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Modeling and Applications of *FGMs* in Aerospace Structures

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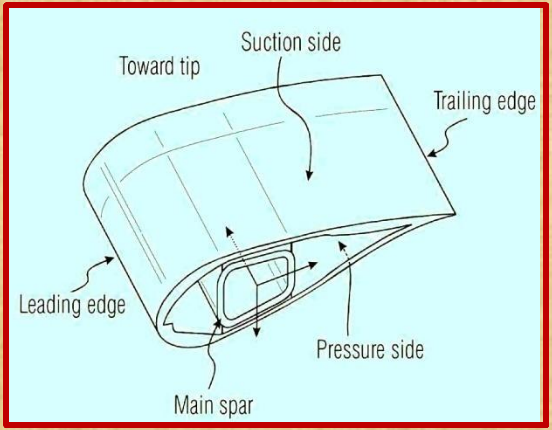
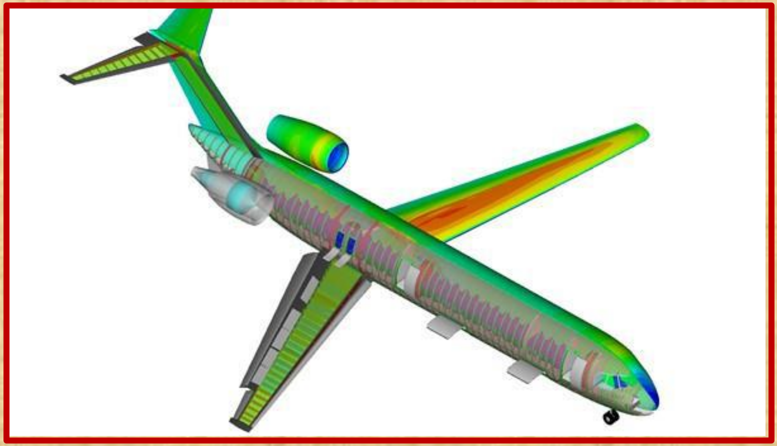




Main Aim of This Presentation

- **Use of *FGM concept* for improving dynamic, stability and aero-elastic performance of typical aerospace structures. The properties of the material of construction are optimized using either continuous or piecewise variations of the volume fractions in predetermined directions .**

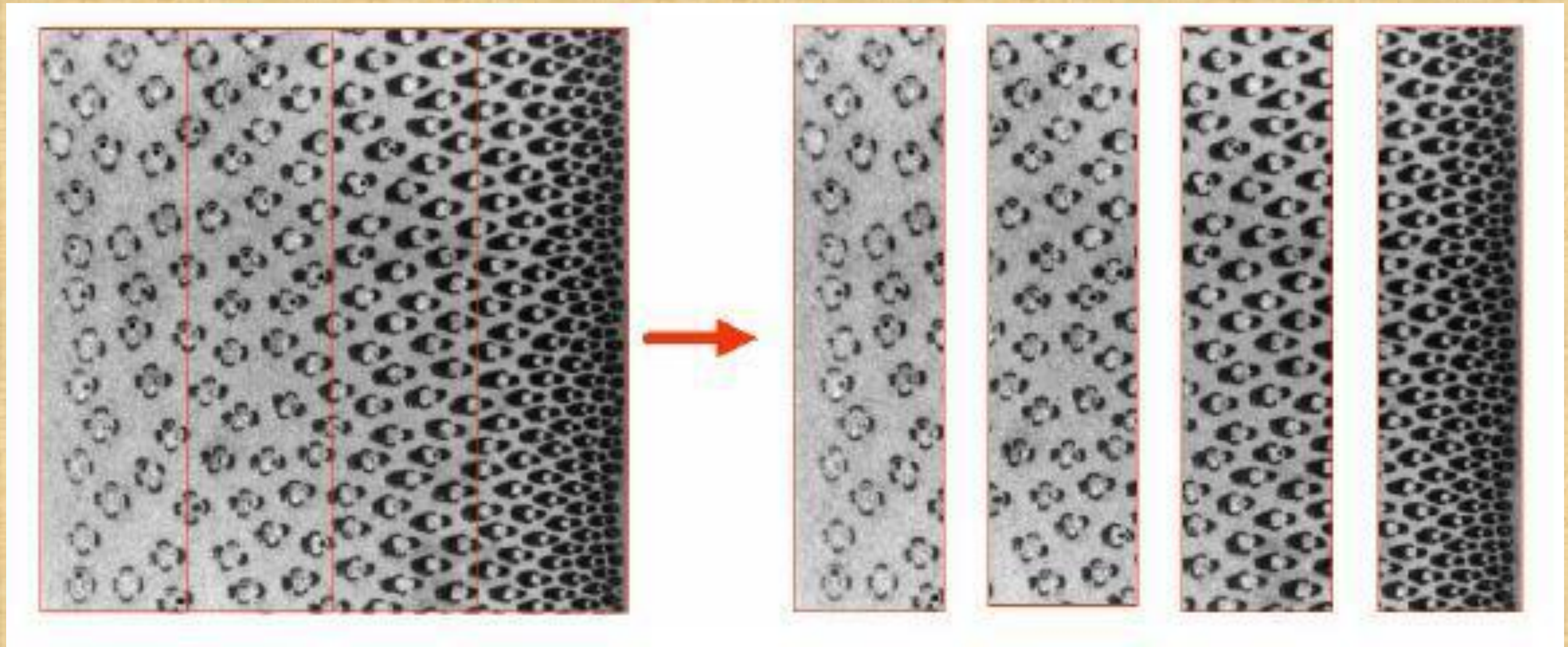
- **The major aim is to tailor the mass and stiffness distributions so as to maximize a desired eigenvalue without the penalty of increasing structural mass.**

- **Case studies include frequency optimization of box spar beams, divergence of aircraft wings, whirling & torsional buckling of spinning thin beams.**

- **The mathematical formulation is based on dimensionless quantities, which leads to a naturally scaled optimization models and valid analysis for different configurations and sizes.**

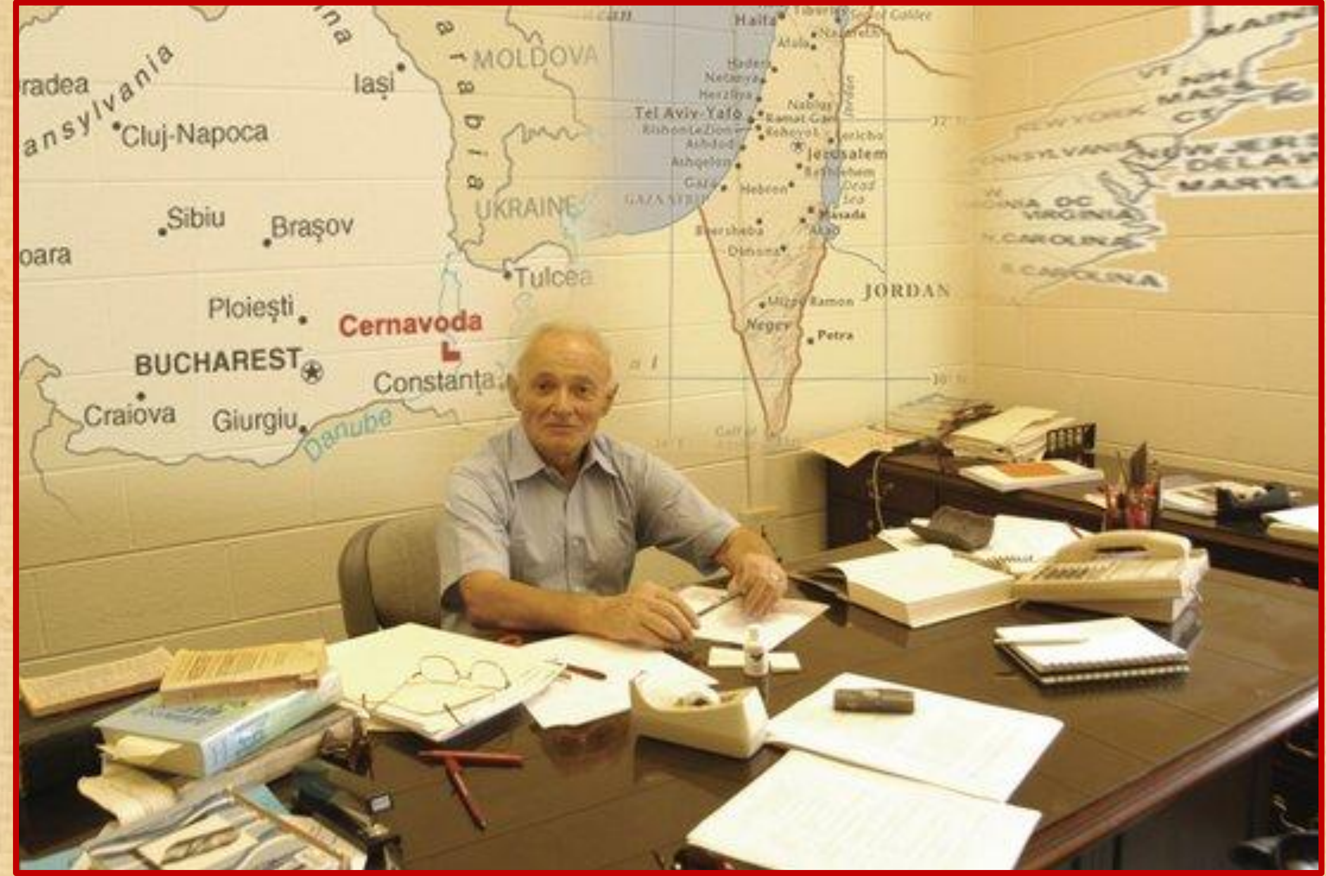
What is FGM ?

- *FGMs* may be defined as advanced composite materials that fabricated to have graded variation of the relative volume fractions of the constituent materials.
- Example: A composite material made from a mixture of ceramic and metal. Ceramic provides high temperature resistance because of its low thermal conductivity while metal secures the necessary strength and stiffness.



