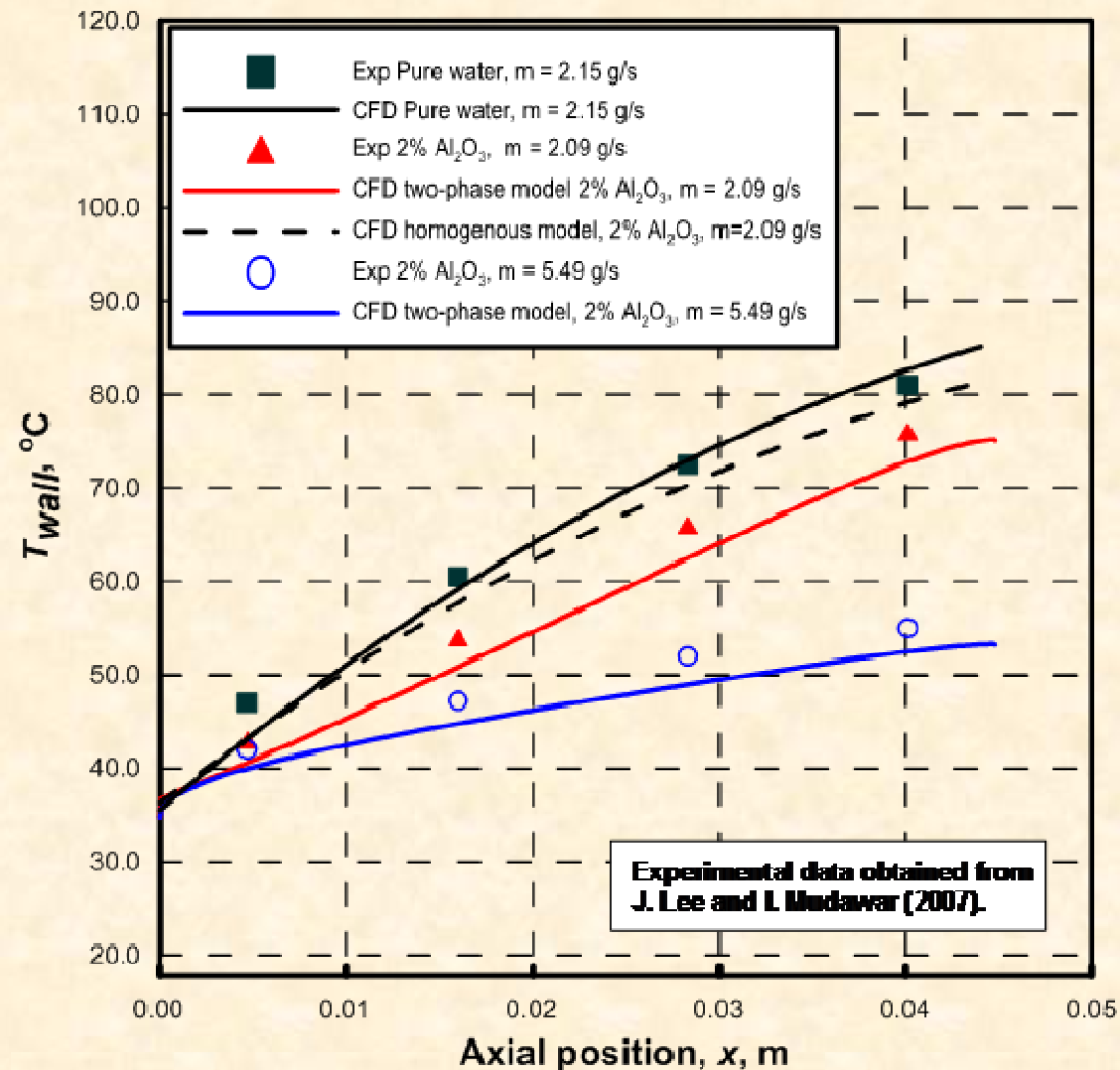
Velocity and temperature profiles for the phases of 2%  $\text{Al}_2\text{O}_3$ /water nanofluid using the Eulerian-Eulerian two-phase flow model for  $Re = 530$ ,  $m = 5.49 \text{ g/s}$ , and heat input ( $Q$ ) = 300W.

