Mechanical Effects of Light: Radiation Pressure, Photon Momentum, and the Lorentz Force Law

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Radiation pressure is (partially) responsible for the tails of the comets pointing away from the Sun

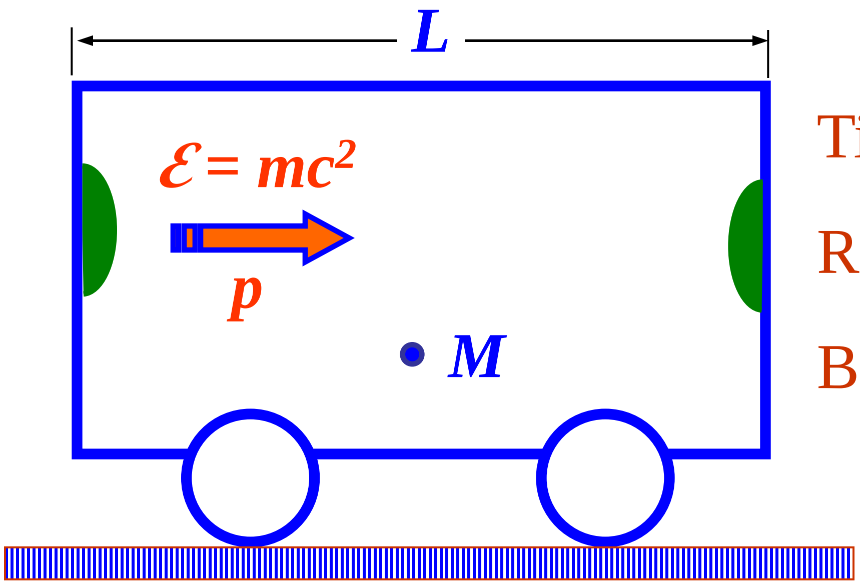
Johannes Kepler (1571-1630)

West (1976)

Kohoutek (1974)

First suggested by Johannes Kepler in his treatise "De Cometis." According to this hypothesis the solar ray pressure is responsible for the deflection of the comet tails. Although the observed deflections could not be explained solely on the basis of light pressure, this hypothesis played a significant role in understanding the effect of light pressure in the universe.

Einstein Box "Thought Experiment"



