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About OMICS Group Conferences

OMICS Group International is a pioneer and leading science event organizer, which publishes around 400 open access journals and conducts over 300 Medical, Clinical, Engineering, Life Sciences, Pharma scientific conferences all over the globe annually with the support of more than 1000 scientific associations and 30,000 editorial board members and 3.5 million followers to its credit.

OMICS Group has organized 500 conferences, workshops and national symposiums across the major cities including San Francisco, Las Vegas, San Antonio, Omaha, Orlando, Raleigh, Santa Clara, Chicago, Philadelphia, Baltimore, United Kingdom, Valencia, Dubai, Beijing, Hyderabad, Bengaluru and Mumbai.

The Theoretical Optimization of a Cylindrical Body of Rotation Using Magnus Effect Lift

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Background

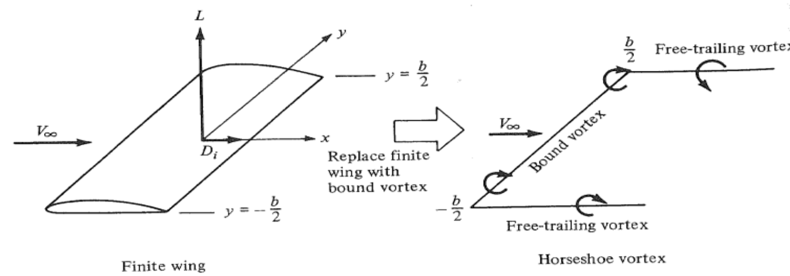
- Kutta-Joukowski theorem gives lift as a function of circulation
- Prandtl's lifting line theory leads to an optimized circulation distribution
- Robins-Magnus effect produces lift via friction driven circulation

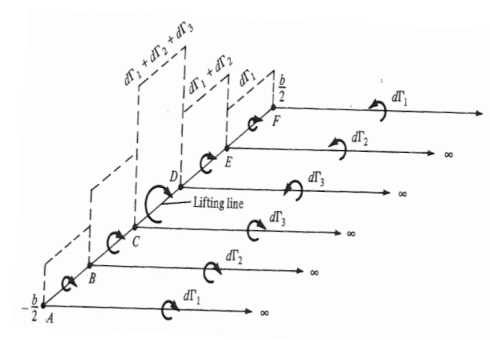
$$\mathrm{L} = \rho_\infty V_\infty \Gamma$$

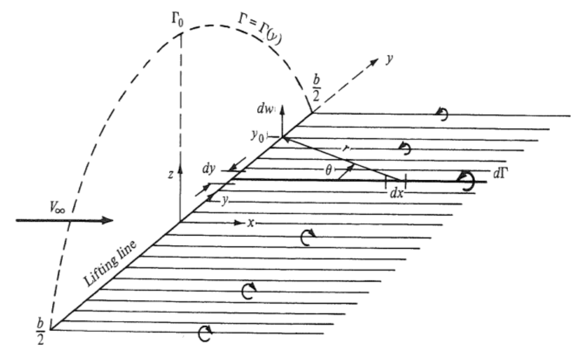
$$\Gamma(\mathrm{y}) = \Gamma_0 \sqrt{1 - \left(\frac{2\mathrm{y}}{\mathrm{b}}\right)^2}$$

Lifting Line Theory

- Finite wing represented by horseshoe vortex
- Superposition of a finite number of horseshoe vortices
- Superposition of an infinite number of horseshoe vortices yielding a distribution of circulation







Figures from Anderson's *Fundamentals of Aerodynamics*

Lifting Line Theory cont.