# Comparison of CFD results with experimental data



Variation of local Nusselt Number along the micro-channel flow axial direction for different  $\text{Al}_2\text{O}_3$  concentrations. ( $m = 5.49 \text{ g/s}$ ,  $Q=300 \text{ W}$ )

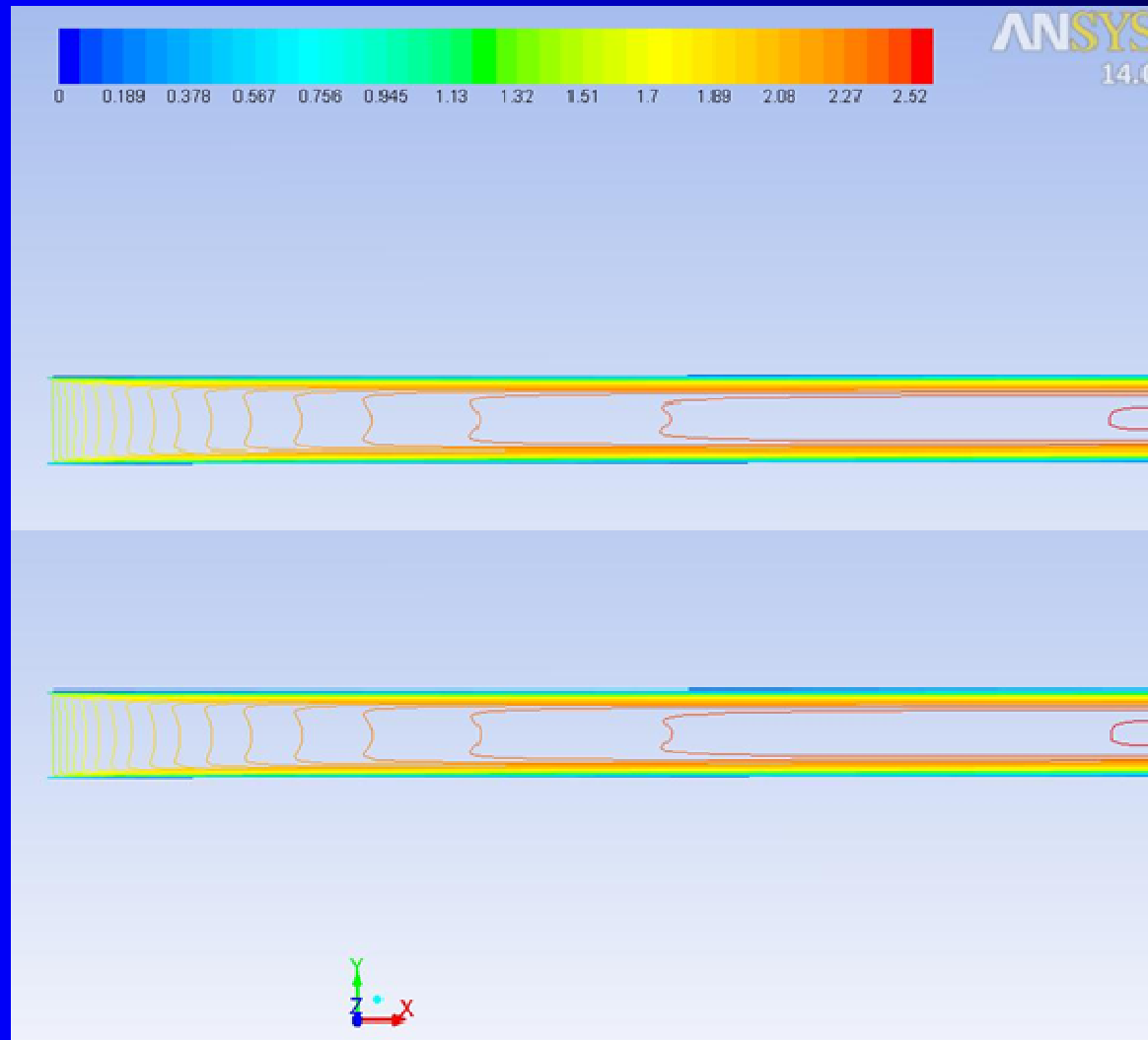
# Comparison of CFD results with experimental data