Material Grading

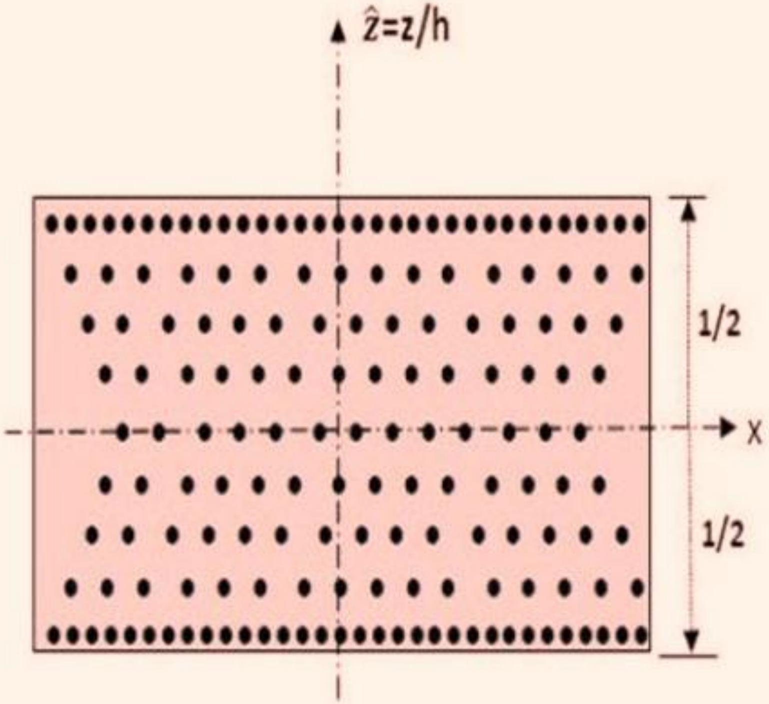
VT
VIRGINIA TECH

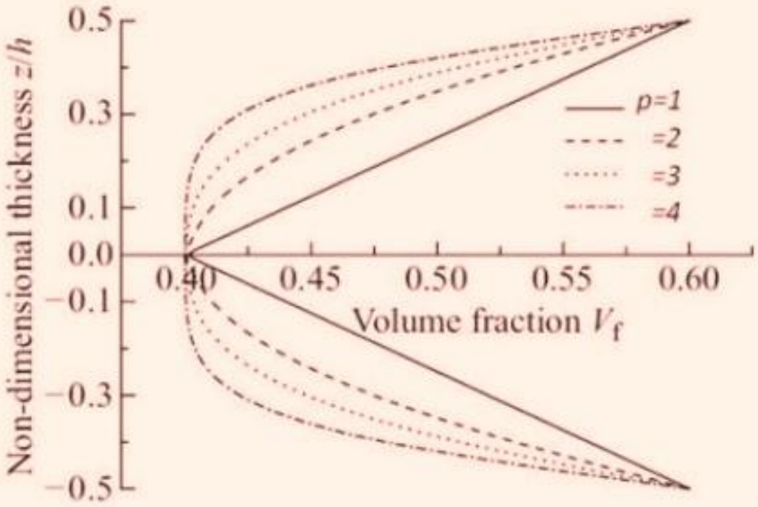


Librescu L. & Maalawi K., "Material grading for improved aeroelastic stability in composite wings", *Journal of Mechanics of Materials and Structures*, 2(7), pp.1381-1394, 2007.

$$v_f(\hat{z}) = v_f(0) + \left[v_f(0.5) - v_f(0)\right](2|\hat{z}|)^P,$$

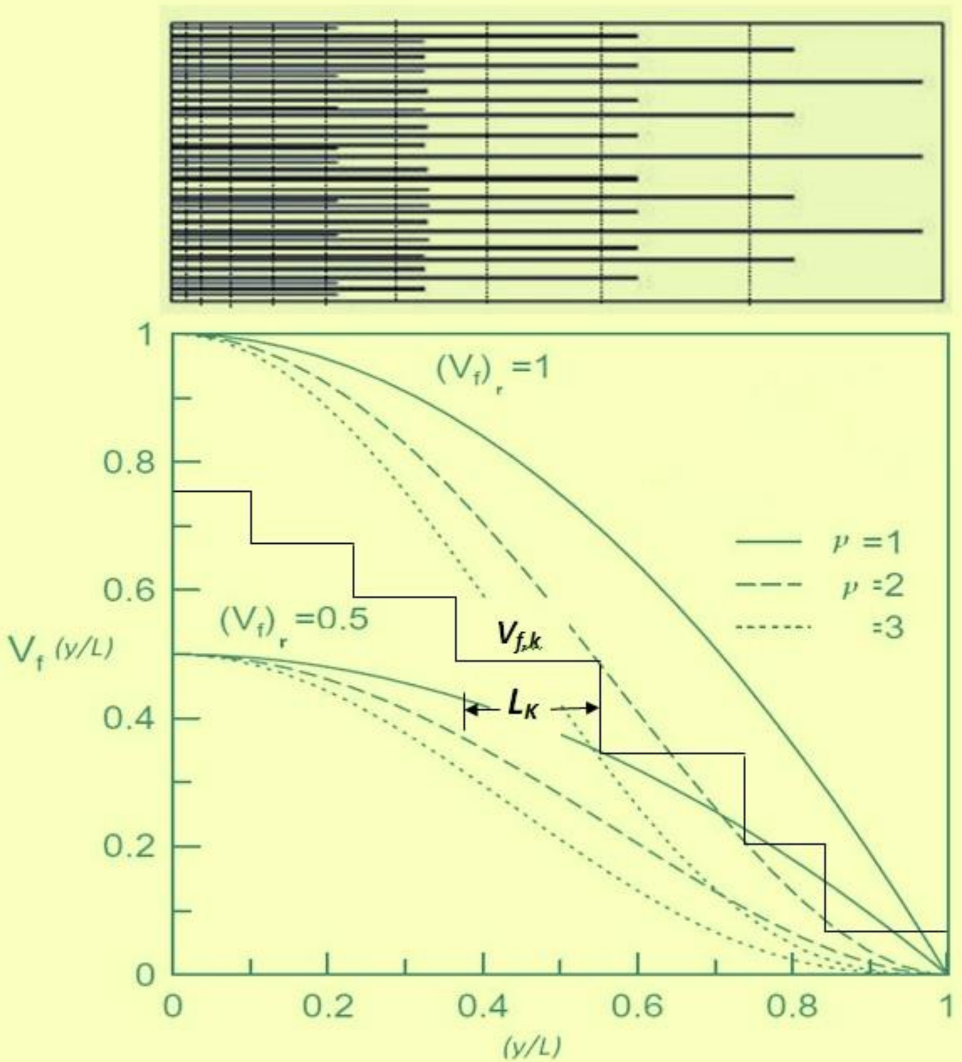
$$-0.5 \le \hat{z}(=\frac{z}{h}) \le 0.5, \quad p \ge 0$$





Thickness distribution of the fiber volume fraction in FGM beam, $v_f(0)=40\%$, $v_f(1/2)=60\%$

Bedjilili Y, Tounsi A, Berrabah H.M, Mechab I: Natural frequencies of composite beams with a variable fiber volume fraction including rotary inertia and shear deformation. Applied Mathematics and Mechanics. 2009; 30(6): 717-726.



$$v_f(\hat{y}) = v_{fr}[\beta_f(1-\hat{y}^n)^p + \Delta_f]\,,\ n=1,2,3\quad p\geq 0$$

Spanwise grading of fibers in a fibrous composite plate

Shin-Yao K: Flutter of rectangular composite plates with variable fiber spacing. Composite Structures. 2011; 93: 2533-2540

Advantages of Fabricating Aerospace Structures from Composites

- Higher stiffness-to-weight ratio.
- Superior fatigue characteristics.
- Corrosion resistant.
- Material anisotropy provides direct bending-axial-torsion elastic coupling.
- Use of aeroelastic tailoring to improve structural design.

Eigenvalue Maximization

- The eigenvalues of free vibration, critical buckling load, critical flow velocity, have been used widely as a performance measure of aerospace structures.

- The maximization problem of the minimal eigenvalue (E) of a structure under mass and side constraints may be cast in the following:

$$\text{Maximize:} \quad E(\underline{X})$$

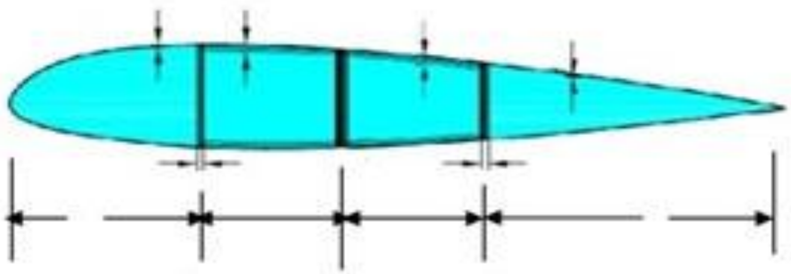
$$\text{Subject to mass constraint:} \quad M(\underline{X}) = M_o$$

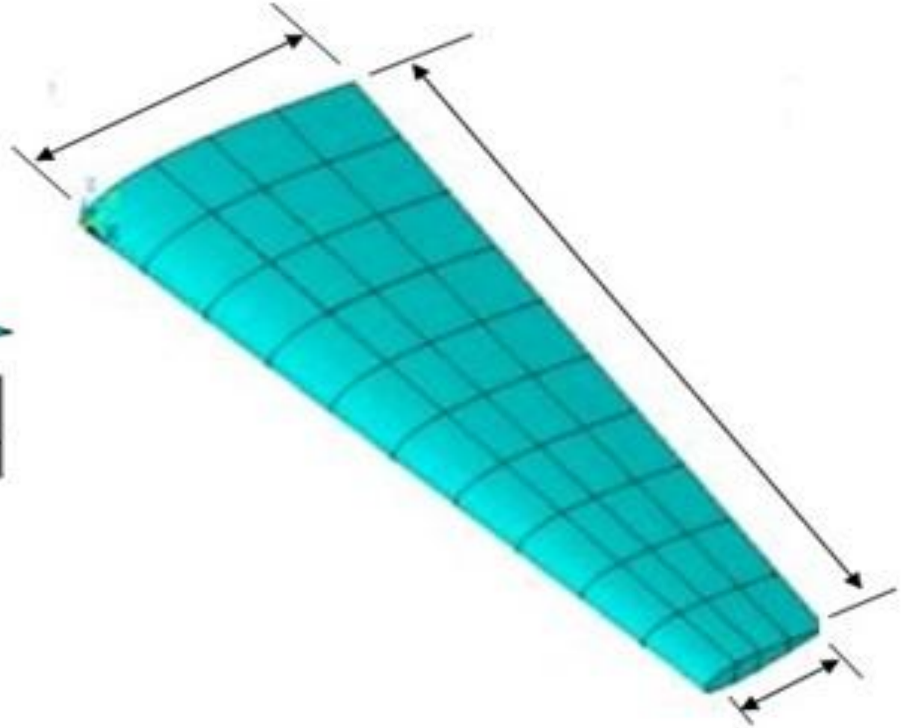
$$\text{and side constraints:} \quad \underline{X}_L \leq \underline{X} \leq \underline{X}_U$$

- E(X) : Fundamental natural frequency; Critical buckling load; Divergence and Flutter speeds; …….