- Circulation distribution

$$\Gamma = \oint_C \boldsymbol{V} \cdot \boldsymbol{ds}$$

$$\Gamma(y) = \oint_{C(y)} \boldsymbol{V}(y) \cdot \boldsymbol{ds}(y)$$

- Lift from integration

$$\mathrm{L} = \rho_\infty V_\infty \int_{-b/2}^{b/2} \Gamma(y) dy$$

Minimum Induced Drag

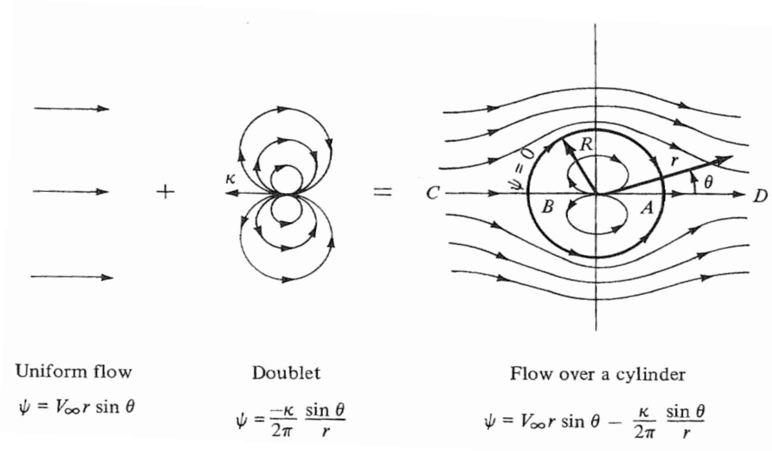
- Induced drag coefficient from lifting line theory
  - Span efficiency factor
    - General circulation distribution
      $0 \le e \le 1$
    - Elliptical circulation distribution
      $e = 1$
- Munk derived the theoretically optimum chord distribution for a finite wing

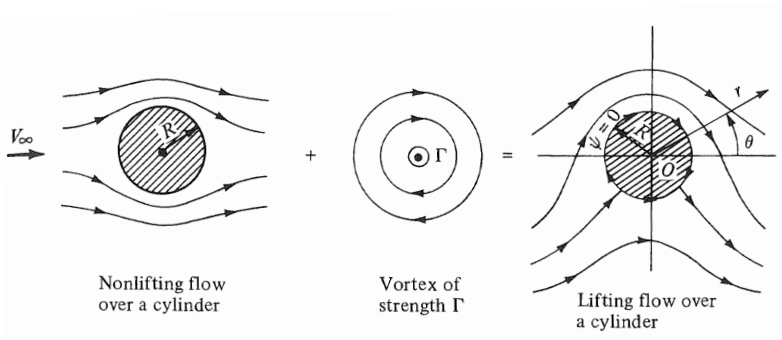
$$C_{D_i} = \frac{{C_L}^2}{\pi A R e}$$

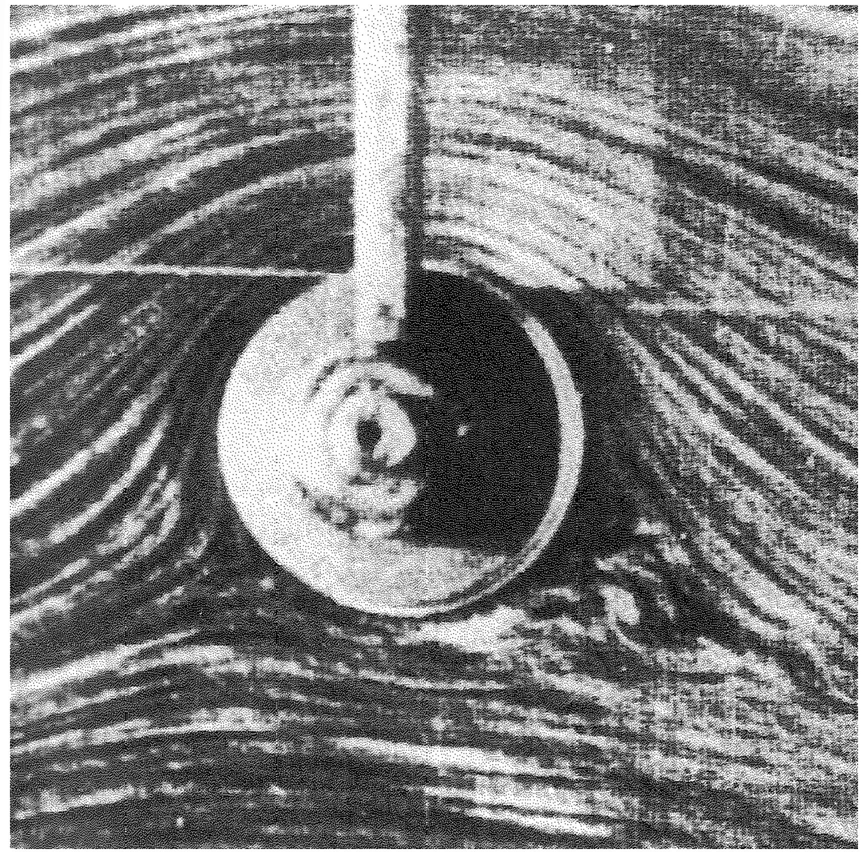
$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