Time of flight = L/c

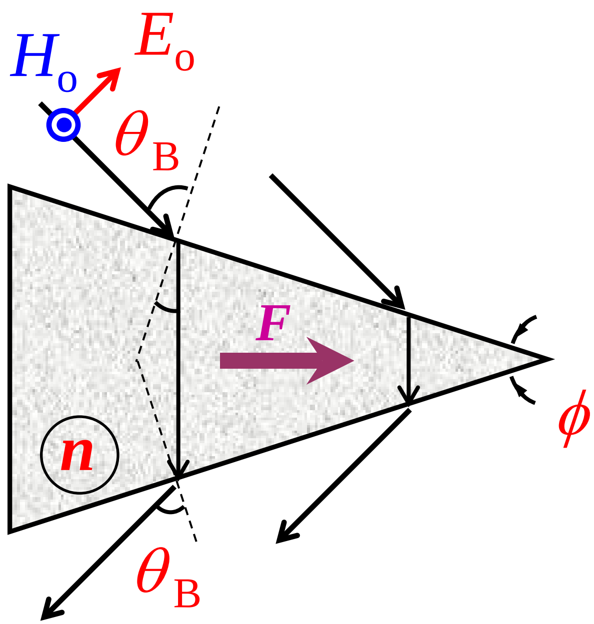
Recoil velocity = $-\,\boldsymbol{p}/M$

Box displacement = $-(\boldsymbol{p}/M)(L/c)$

Center-of-mass displacement = $(\mathcal{E}/c^2)L - M\,(p/M)(L/c) = 0$

$$\boldsymbol{p = \mathcal{E}/c}$$

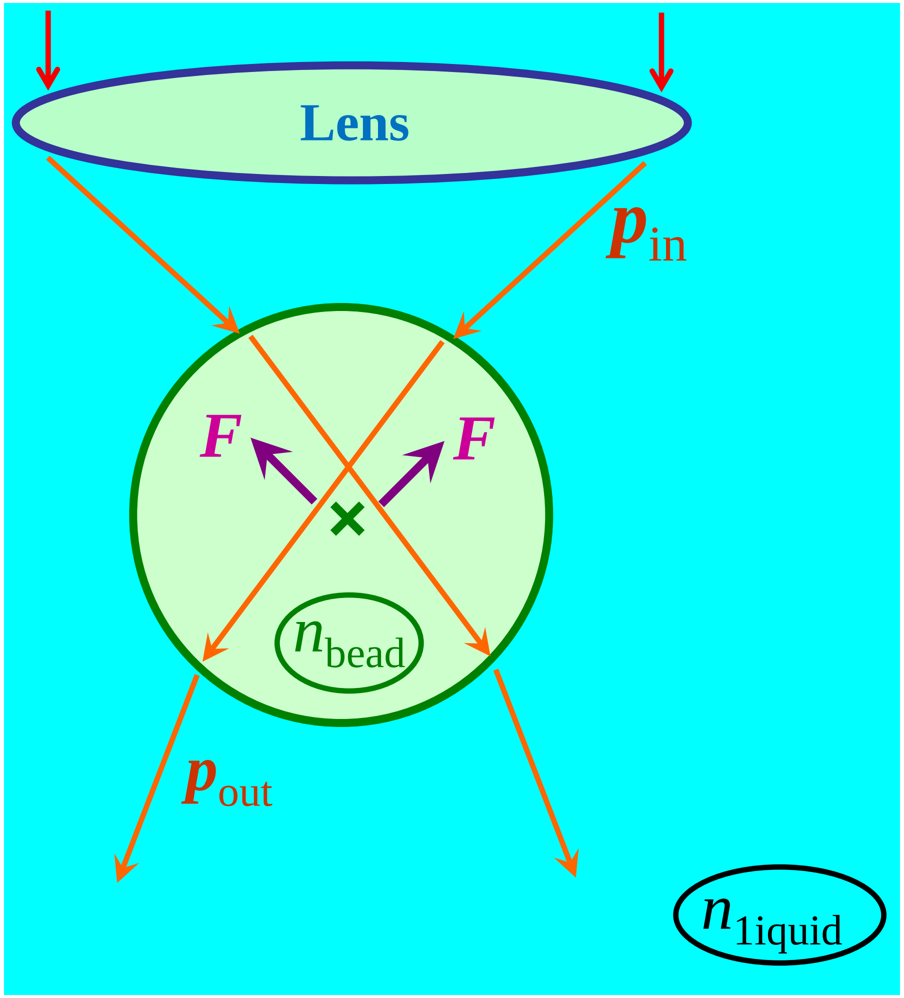
Radiation Pressure on Dielectric Wedge



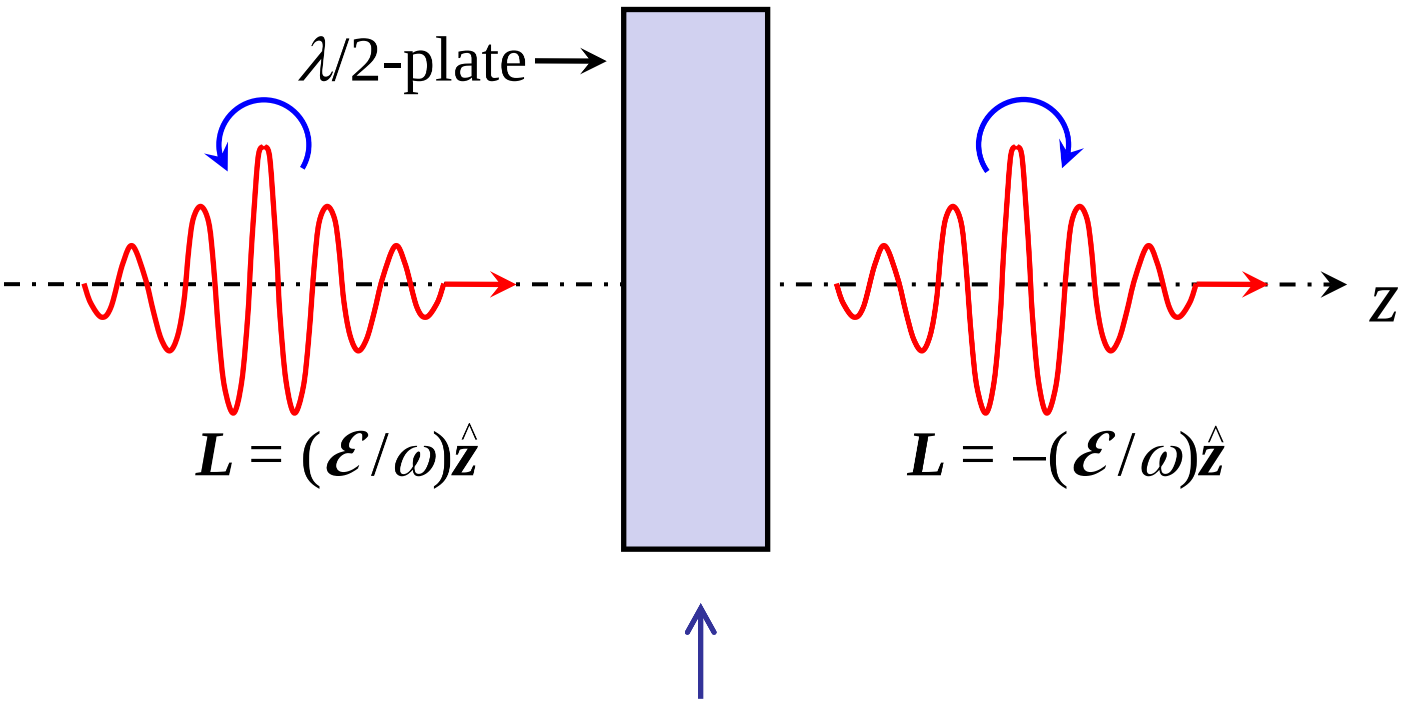
At Brewster's angle of incidence, where $\tan\theta_B = n$, reflectance of the surface for p-polarized light is exactly zero.

Optical tweezers

The first optical traps were built by Arthur Ashkin at AT&T Bell Labs in 1970. "Levitation traps" used the upward-pointing radiation pressure to balance the downward pull of gravity, whereas "two-beam traps" relied on counter-propagating beams to trap particles. Then, in 1986, Ashkin and colleagues realized that the gradient force alone would be sufficient to trap small particles. They used a single tightly focused laser beam to trap a transparent particle in three dimensions.