- Optimization is performed with respect to a <u>known baseline design that is conservative regarding other structural and aerodynamic requirements.</u>

Design Variables

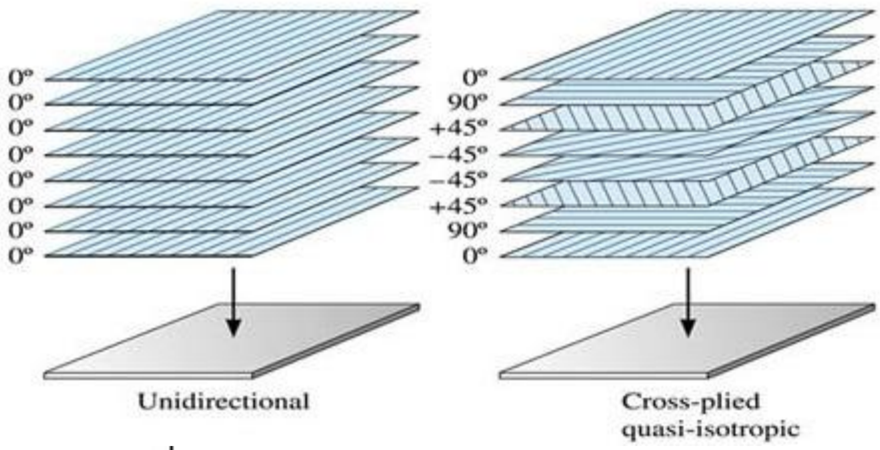




Design variables (X)

-Geometry, Cross sectional dimensions

- Properties of fibers and resins
    - No. of layers.
    - Thickness of each layer.
    - Fiber orientation in each layer.
    - Fiber volume fraction in each layer.



Determination of Material Properties

A variety of approaches have been developed to predict the mechanical properties of fibrous composite materials. The common approaches fall into the following general categories:

1. **Mechanics of materials**
2. **Numerical methods**
3. **Variational approach**
4. **Semi-empirical**
5. **Experimental**

Semi-empirical relationships have been developed to avoid the difficulties with the above theoretical approaches and to facilitate computations. The so-called Halpin-Tsai relationships have consistent forms for all properties of fibrous composite materials.

Halpin-Tsai Semi-empirical relations

Elastic property Mathematical formula	
$E_{11}=E_m V_m + E_{1f} V_f$	
$E_{22}=E_m (1 + \xi\eta V_f)/(1 - \eta V_f)$	$\eta = (E_{2f}-E_m)/(E_{2f} + \xi E_m)$
$G_{12}=G_m (1 + \xi\eta V_f)/(1 - \eta V_f)$	$\eta = (G_{12f}-G_m)/(G_{12f} + \xi G_m)$
$\nu_{12}=\nu_m V_m + \nu_{12f} V_f$	

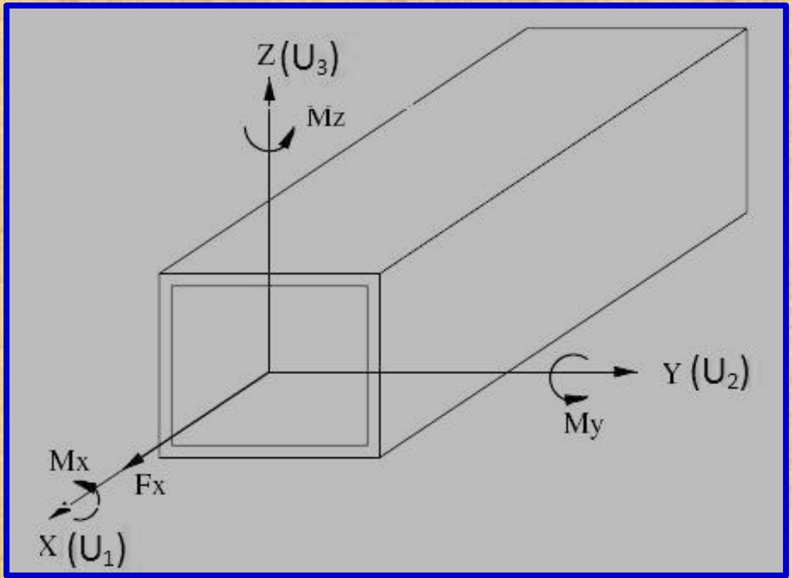
G_{12} denotes the shear modulus, ν_{12} the major Poisson's ratio and E_{22} and E_{11} and are the Young's moduli in the principal material directions. V denotes volume fraction. Subscripts "*m*" and "*f*" denote properties of matrix and fiber materials, respectively. The sum $V_m+V_f= 1$, assuming no voids are present.

The factor ξ is a measure of reinforcement of the composite material that depends on the fiber geometry, packing geometry and loading conditions . It is used to make Halpin–Tsai relations conform to the experimental data.

Case Study (I)

Frequency Optimization of Thin-Walled Box Beam

- **Natural frequencies are the most representative of the overall Stiffness/mass level of a structure,**



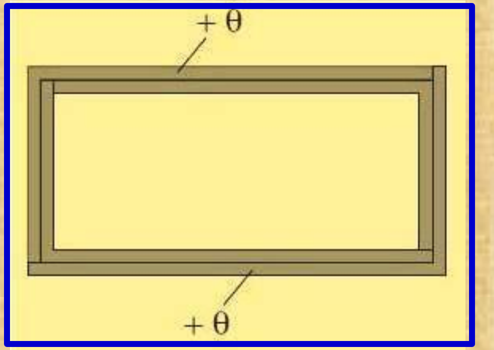
$$Max.\ \ F(\vec{X}) = \sum_i w_{fi}\,\omega_i$$

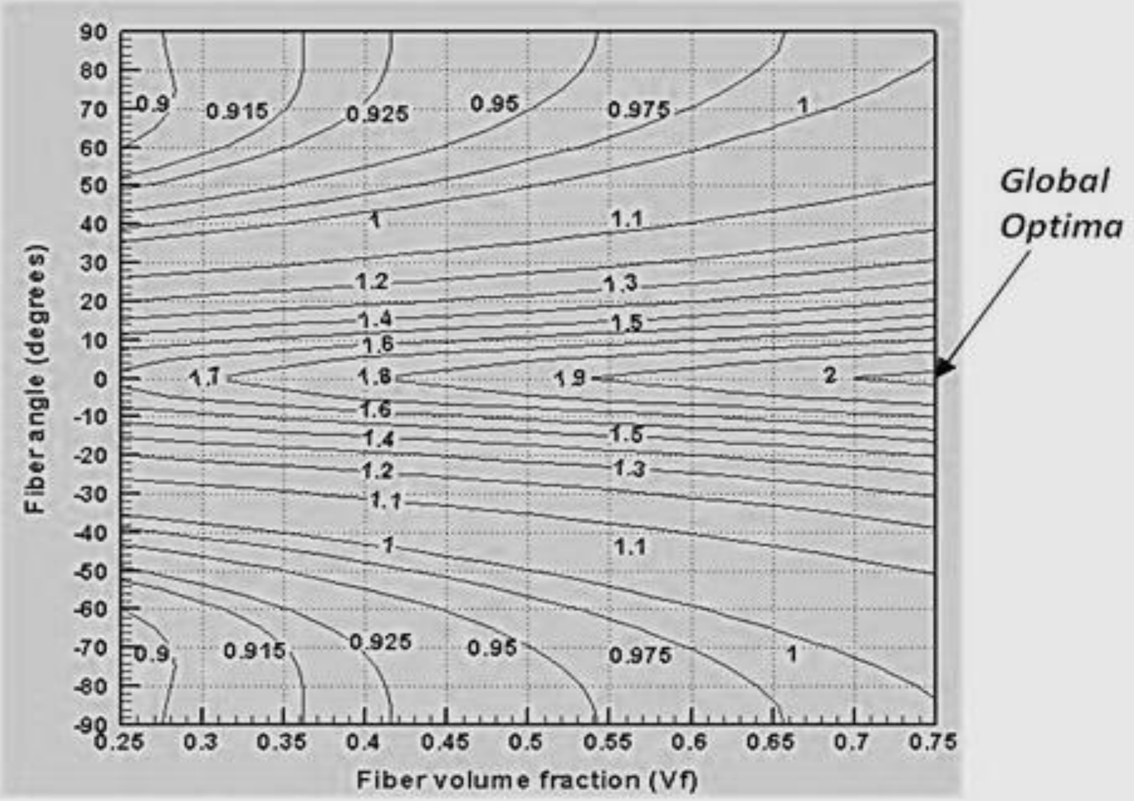
$$Min.\ \ F(\vec{X}) = \sum_i w_{fi}(\omega_i - \omega_i^*)^2$$

Karam Y. Maalawi, "Dynamic Optimization of Functionally Graded Thin-Walled Box Beams," International Journal of Structural Stability and Dynamics, Vol. 17, No.9, 2017

CUS-"Helical" Lay-up
(Carbon -AS4 / Epoxy-3501-6)

(axial/twisting coupling)





One-panel, Thin-walled cantilevered spar beam made of one 'carbon/epoxy' lamina ($\hat{L} = \hat{D} = 1$)

Level curves of $(\sqrt{\hat{\omega}_1})$ frequency function augmented with the equality mass constraint $\hat{M} = 1$

$(\sqrt{\hat{\omega}_1})_{max}$ = 2.02589 (Gain w.r.t baseline design=*8.04%*), $(V_f, \hat{H}, \theta)_{opt.}$ = *(0.75, 0.92, 0)*