$$c(y) = c_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

Robins-Magnus Effect





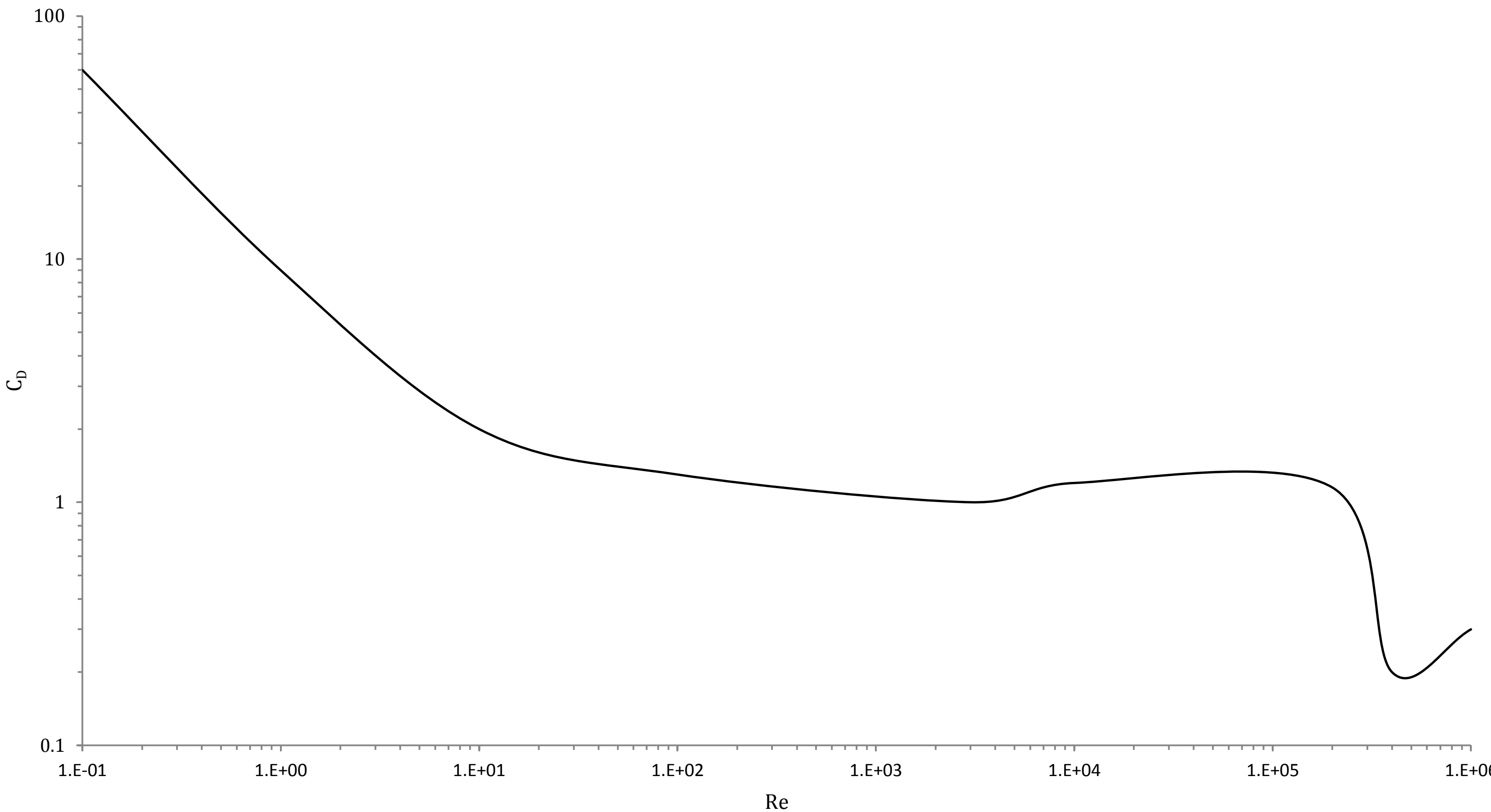


Figures from Anderson's *Fundamentals of Aerodynamics* and Prandtl's and Tietjen's *Applied Hydro- and Aeromechanics*