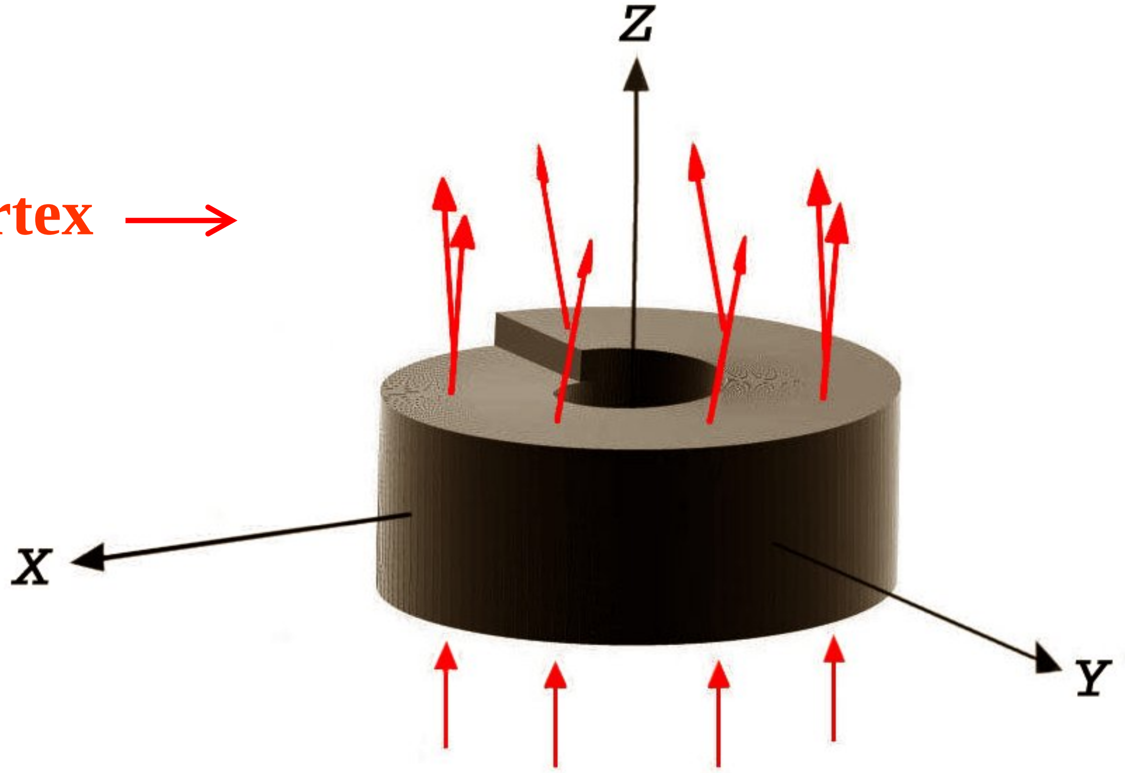
Circularly-polarized light passing through a half-wave plate



Birefringent crystal (transparent)

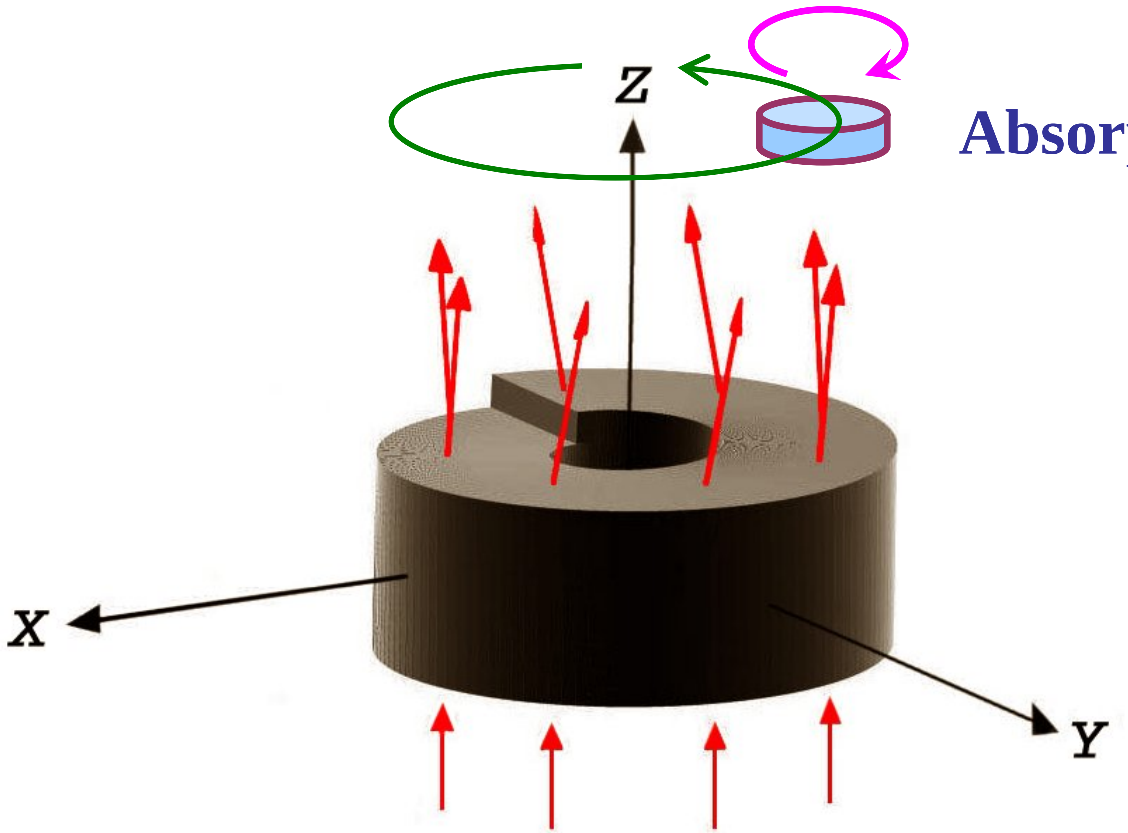
Collimated beam of light passing through a transparent spiral ramp