Independent on units and dimensions. Target frequency can be easily chosen without mass penalty

$$v_f(\hat{y}) = v_{fr}\left(1 - \beta_f \hat{y}^P\right),\ 0 \le \hat{y}\left(=\frac{y}{L}\right) \le 1$$

$$\beta_f = (1 - \Delta_f)\ ,\Delta_f = v_{ft}/v_{fr}$$

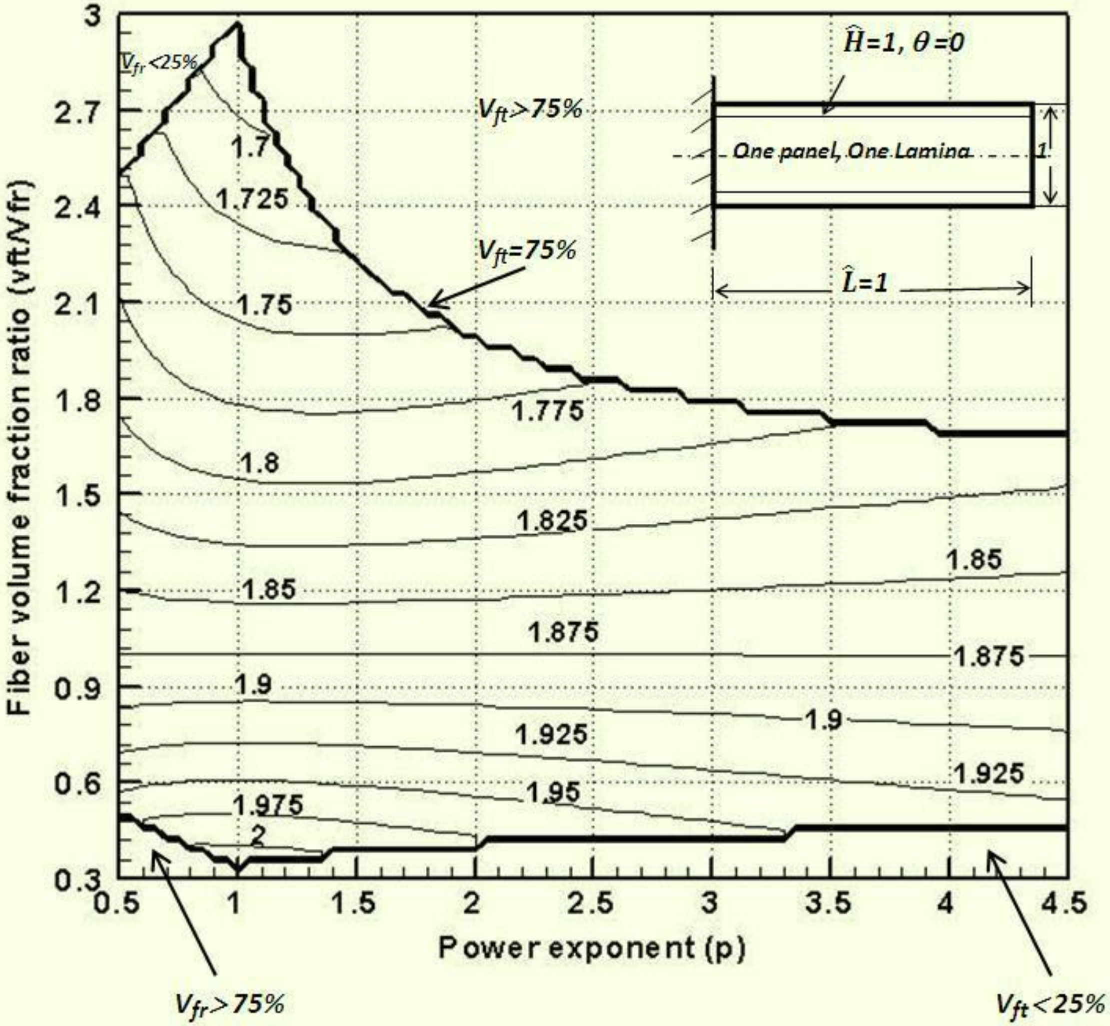
Continuous Grading

$$F(\underline{X}) = -\sqrt{\widehat{\omega}_1}$$

$$\widehat{M}_s = 1$$

$$0.33 \le \Delta_f \le 3.0$$

$$P \ge 0$$



$\sqrt{\widehat{\omega}_1}$ - Level curves under mass constraint $\widehat{M} = 1$

$(\sqrt{\widehat{\omega}_1})_{max} = 2.01875$ (7.66% *gain*), $(p, \Delta_f)opt = (1.01, 0.34), V_{fr} = 75\%, V_{ft} = 25\%$

Optimal solutions using spanwise grading *($\hat{M}$=1)*

'S-1' model: $v_f(\hat{x}) = v_{fr}(1 - \beta_f \hat{x}^P)$,

'S-2' model: $v_f(\hat{x}) = v_{fr}[\beta_f(1 - \hat{x}^n)^p + \Delta_f]$, *n=1, 2, 3*

$\beta_f = (1 - \Delta_f)$, $\Delta_f = v_{ft}/v_{fr}$ $0 \le \hat{x}(= x/L) \le 1$ $p \ge 0$

Grading model		$(\Delta_f, p)_{opt.}$, $(\sqrt{\omega_1})_{max}$, **Gain %**
(S-1)		***(0.34, 1.01), 2.01875, 7.66%***
(S-2)	*n=1*	***(0.34, 1.02), 2.01938, 7.70%***
	n=2	***(0.34, 2.425), 2.04813, 9.23%***
	n=3	***(0.34, 5.175), 2.06125, 9.93%***

Piecewise Grading
(Gain= 10.1 %)

(No longer restricted with power law)

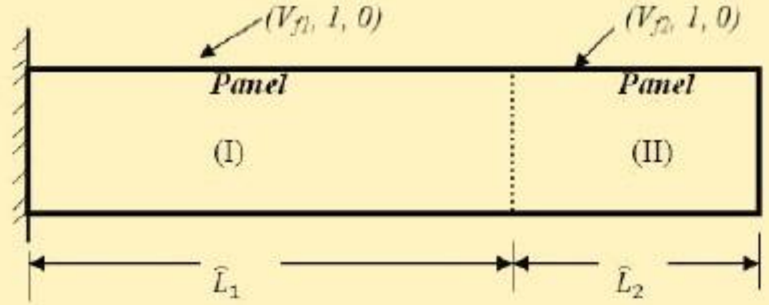
$$Minimize\ F(\underline{X}) = -\left(\sqrt{\widehat{\omega}_1}\right)$$

$$Subject\ to \quad \widehat{M}_s = 1$$

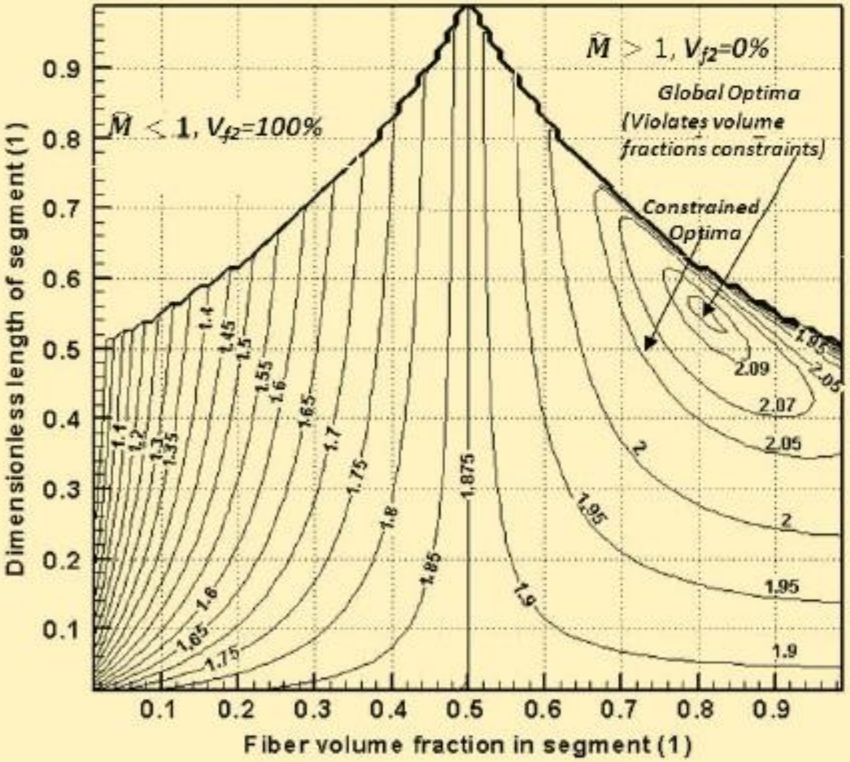
$$\sum_1^{Ns} \hat{L}_j = 1$$

$$0.25 \le V_{fj} \le 0.75 \quad j=1,2$$

$$0.0 \le \hat{L}_j \le 1.0 \quad j=1,2$$



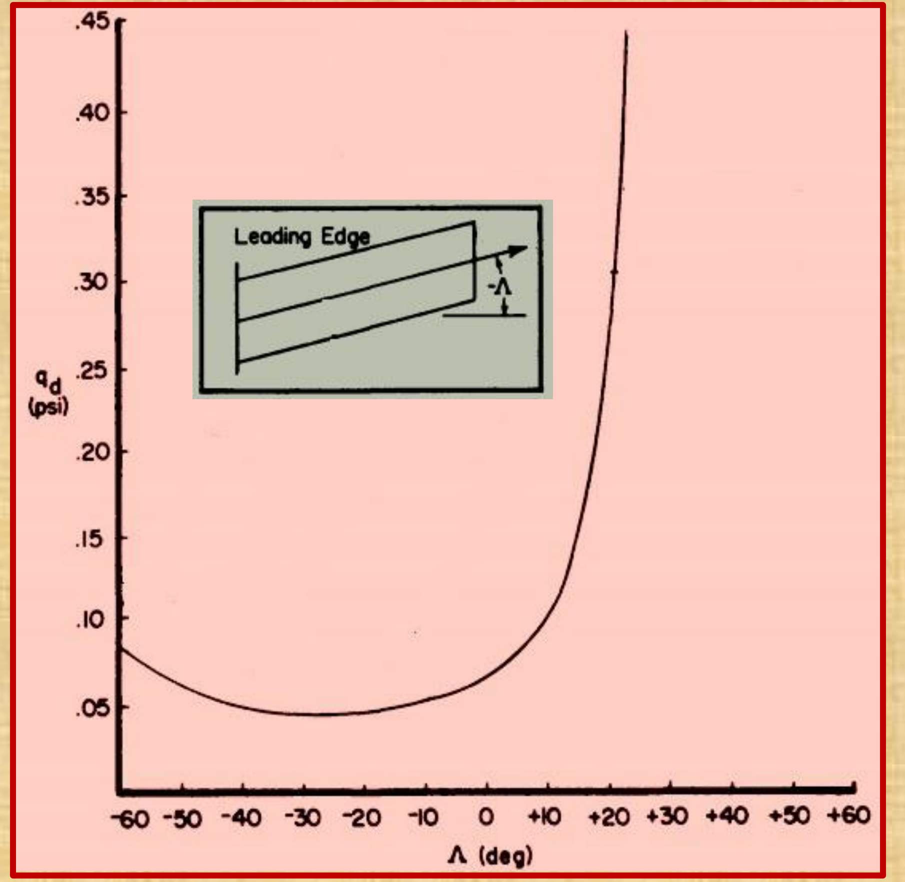
Two-segment, one unidirectional lamina spar beam.