Current State of the Art

- Nonrotating infinite cylinders
- Rotating 2D cylinders

Nonrotating Circular Cylinders



Rotating Circular Cylinders

- Prandtl's theoretical maximum lift coefficient

$$C_{L_{max}} = 4\pi$$

- Experiment has shown higher values

$$C_{L_{max}} \sim 15.4$$

- Numerical studies have shown values even higher

$$C_{L_{max}} \sim 34$$

Rotating Circular Cylinders

Definitions:

- Uniform stream: V_∞
- Tangential velocity of the cylinder's surface: V_r
- Ratio of surface velocity to uniform stream: α

$$\alpha = \frac{V_r}{V_\infty}$$

Rotating Circular Cylinders

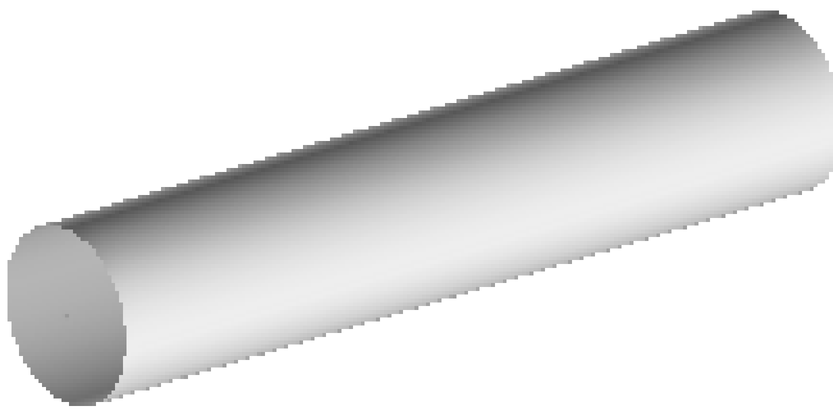
- Studies to date have been two dimensional
  - Analytical:
    - 2-D
  - Experimental:
    - Full span
    - End plates
  - Numerical:
    - 2-D
    - End plates

Researcher(s)	$C_{L_{max}}$	$(L/D)_{max}$	Re	α	Investigation Type
Prandtl	4π	-	-	2	Analytical, 2D
Reid	9.5*	7.8	4×10^4	2.5	Experimental, 2D
Ou & Burns	6.25	4.6	2×10^2	2.38	Numerical, 2D
Karabelas *et al.*	2.3*	4.7	5×10^6	2	Numerical, 2D
Chew, Cheng, &Luo	9.1	4	1×10^3	2	Numerical, 2D
Stojkovic, Breur, & Durst	$2\pi\alpha$	-	1×10^2	≥ 5.5	Numerical, 2D
Tokumaru & Dimotakis	15.4*	-	3.8×10^3	10	Experimental, 2D
Mittal & Kumar	27	-	2×10^2	5	Numerical, 2D
Padrino & Joseph	34	-	4×10^2	6	Numerical, 2D

* This was the highest achieved in testing but no absolute maximum was identified.

Rotating Circular Cylinders

- Constant Diameter Circular Cylinders (CDCC)



Theoretical Goal

The goal is to theoretically optimize the lifting, rotating cylinder by developing the three dimensional cylindrical geometry that will create an elliptical circulation distribution.

Bodies of Revolution

- Spheroids were considered first
  - Elliptic planform
  - Elliptic velocity distribution at the surface due to rotation

however…

Bodies of Revolution cont.