Optical Vortex



Emergent beam has (orbital) angular momentum

Spin and Orbital Angular Momentum



Circularly polarized beam of light

Feynman Lectures on Physics (Vol. II)

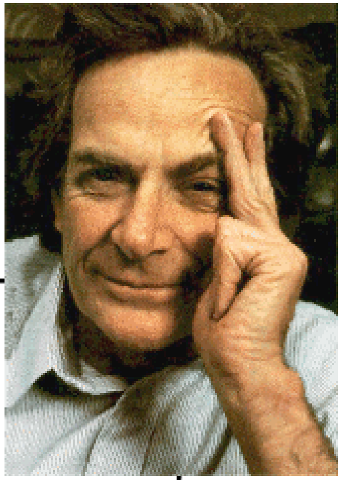


Table 18–1 Classical Physics

Maxwell's equations

I. $\nabla \cdot \boldsymbol{E} = \dfrac{\rho}{\epsilon_0}$ (Flux of $\boldsymbol{E}$ through a closed surface) = (Charge inside)/ϵ_0

II. $\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$ (Line integral of $\boldsymbol{E}$ around a loop) = $-\dfrac{d}{dt}$(Flux of $\boldsymbol{B}$ through the loop)

III. $\nabla \cdot \boldsymbol{B} = 0$ (Flux of $\boldsymbol{B}$ through a closed surface) = 0

IV. $c^2 \nabla \times \boldsymbol{B} = \dfrac{\boldsymbol{j}}{\epsilon_0} + \dfrac{\partial \boldsymbol{E}}{\partial t}$ c^2(Integral of $\boldsymbol{B}$ around a loop) = (Current through the loop)/ϵ_0 $+\dfrac{\partial}{\partial t}$(Flux of $\boldsymbol{E}$ through the loop)

[Conservation of charge

$\nabla \cdot \boldsymbol{j} = -\dfrac{\partial \rho}{\partial t}$ (Flux of current through a closed surface) = $-\dfrac{\partial}{\partial t}$(Charge inside)]

Force law

$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

Law of motion

$\dfrac{d}{dt}(\boldsymbol{p}) = \boldsymbol{F}$, where $\boldsymbol{p} = \dfrac{m\boldsymbol{v}}{\sqrt{1 - v^2/c^2}}$ (Newton's law, with Einstein's modification)

Gravitation

$\boldsymbol{F} = -G\dfrac{m_1 m_2}{r^2}\boldsymbol{e}_r$

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Trouble with the Lorentz Law of Force: Incompatibility with Special Relativity and Momentum Conservation