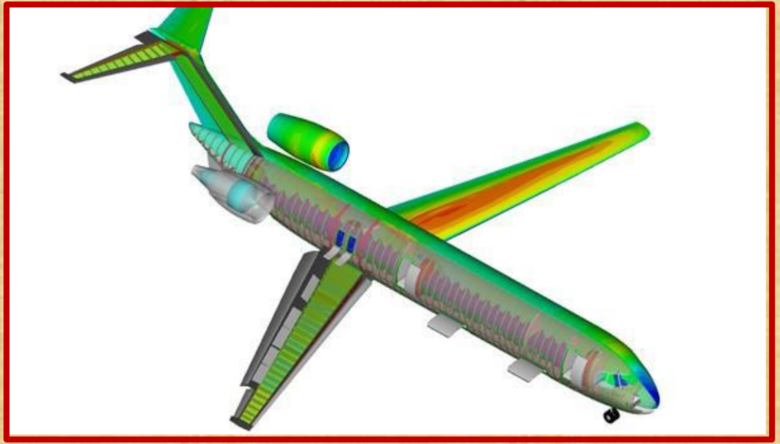


Level curves of $\sqrt{\widehat{\omega}_1}$ –function augmented with $\widehat{M}_s = 1$ in $(V_{f1}, \hat{L}_1)$ design space (*Ns*=2).

Case Study (II)

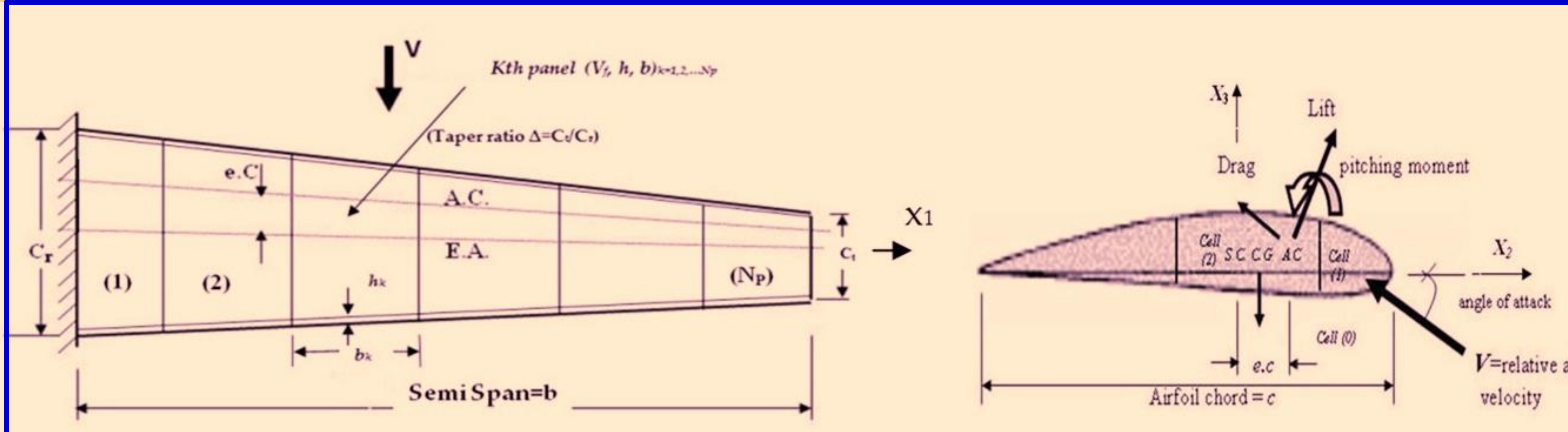
Optimization of Aircraft Wings against Divergence





Forward-swept wings can have some aerodynamic and stability advantageous over the back-swept wings. In addition, the rearward location of the main spar would lead to a more efficient interior arrangement with more usable space inside the passenger cabin. However, the large structural weights required to preclude aeroelastic divergence of forward-swept wings are unfavorable when compared to similar swept back designs. Thus, proper aeroelastic tailoring is necessary to lessen the severity of the aeroelastic divergence problem of such wing configuration.

1- Basic Model: Unswept Slender Wing



(a) Multi-panel, piecewise wing model (b) Airfoil section and applied airloads

Trapezoidal wing planform and cross section geometry

Chord distribution: $C(x) = C_r\,(1\text{-}\beta_c x), \quad \beta_c = (1\text{-}\Delta_c)$

Same lay-up [0º / ±90º / ±45º]s ‘Quasi-isotropic material)

The various parameters and variables are normalized with respect to known baseline design, which is constructed from the same material with V_f=V_m=50%. Optimized wing designs, shall have the same wing area, airfoil, span, and total mass.

The governing differential equation of wing divergence is:

$$(G_e J\, \alpha')' + \frac{1}{2}\rho V^2 C^2 e a \alpha = 0$$

Equivalent modulus of rigidity: $G_e = \sum_{k=1}^{N_L} \bar{Q}^{(k)}{}_{66} \bar{h}_k \quad (\bar{h}_k = h_k/H)$

Torsional constant of the cross section: $J = 4A_e^2 / \oint \frac{ds}{H(s)}$.

For the i^{th} panel extending from x_i to x_{i+1}, and having uniform wall thickness H_i and volume fraction V_{fi}, the governing D.E. has the form:

$$(1 - \beta\hat{x})\alpha'' - 3\beta\alpha' + \lambda_i^2 \alpha = 0, \quad (\beta = 1 - \Delta)$$

$$\text{where } \hat{x} = \frac{x}{b}, \quad \lambda_i^2 = \frac{\hat{V}^2}{\hat{G}_{ei}\hat{H}_i}, \quad \hat{G}_{ei} = \frac{G_{ei}}{G_{eo}}, \quad \hat{H}_i = H_i/H_o$$

$$\hat{V}^2 = \left(\frac{\rho e a b^2 C_0^2}{2 G_{eo} J_o}\right) V^2$$

<u>Power series solution</u>: $\alpha(\hat{x}) = \sum_{J=0}^{\infty} A_J \hat{x}^{(r+J)}$

$$\alpha(\hat{x}) = C_1 f(\hat{x}) + C_2 g(\hat{x})$$

$$f(\hat{x}) = \sum_{J=0}^{\infty} A_J^{(0)} \hat{x}^{(J)} \qquad g(\hat{x}) = \sum_{J=0}^{\infty} A_J^{(1)} \hat{x}^{(J+1)}$$

$$A_J^{(0)} = \{1 \quad 0 \quad -\lambda_i^2/2 \quad -2\beta\lambda_i^2/3 \quad \lambda_i^2(\lambda_i^2/2 - 10\beta^2)/12 \; \ldots\}$$

$$A_J^{(1)} = \{1 \quad 3\beta/2 \quad (12\beta^2 - \lambda_i^2)/6 \quad \ldots\ldots\ldots\ldots\}$$

$$\begin{Bmatrix}\alpha_{i+1}\\ T_{i+1}\end{Bmatrix} = \begin{bmatrix} f(\hat{x}_{i+1}) & g(\hat{x}_{i+1}) \\ \hat{G}_{ei}\hat{J}(\hat{x}_{i+1}) f'(\hat{x}_{i+1}) & \hat{G}_{ei}\hat{J}(\hat{x}_{i+1}) g'(\hat{x}_{i+1}) \end{bmatrix} \begin{bmatrix} f(\hat{x}_{i}) & g(\hat{x}_{i}) \\ \hat{G}_{ei}\hat{J}(\hat{x}_{i}) f'(\hat{x}_{i}) & \hat{G}_{ei}\hat{J}(\hat{x}_{i}) g'(\hat{x}_{i}) \end{bmatrix}^{-1} \begin{Bmatrix}\alpha_{i}\\ T_{i}\end{Bmatrix}$$

Unswept Wing

$$\begin{Bmatrix} \alpha_{Np+1} \\ 0 \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix} \begin{Bmatrix} 0 \\ T_1 \end{Bmatrix}$$