Variation of wall temperature along micro-channel axial flow direction at symmetry plane for different flow rates. ( $Q = 300W$ )

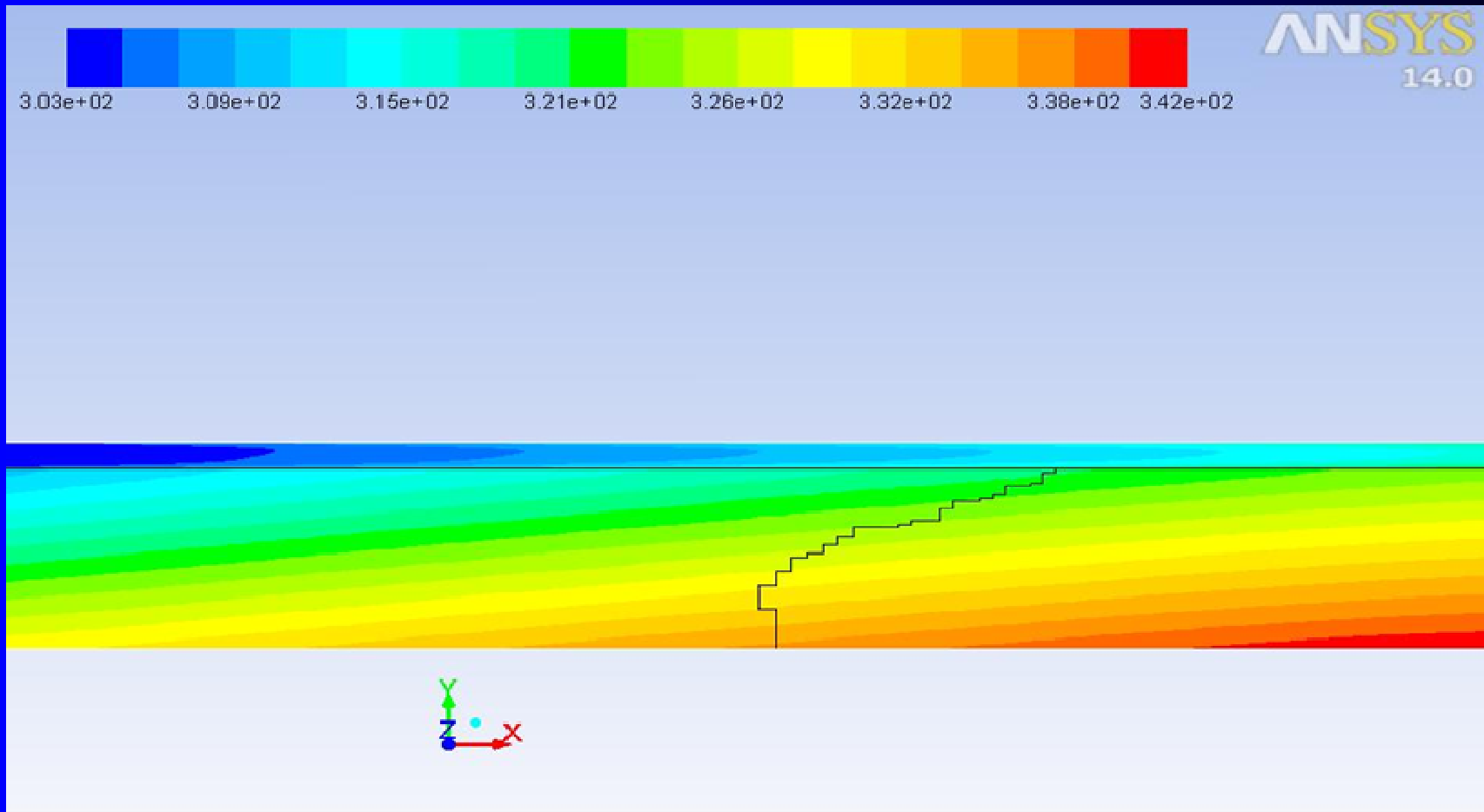
# Velocity contours (m/s) of the two-phases near the inlet region