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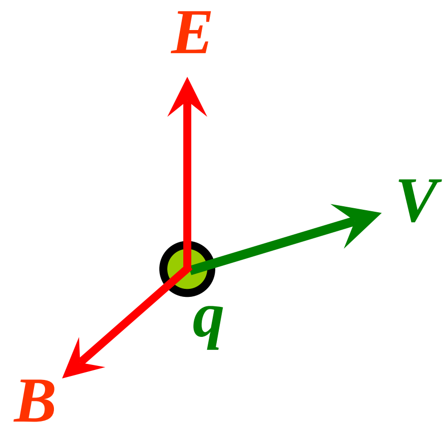
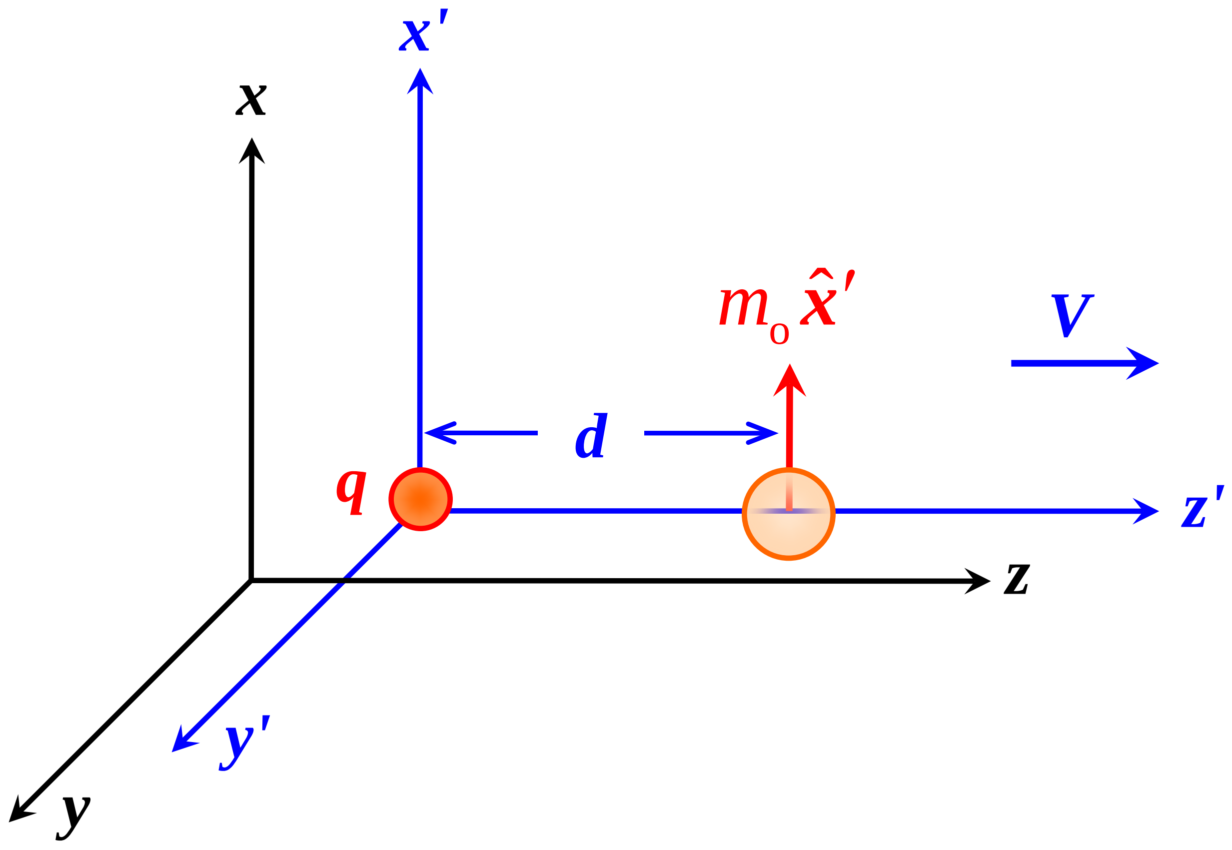
Hendrik Lorentz
(1853-1928)

$$\boldsymbol{f} = q\,(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$$



Charge-Dipole Paradox



In the rest frame $x'y'z'$ there is neither force nor torque acting on either particle.