The non-trivial solution: *$E_{22}=0$ (The smallest root gives $\hat{V}_{div}$)*

Optimization Model

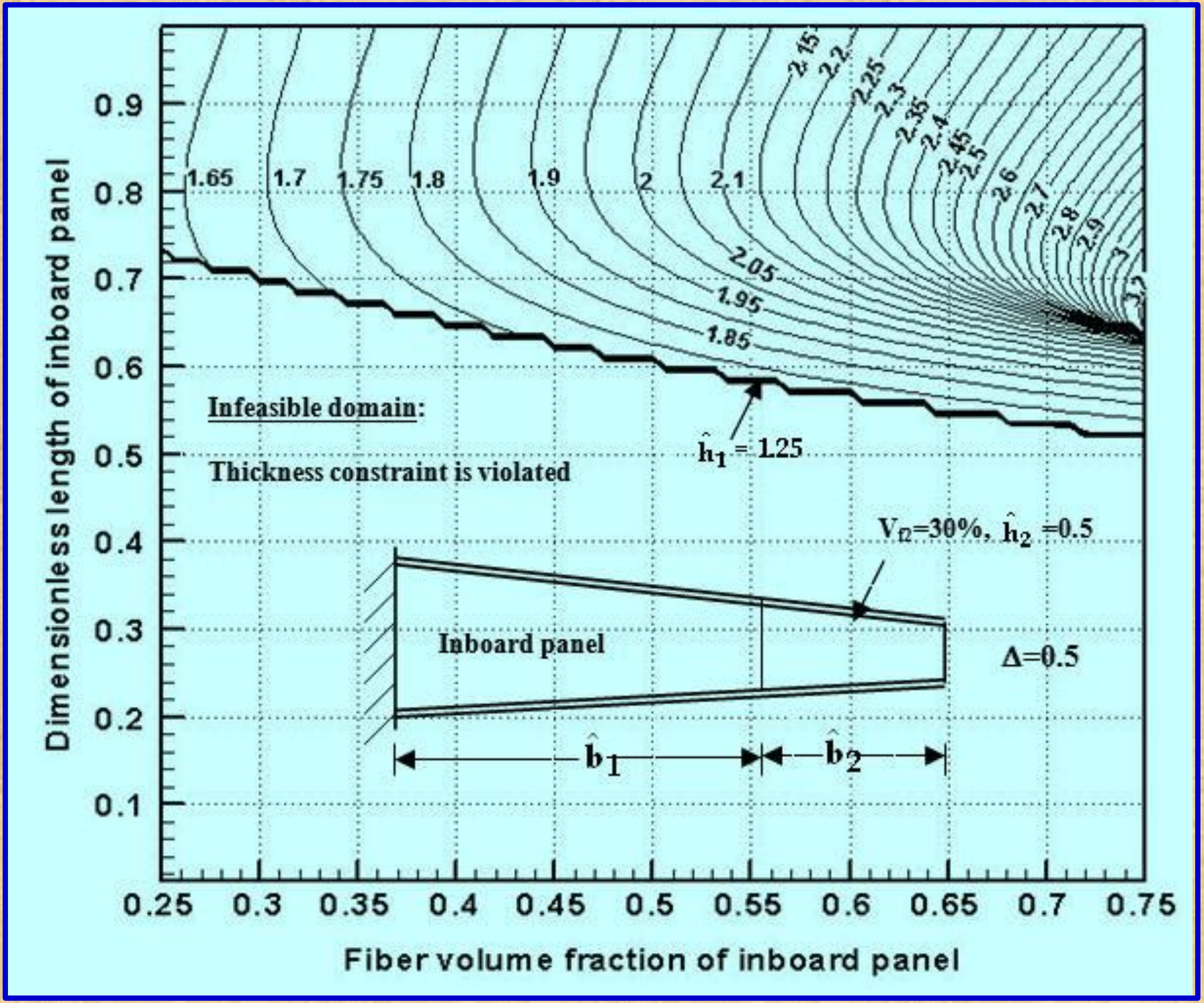
$$\text{Minimize} \qquad -\hat{V}_{div}$$

$$\text{Subject to} \qquad \hat{M}_s - 1.0 = 0$$

$$(0.3, 0.5, 0.0) \leq (V_f, \hat{H}, \hat{b})_{i=1,2,..Np} \leq (0.7, 1.25, 1.0)$$

$$\sum \hat{b}_k = 1.0$$

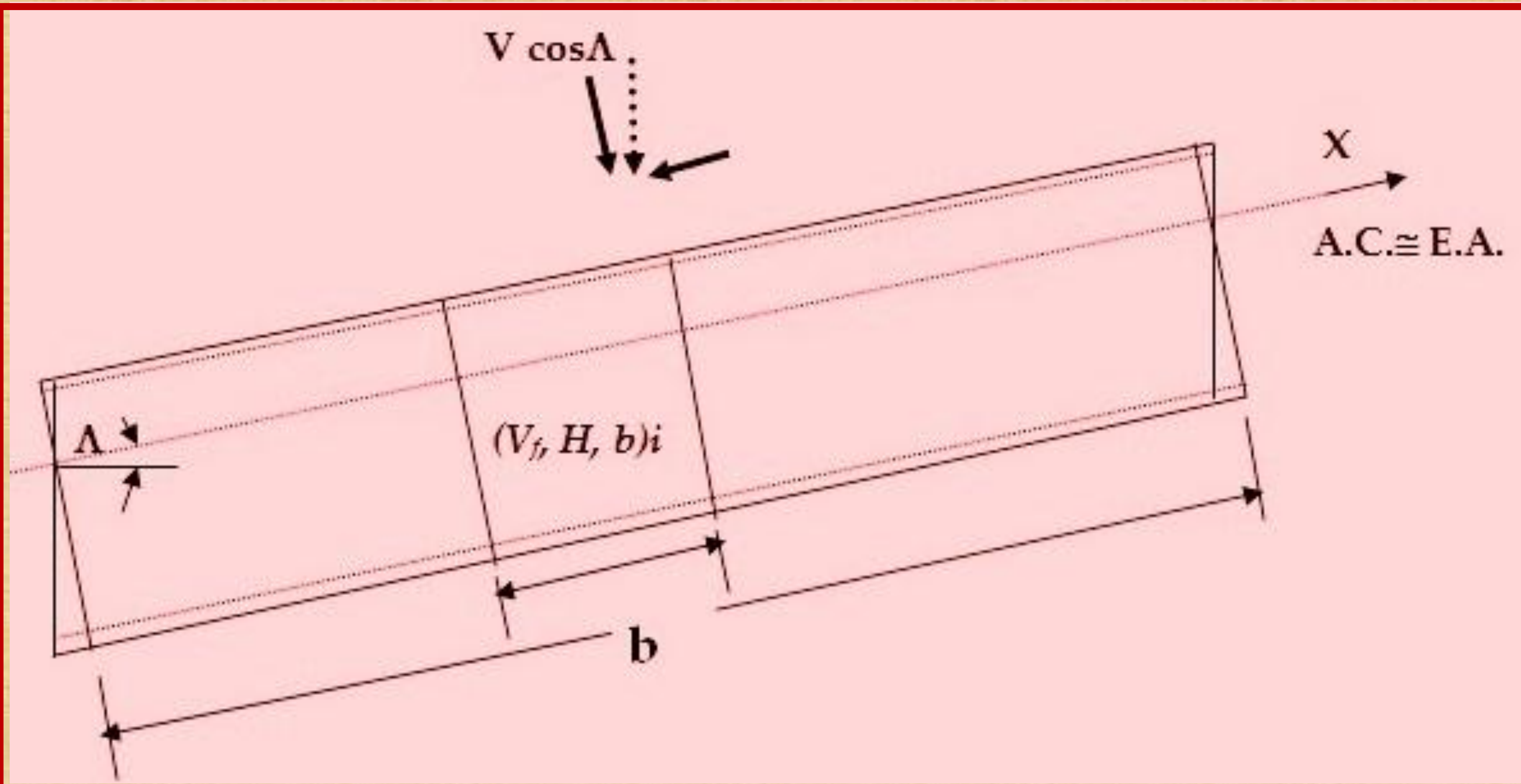
Two-panel Wing with Combined Material and Thickness Grading



<u>Optimum solution</u>: $(V_f H, b)_{i=1,2}$ =(0.7, 1.0662, 0.725), (0.3, 0.5, 0.275)

V_{div}=2.76725 (i.e. 40.13% gain over 1.9747)

2- Bending Divergence of Swept Forward Rectangular Wing



Swept Forward Wing

For the i^{th} panel extending from x_i to x_{i+1}, and having uniform wall thickness H_i and volume fraction V_{fi}, the governing D.E. has the form:

$$W'''' + \lambda_i W' = 0, \quad x_i \le x \le x_{i+1}$$

$$\lambda_i = \hat{V}^2 \frac{sin2\Lambda}{\hat{E}_{ei}\hat{I}_i}$$

$$\hat{V}^2 = \left(\frac{\rho a C_0 b^3}{4E_{eo}I_o}\right) V^2$$

Equivalent modulus of elasticity: $E_e = \sum_{k=1}^{N_L} \bar{Q}^{(k)}{}_{11}\bar{h}_k - \frac{(\sum_{k=1}^{N_L} \bar{Q}^{(k)}{}_{12}\bar{h}_k)^2}{\sum_{k=1}^{N_L} \bar{Q}^{(k)}{}_{22}\bar{h}_k}$ $(\bar{h}_k = h_k/H)$

2nd moment of area: $I = \oint H(s) Z^2 ds$.

Swept Forward Wing

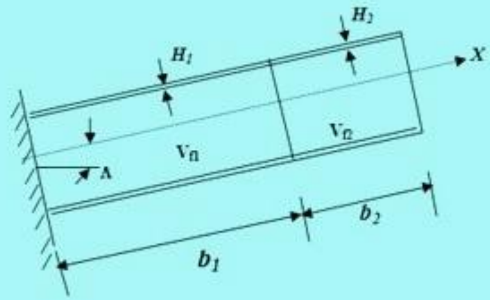
$$\begin{Bmatrix} W_{i+1} \\ W'_{i+1} \\ M_{i+1} \\ Q_{i+1} \end{Bmatrix} = \begin{bmatrix} e^{r_1\hat{b}_i}/r_1 & e^{r_2\hat{b}_i}/r_2 & e^{r_3\hat{b}_i}/r_3 & 1 \\ e^{r_1\hat{b}_i} & e^{r_2\hat{b}_i} & e^{r_3\hat{b}_i} & 0 \\ r_1 EI_i e^{r_1\hat{b}_i} & r_2 EI_i e^{r_2\hat{b}_i} & r_3 EI_i e^{r_3\hat{b}_i} & 0 \\ r_1^2 EI_i e^{r_1\hat{b}_i} & r_2^2 EI_i e^{r_2\hat{b}_i} & r_3^2 EI_i e^{r_3\hat{b}_i} & 0 \end{bmatrix} \begin{bmatrix} 1/r_1 & 1/r_2 & 1/r_3 & 1 \\ 1 & 1 & 1 & 0 \\ r_1 EI_i & r_2 EI_i & r_3 EI_i & 0 \\ r_1^2 EI_i & r_2^2 EI_i & r_3^2 EI_i & 0 \end{bmatrix}^{-1} \begin{Bmatrix} W_i \\ W'_i \\ M_i \\ Q_i \end{Bmatrix}$$