$\phi = 2\% \text{ Al}_2\text{O}_3$  ,  $Q = 300 \text{ W}$  ,  $m = 5.49 \text{ g/s}$



# Static temperature contours (K) of the two-phases near the inlet region

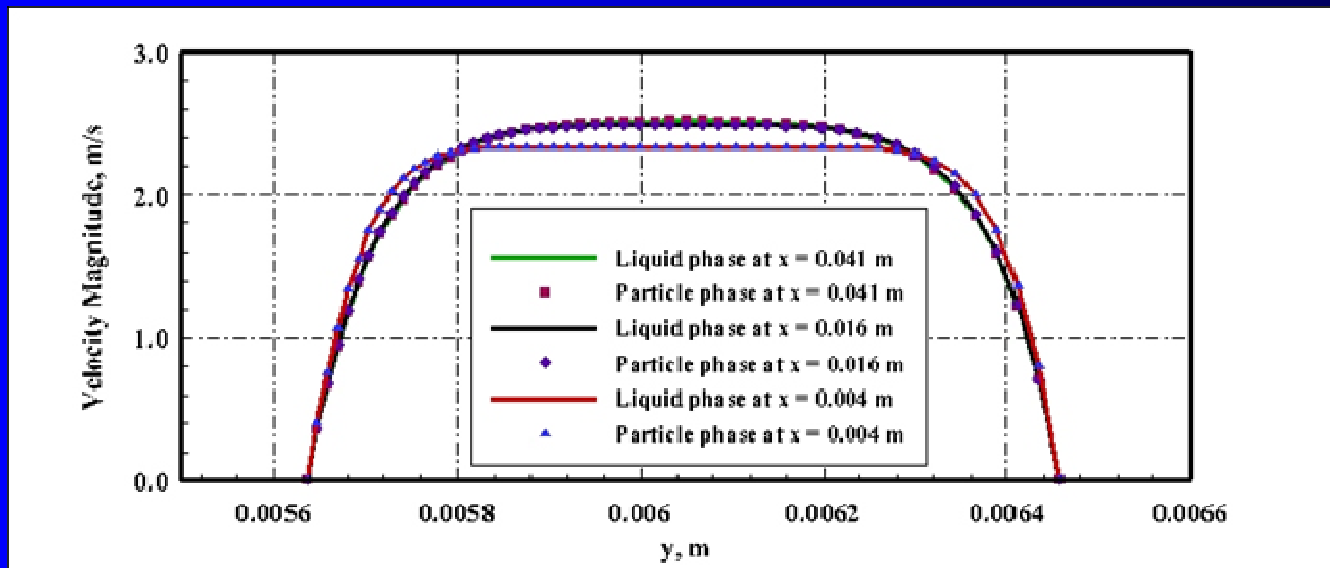
$$\phi = 2\% \text{ Al}_2\text{O}_3, Q = 300 \text{ W}, m = 5.49 \text{ g/s}$$