- When beginning with a prolate spheroid geometry:

$$r(y) = r_0\sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

$$\Gamma(0) = \Gamma_0 = \oint_{C(0)} \boldsymbol{V}(0) \cdot \boldsymbol{ds}(0)$$

$$= \omega r_0 \cdot 2\pi r_0 = 2\pi\omega {r_0}^2$$

$$\Gamma(y) = \oint_{C(y)} \boldsymbol{V}(y) \cdot \boldsymbol{ds}(y)$$

$$= \omega r(y) \cdot 2\pi r(y) = 2\pi\omega r(y)^2$$

$$= 2\pi\omega {r_0}^2\left[1 - \left(\frac{2y}{b}\right)^2\right]$$

$$= \Gamma_0\left[1 - \left(\frac{2y}{b}\right)^2\right]$$

Parabolic circulation distribution

Bodies of Revolution cont.

- Must begin with the elliptic circulation distribution and solve for the appropriate geometry:

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

$$\Gamma(y) = 2\pi\omega r(y)^2$$

$$2\pi\omega r(y)^2 = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

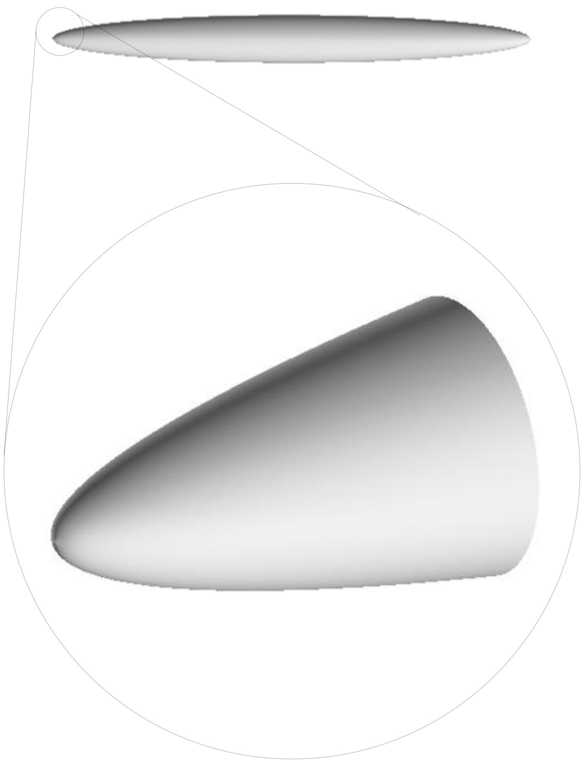
$$r(y) = r_0 \left[1 - \left(\frac{2y}{b}\right)^2\right]^{\frac{1}{4}}$$

$$\left(\frac{r(y)}{r_0}\right)^4 + \left(\frac{y}{b/2}\right)^2 = 1$$

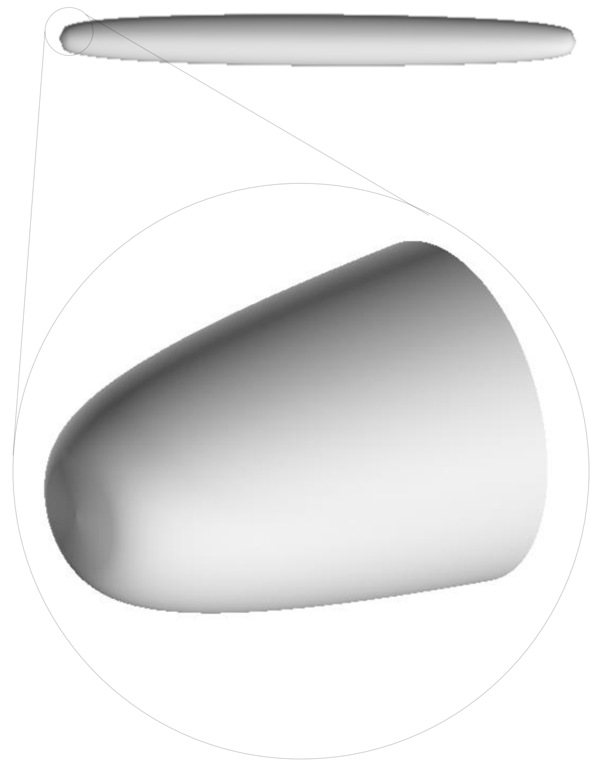
Biquadratic Body of Revolution (BBOR)

Bodies of Revolution cont.

Prolate Spheroid



Biquadratic Body of Revolution



Circulation Distributions