$$r_1 = -(\lambda_i)^{1/3}$$

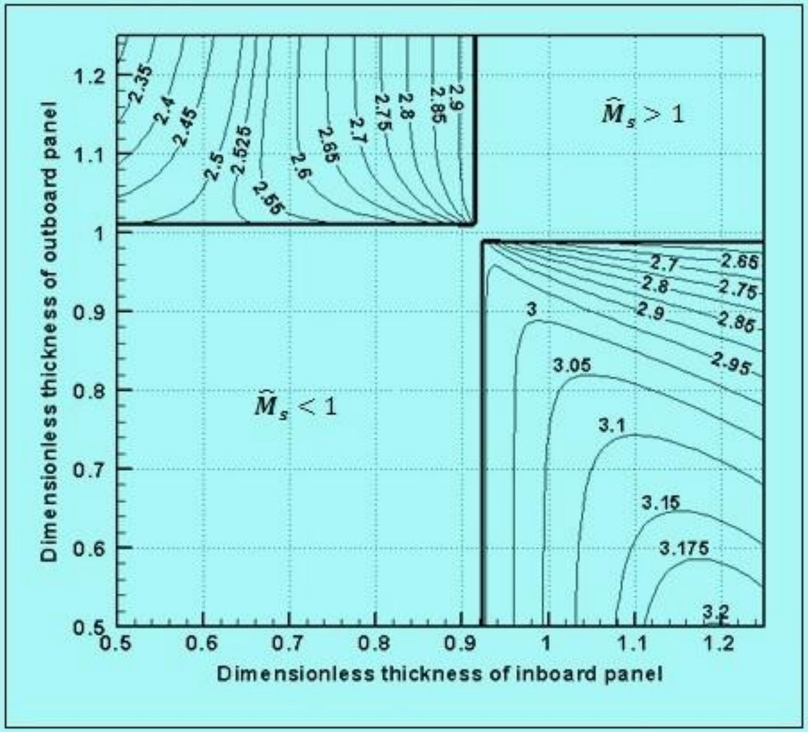
$$r_2 = \frac{1}{2}(1 + i\sqrt{3})(\lambda_i)^{1/3}$$

$$r_3 = \frac{1}{2}(1 - i\sqrt{3})(\lambda_i)^{1/3}$$

<u>The non-trivial solution</u>: $E_{33}\, E_{44} - E_{34}\, E_{43} = 0$ *(The smallest root gives* $\hat{V}_{div}$*)*



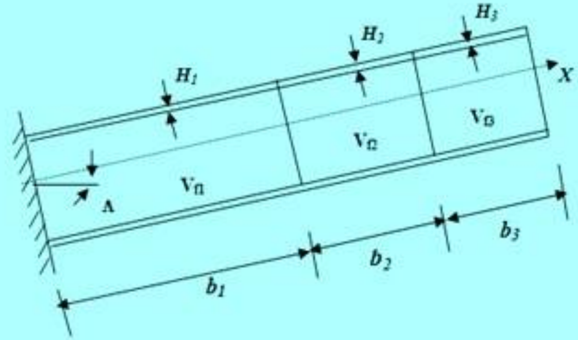
Two- Panel wing with combined material and thickness grading



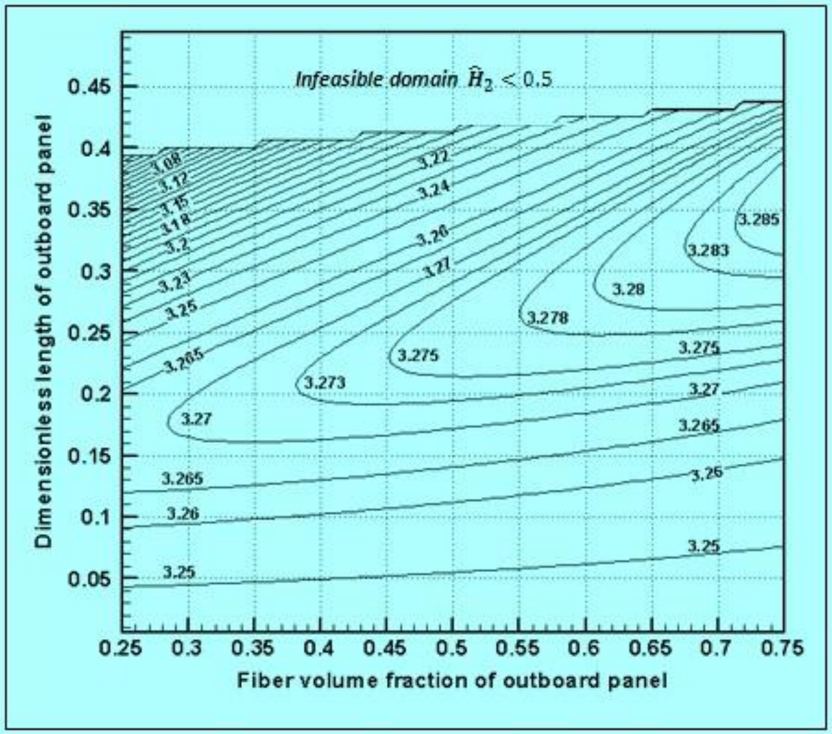
Level curves of $\widehat{V}_{div}\sqrt{sin2|\Lambda|}$ augmented with the mass constraint $\widehat{M}_s = 1$ ($\widehat{H}_1$-$\widehat{H}_2$)-Design space

Optimal solution: $(V_{R}, \widehat{H}, \widehat{b})_{i=1,2}$=(0.75, 1.18125, 0.6375), (0.50, 0.50, 0.3625) $\widehat{V}_{div,max}\sqrt{sin2|\Lambda|} = 3.2011$

Optimization Gain = (3.2011-2.516)/2.516 = 27.23 %



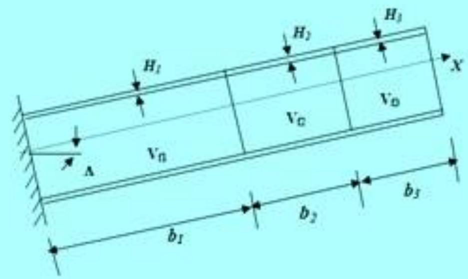
Three- Panel wing with combined material and thickness grading



Level curves of $\widehat{V}_{div}\sqrt{sin2|\Lambda|}$ augmented with the mass constraint $\widehat{M}_s = 1$ (V_{f3}-$\widehat{b}_3$)-Design space

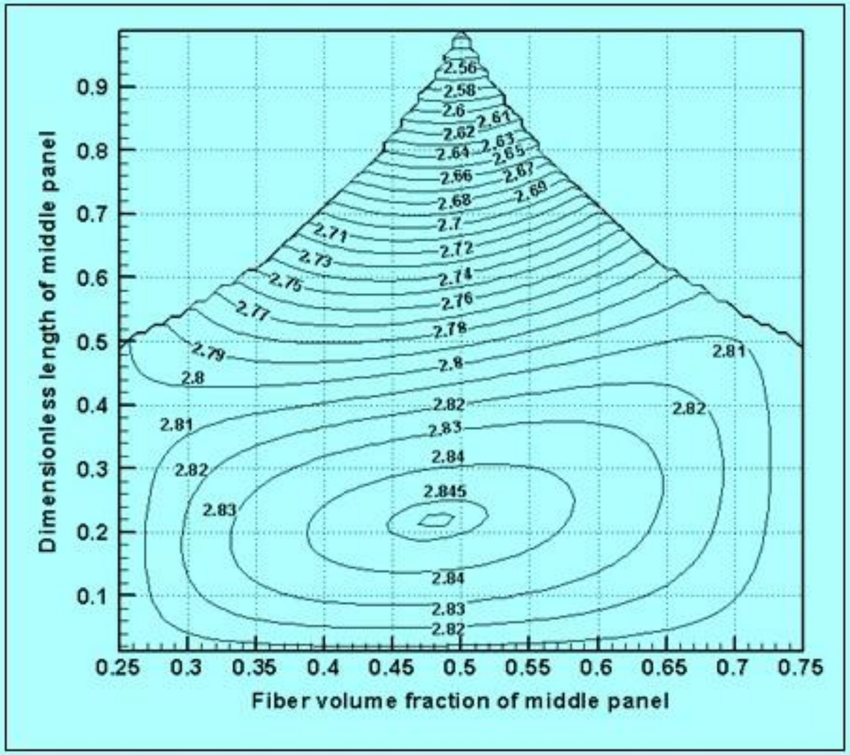
Optimal solution: $(V_{f_i}, \widehat{H}, \widehat{b})_{i=1,2,3}$ = (0.75, 1.25, 0.475), (0.75, 0.8625, 0.175), (0.75, 0.5, 0.35), $\widehat{V}_{div,max}\sqrt{sin2|\Lambda|} = 3.288$

Optimization Gain = (3.288-2.516)/2.516 = 30.7 %



Three- Panel wing with only material grading (Wall thickness is kept consta

($\widehat{H}_i$ = 1.0, i=1, 2, 3)



Level curves of $\widehat{V}_{div}\sqrt{sin2|\Lambda|}$ augmented with the mass constraint $\widehat{M}_s = 1$ (V_{f2}-$\widehat{b}_2$)-Design space

Optimal solution: $(V_f, \widehat{b})_{i=1,2,3}$=(0.75, 0.40625), (0.475, 0.215), (0.25, 0.37875), $\widehat{V}_{div,max}\sqrt{sin2|\Lambda|} = 2.845$

Optimization Gain = (2.845-2.516)/2.516 = 13.08 %

Torsional buckling optimization problem

The governing differential equations of torsional buckling are:

$$N_{x,x} + N_{yx,y} - 2Tu_{,xy} = 0$$
$$N_{xy,x} + N_{y,y} + (M_{xy,x}/R) + (M_{y,y}/R) - 2T(v_{,y} + w_{,x}/R) = 0$$
$$M_{x,xx} + (M_{xy} + M_{yx})_{,xy} + M_{y,yy} - N_y/R + 2T(v_{,x}/R - w_{,xy}) = 0$$

N_x and N_y are the normal forces, N_{xy} and N_{yx} are shear forces, M_x and M_y are bending moments, and M_{xy} and M_{yx} are torsional moments applied to the mid-surface per unit wall thickness of the spinning beam. T is the applied torque, R is the mean radius and (u, v, w) the displacements of a generic point on the middle-surface of the beam wall.

Typical torsional buckling mode shape of a rotating shaft

There are other simple empirical equations based on experimental studies that can give a reasonable estimate of the buckling torque. The most commonly used formula for the case of simply supported beam is:

$$T_{cr} = (2\pi R^2 H)(0.272)(E_x)^{0.25}(E_y)^{0.75}(H/R)^{1.5}$$

T_{cr} = critical torque at which torsional buckling occurs

H = total wall thickness of the spinning beam.

$E_x = \frac{1}{H}\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)$ equivalent modulus of elasticity in the axial direction

$E_y = \frac{1}{H}\left(A_{22} - \frac{A_{12}^2}{A_{11}}\right)$ equivalent modulus of elasticity in the hoop direction

A_{mn} = elements of the extensional stiffness matrix.