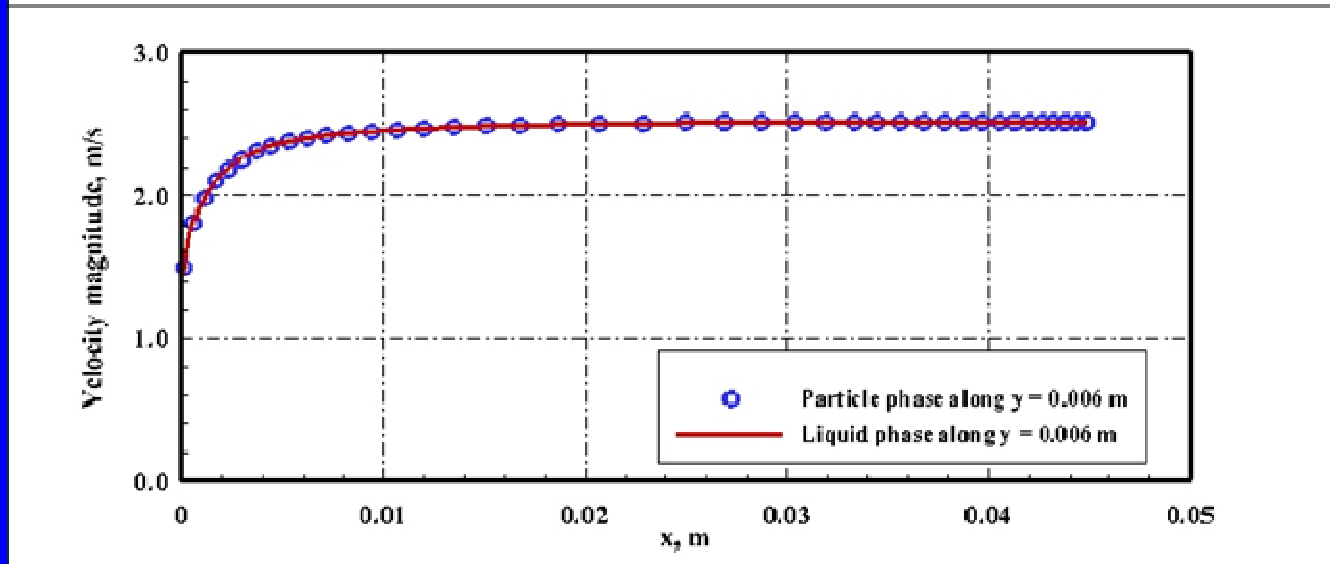
Contours of Static Temperature (phase-1) (K)

Sep 29, 2014  
ANSYS FLUENT 14.0 (3d, dp, pbns, eulerian, lam)

# Velocity magnitude of the two-phases at Symmetry plane $f = 2\% \text{ Al}_2\text{O}_3$ , $Q = 300 \text{ W}$ , $m = 5.49 \text{ g/s}$