In the moving xyz frame a torque $\boldsymbol{T} = (Vqm_o/4\pi d^2)\,\hat{\boldsymbol{x}}$ acts on the magnetic dipole.

The Einstein-Laub Force and Torque Density Equations

Albert Einstein (1879-1955)

$$\boldsymbol{F}(\boldsymbol{r},t) = \rho_{\text{free}}\boldsymbol{E} + \boldsymbol{J}_{\text{free}} \times \mu_{\text{o}}\boldsymbol{H} + (\boldsymbol{P}\cdot\nabla)\boldsymbol{E} + (\partial\boldsymbol{P}/\partial t)\times\mu_{\text{o}}\boldsymbol{H} + (\boldsymbol{M}\cdot\nabla)\boldsymbol{H} - (\partial\boldsymbol{M}/\partial t)\times\varepsilon_{\text{o}}\boldsymbol{E}$$

$$\boldsymbol{T}(\boldsymbol{r},t) = \boldsymbol{r}\times\boldsymbol{F}(\boldsymbol{r},t) + \boldsymbol{P}(\boldsymbol{r},t)\times\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{M}(\boldsymbol{r},t)\times\boldsymbol{H}(\boldsymbol{r},t)$$

In the rest frame *x'y'z'*, and also in the moving frame *xyz*, there is neither force nor torque acting on either particle.

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NEWS&ANALYSIS

PHYSICS

Textbook Electrodynamics May Contradict Relativity

A basic equation of electricity and magnetism is wrong, one scientist claims. The classic formula for the force exerted by electric and magnetic fields—the so-called Lorentz force—clashes with Einstein's special theory of relativity, says Masud Mansuripur, an electrical engineer at the University of Arizona in Tucson. Others doubt the claim but have not found a flaw in the simple argument that challenges century-old textbook physics.

"If it's true, it's astonishing," says Stephen Barnett, a theorist at the University of Strathclyde in Glasgow, U.K. "I suspect there is something subtle going on here" that doesn't contradict relativity. But Rodney Loudon, a theorist retired from the University of Essex in the United Kingdom, says, "As far as I can tell, [the analysis] is right."

The Lorentz force formula describes how electric and magnetic fields push around a charged particle. The electric field pushes the particle with a force proportional to the particle's charge and the field's strength. (A negatively charged particle feels a pull.) The magnetic field shoves the particle sideways in a direction perpendicular to both the field and the particle's velocity. That magnetic force is proportional to the charge, the velocity, and the field strength.

Ironically, physicists invoke the Lorentz force in the textbook example of how electrodynamics and relativity mesh. A positively charged particle moves parallel to a wire carrying current in the same direction (see figure, top left). The current produces a magnetic field that wraps around the wire. As the particle crosses the field, it feels a magnetic force pulling it toward the wire.

From the particle's perspective, things look very different. In that "frame of reference," the particle stands still while the wire moves. The wire still exudes a magnetic field, but because the particle has no velocity it feels no magnetic force. Yet relativity demands that if an observer in one frame of reference sees a force, an observer in another frame should see an equal force.

A contradiction? Not quite, thanks to special relativity's weird prediction that observers moving at different speeds perceive lengths differently. Those lengths include the distances between the positively charged ions that form the wire and the negatively charged electrons that flow to produce the current. In the lab frame, the wire is stationary, and the ions and electrons are equally spaced. In the particle's frame, however, the wire moves and its ions appear more closely spaced than they are in the lab frame. But the oncoming electrons move faster still and appear even closer together. The wire thus has a net negative charge (see figure, top right). That charge draws the particle with *electric* force equal to the magnetic force seen in the lab frame. Paradox averted.

Now, an equally simple example shows how the Lorentz force trips up when applied to magnetic particles, Mansuripur argues in a paper in press at *Physical Review Letters*. A charged particle and a tiny magnet sit apart in the lab frame (see figure, bottom left). The uncharged magnet cannot feel the charged particle's electric field, and the motionless particle cannot feel the magnet's magnetic field. So no forces are at work.