Optimization Model

Find the design variables vector $\vec{X} = \left(V_f, \theta, \hat{h}\right)_{k=1,2,\ldots N_L}$ which minimizes the objective function:

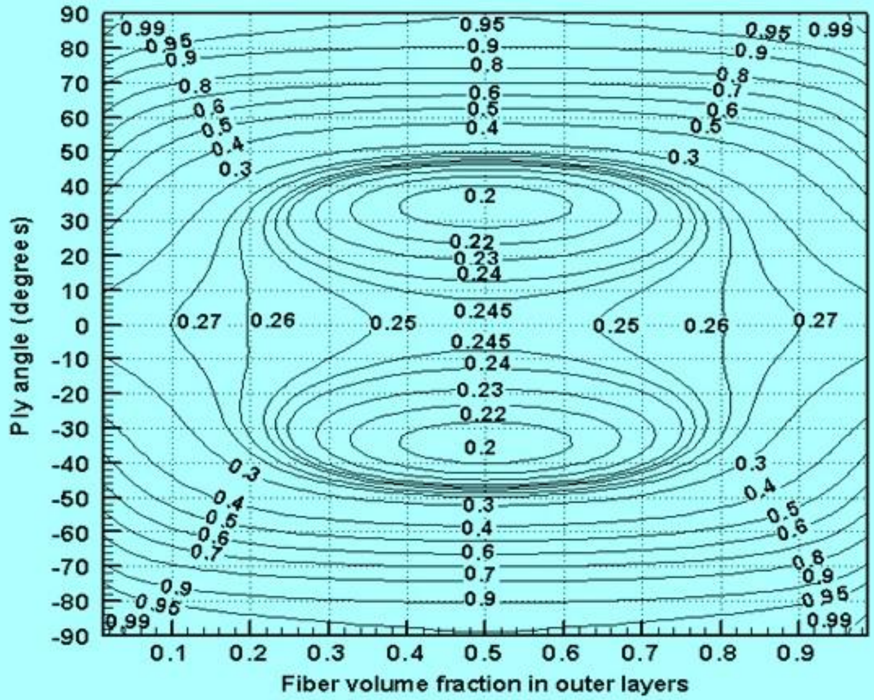
	Minimize	$F = -\hat{T}_{cr}$
subject to	Mass limitation :	$\hat{M} - 1 \leq 0$
	Torsional strength:	$\left(\frac{\tau_{max}}{\tau_{allow}}\right) - 1.0 \leq 0$
	Whirling :	$\hat{\Omega}_{max} - \hat{\Omega}_{cr} \leq 0$
	Side constraints :	$\vec{X}_L \leq \vec{X} \leq \vec{X}_U$
		$\hat{H}_L \leq \sum_{k=1}^{N_L} \hat{h}_k \leq \hat{H}_U$

Maximum shear stress: $\tau_{max} = (T_{max}/2\pi R^2 H)$

Maximum applied torque: T_{max}

Allowable shear stress: τ_{allow} (to be calculated according to the embedded material properties and volume fraction of the fiber).

<u>Example (1)</u>: Rotating thin-walled beam made of carbon/epoxy composites with discrete thickness grading constructed from eight symmetric, balanced plies $(\pm\vartheta\ \pm\vartheta)_s$ with same thicknesses.



$\widehat{T}_{cr}$ - contours in $(V_{f1}$-$\theta)$ design space under mass constraint $\widehat{M} = 1$.

A local maximum of $\widehat{T}_{cr}$ can be observed near the design point $(V_{f1}, \theta) = (0.7, 90^o)$. This illustrates that the maximum critical buckling torque can be achieved when the fiber orientation angle is close to 90^o.

<u>Example (2)</u>: Cross ply layup $[90^0/0^0]_4$

$V_{f,opt.,i}$=$(70\%)_{i=1\text{-}8}$, $\widehat{h}_{opt,i}$= $[0.1994, 0.0967, 0.152, 0.019]_s$

$\widehat{T}_{cr,max} = 1.321$ (*i.e.* optimization gain=32.1% above the baseline design).

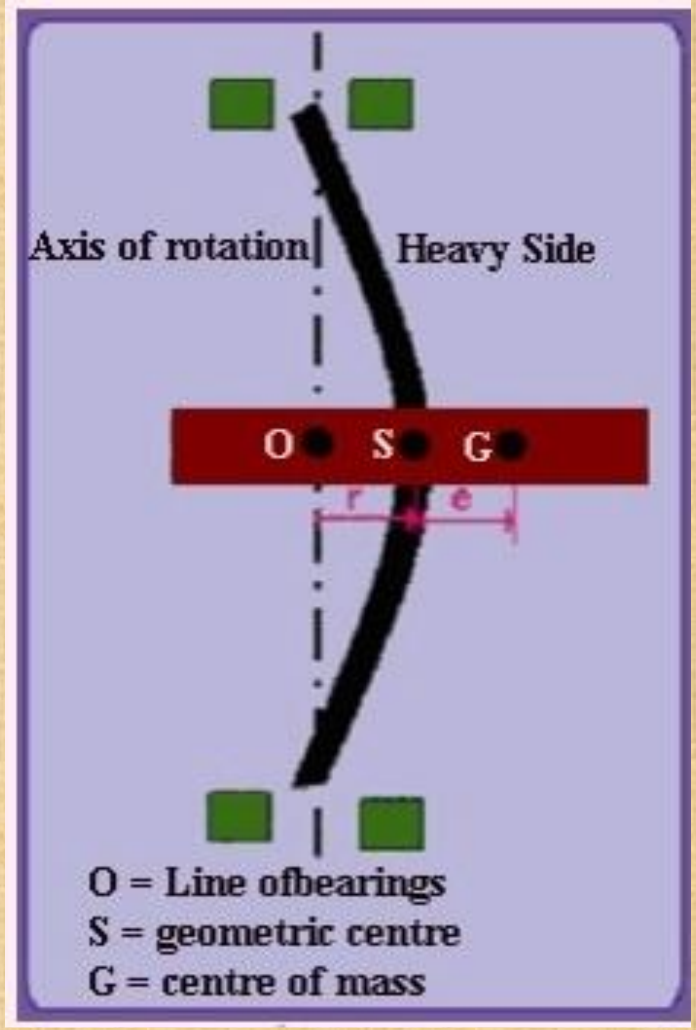
Mass and whirling constraints became active at the achieved optimum design point.

Whirling

$$K.r = m\Omega^2 (r + e)$$

$$r = \frac{\Omega^2}{\omega_n^2 - \Omega^2}, \quad \omega_n^2 = K/m$$

When the rotational speed coincides with the natural frequency of bending vibration, the beam tends to bow out with large amplitude. Such a resonance situation should be avoided in actual practice.



Whirling optimization problem

Two alternatives may be considered regarding the whirling optimization problem:

(a) Direct maximization of the critical rotational speed

Find the design variables vector $\vec{X} = \left(V_f, \theta, \hat{h}\right)_{k=1,2,\ldots N_L}$ which minimizes the objective function:

$$\text{Minimize} \qquad F = -\hat{\Omega}_{cr}$$

$$\text{Subject to} \qquad \hat{M} - 1 \le 0$$

$$\left(\frac{\tau_{max}}{\tau_{allow}}\right) - 1.0 \le 0$$

$$\left(\frac{T_{max}}{T_{cr,o}}\right) - \hat{T}_{cr} \le 0$$

$$\vec{X}_L \le \vec{X} \le \vec{X}_U$$

$$\hat{H}_L \le \sum_{k=1}^{N_L} \hat{h}_k \le \hat{H}_U$$

(b) Placement of the critical speed

The other alternative of the objective function is defined by:

$$\text{Minimize} \qquad F = (\hat{\Omega}_{cr} - \hat{\Omega}^*)^2$$

$\hat{\Omega}^*$ is a dimensionless target rotational speed ($> \Omega_{max}$, by a reasonable margin (e.g. *10-20%*).

Example 1: Thickness grading pattern

$$v_f(\hat{z}) = v_f(0) + \left[v_f(0.5) - v_f(0)\right](2|\hat{z}|)^P,$$
$$-0.5 \leq \hat{z}(=\tfrac{z}{h}) \leq 0.5, \;\; p \geq 0$$

$\vec{X}_{opt} = \left(V_f(0),\, V_f\left(\frac{1}{2}\right), p, \hat{H}\right) = (0.7, 0.3, 5.61, 0.955)$ at which $\hat{\Omega}_{cr}$ increased by *14%* above that of the baseline design with active mass and torsional buckling constraints.

Example 2: A last optimization strategy to be addressed here is to combine the two criteria in a single objective function subject to the mass, strength and side constraints.

Minimize $\quad F = -(\hat{\Omega}_{cr} + \hat{T}_{cr})$

Subject to $\quad \hat{M} - 1 \leq 0$

$$\left(\frac{\tau_{max}}{\tau_{allow}}\right) - 1.0 \leq 0$$
$$\vec{X}_L \leq \vec{X} \leq \vec{X}_U$$
$$\hat{H}_L \leq \sum_{k=1}^{N_L} \hat{h}_k \leq \hat{H}_U$$

It is assumed that whirling and torsional buckling instabilities are of equal relative importance. This model resulted in a balanced improvement in both stabilities with active mass constraint. The attained optimal solution was found to have a uniform distribution of the fiber volume fraction with its upper limiting value of 70% and wall thickness = 0.935. The corresponding optimal values of the design objectives were $\hat{\Omega}_{cr} = 1.135$ and $\hat{T}_{cr} = 1.161$, representing optimization gains *13.5%* and *16.1%*, respectively as measured from the baseline design.

CONCLUSIONS

- Efficient models for enhancing dynamics & aeroelastic stability of composite wings using the concept of FGM have been formulated. Optimization against torsional buckling and whirling of rotating beams have been also addressed.
- Exact solutions have been given analytically using differential equation and power series methods.
- The model formulation is independent on structural geometry and type of material.
- Cross-ply lay-up is efficient in both bending and torsion modes.
- The attained solutions using continuous FGM depend entirely upon the form of the power-law expression, which represents an additional constraint on the proposed optimization model. The problem of determining the actual optimal distribution of the volume fraction may be treated using advanced optimal control theories.
- Coupled bending-torsion divergence of slender wings is currently investigated. Extension of this work shall consider optimization against flutter using grading in both wall thickness and span directions

Thank You

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