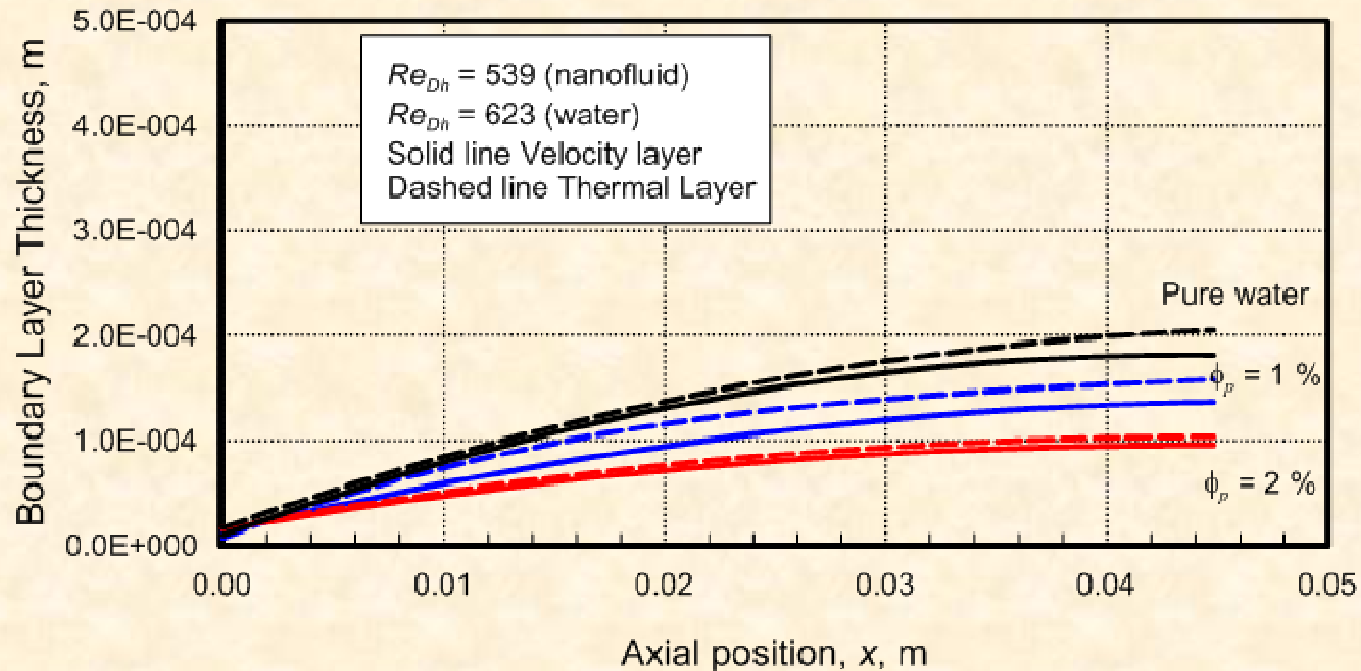


Liquid Phase  
water



Particle Phase  
 $\text{Al}_2\text{O}_3$

# Thermal and hydraulic Boundary Layers

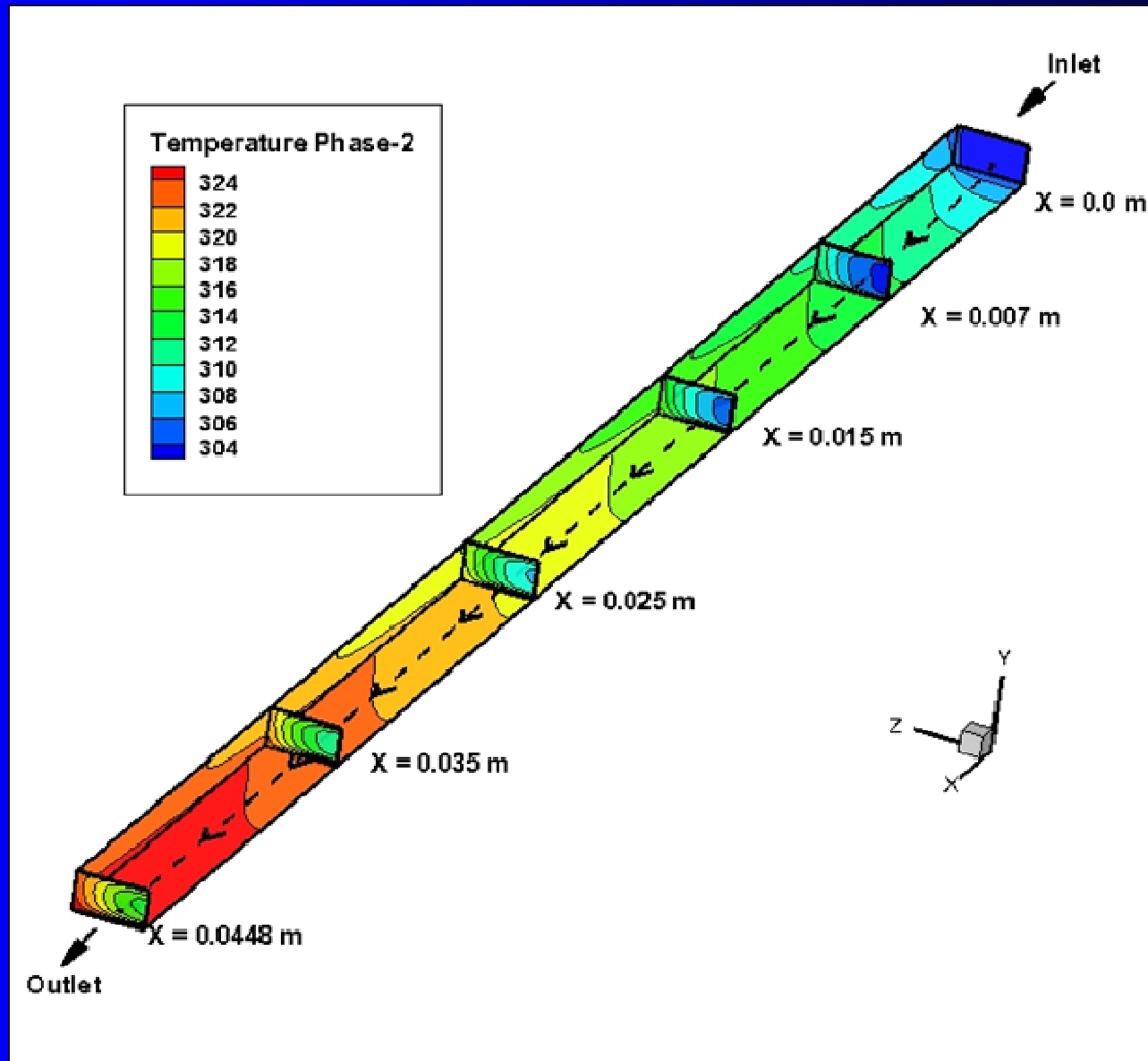


Thermal and Velocity boundary layer thickness along the micro-channel flow direction for different  $Al_2O_3$  concentrations ( $m = 5.49$  g/s,  $Q=300$  W).



# Temperature contours of the two-phases

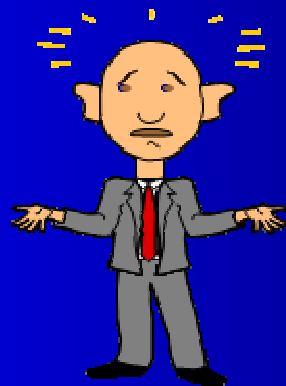
$$\phi = 2\% \text{ Al}_2\text{O}_3, Q = 300 \text{ W}, m = 5.49 \text{ g/s}$$



## Concluding Remarks

- Heat transfer due to alumina-water nanofluid flow inside a rectangular micro-channel heat sink is studied numerically using kinetic theory modeling.
- The two-phase Eulerian-Eulerian method is used to simulate the nanofluid flow. Also, homogeneous modeling is done to compare with the experimental and two-phase results.
- Computations are performed for pure water, 1%, and 2%  $\text{Al}_2\text{O}_3$ -water nanofluid at different Reynolds numbers in an open loop and a constant heat flux. It was found that the heat transfer enhancement increases with an increase in Reynolds number and nanofluid volume concentration.
- Comparing the experimental data, homogeneous and two-phase results, it was shown that the two-phase results are in better agreement with the experimental data.
- This implies that the two-phase modeling is more appropriate than the homogeneous model to simulate the nanofluid flow.
- However, based on the two-phase, results show that the velocity and temperature difference between the base liquid and nanoparticle phases is very small and negligible.
- Thus, the nanofluid can be considered as a homogeneous mixture; however, its analysis requires an appropriate modeling for the nanofluid thermophysical property based on kinetic theory. This remark deserves a further study.

THANK YOU



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