Now consider how things appear to an observer in a "moving frame" in which the magnet and the charge glide past together (figure, bottom right). The magnet appears to be electrically polarized, with a positive charge on one side and a negative charge on the other. That's because in classical electrodynamics, magnetism originates from hypothetical loops of "bound" current within a material. So the magnet is equivalent to a ring of wire carrying current in a circle. As the ring coasts by the observer, contraction effects will redistribute the charges in it just as they did in the current-carrying wire in the first example. On the side of the loop in which current flows in the same direction as the loop's motion, a positive charge appears. On the other side, a negative charge appears.

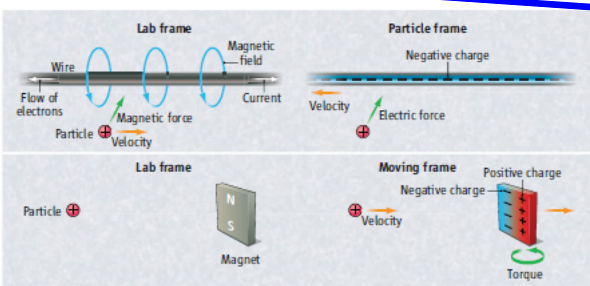
The charged particle interacts with these charges, pulling on one side of the magnet and pushing on the other to create a twisting "torque." The moving charge also produces a magnetic field, but that field does not counteract the twisting. So there's a net torque not seen in the lab frame, Mansuripur calculates. That violates relativity.

There is a way out, Mansuripur says: No torque appears in either frame if he uses a more complicated formula for forces in polarized and magnetized materials that Einstein and Jakob Laub proposed in 1908 but Einstein later repudiated. Some theorists say that's fine with them. "Einstein-Laub is correct—shock and horror!" says Daniel James of the University of Toronto in Canada.

But there's a deeper issue. In classical electrodynamics, physicists assume that magnetization and polarization originate in microscopic bound currents and charges within materials. If that's true and the Lorentz formula is correct on the microscopic level, then they must apply it to macroscopic materials, too, and run afoul of relativity, Mansuripur argues. So, he says, physicists must scrap bound charges and currents and consider polarization and magnetization fundamental entities themselves.

Them's fighting words to some. "The microscopic picture of electrodynamics is clear," James says, "and if the macroscopic picture of electrodynamics doesn't follow from that, I'd be surprised." Somehow, the Einstein-Laub equation for macroscopic materials must follow from the Lorentz force applied on the microscopic level, he says. Barnett says "there's going to be a heated debate about this result." Undoubtedly.

–ADRIAN CHO



Hit and miss. Simple examples show how the Lorentz force jibes (*top*) and clashes with relativity.

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Foundations of Classical Electrodynamics

1) There is more to Maxwell's ***macroscopic*** equations than meets the eye. Take them seriously. Make them the starting point of every investigation in classical electrodynamics.

2) The most important thing you will need to know about EM energy is that the Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)=\boldsymbol{E}\times\boldsymbol{H}$ is the rate of flow of energy (per unit are per unit time). Everything else about energy follows from this postulate in conjunction with Maxwell's macroscopic equations.

3) Momentum density of EM fields is $\boldsymbol{p}(\boldsymbol{r},t)=\boldsymbol{S}/c^2$. This is *always* true, in vacuum as well as in material media, irrespective of the nature of the media.

4) Angular momentum density of EM fields is always $\boldsymbol{L}(\boldsymbol{r},t)=\boldsymbol{r}\times\boldsymbol{S}/c^2$. This is true of spin as well as orbital angular momentum of EM waves.

5) If you use the Lorentz force law $\boldsymbol{f}=q(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B})$, you will get into trouble: you will find that momentum is not conserved and special relativity is violated. Use the **Einstein-Laub law** instead!