

# About OMICS Group

**OMICS Group International is an amalgamation of Open Access publications and worldwide international science conferences and events. Established in the year 2007 with the sole aim of making the information on Sciences and technology 'Open Access', OMICS Group publishes 400 online open access scholarly journals in all aspects of Science, Engineering, Management and Technology journals. OMICS Group has been instrumental in taking the knowledge on Science & technology to the doorsteps of ordinary men and women. Research Scholars, Students, Libraries, Educational Institutions, Research centers and the industry are main stakeholders that benefitted greatly from this knowledge dissemination. OMICS Group also organizes 300 International conferences annually across the globe, where knowledge transfer takes place through debates, round table discussions, poster presentations, workshops, symposia and exhibitions.**

# About OMICS Group Conferences

**OMICS Group International is a pioneer and leading science event organizer, which publishes around 400 open access journals and conducts over 300 Medical, Clinical, Engineering, Life Sciences, Pharma scientific conferences all over the globe annually with the support of more than 1000 scientific associations and 30,000 editorial board members and 3.5 million followers to its credit.**

**OMICS Group has organized 500 conferences, workshops and national symposiums across the major cities including San Francisco, Las Vegas, San Antonio, Omaha, Orlando, Raleigh, Santa Clara, Chicago, Philadelphia, Baltimore, United Kingdom, Valencia, Dubai, Beijing, Hyderabad, Bengaluru and Mumbai.**



# **3rd International Conference and Exhibition on Mechanical & Aerospace Engineering, San Francisco, USA.**

**October 05-07, 2015.**

# Alexandre de Macêdo Wahrhaftig

[alixa@ufba.br](mailto:alixa@ufba.br)



**Polytechnic School**  
*Federal University of Bahia,*  
*Salvador-Bahia, Brazil*





# **ANALYTICAL SOLUTION FOR WELDED JOINTS OF PERPENDICULAR PLATES SUBJECTED TO TORSIONAL MOMENT**



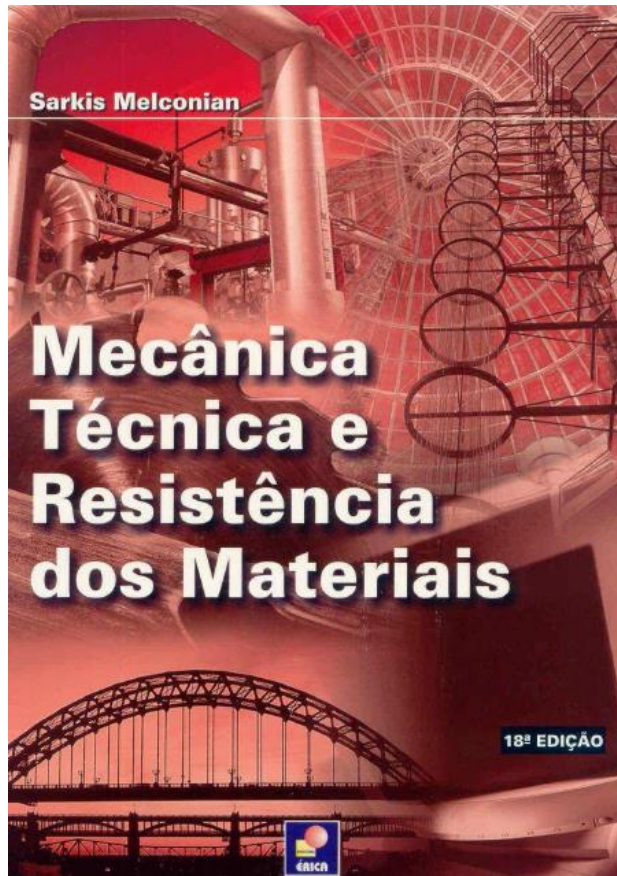
What is the specific objective of this work?

**Specifically, the objective of this work is to present an analytical solution based on shearing stress for welded joints of perpendicular plates subjected to torsional moment.**





# Solution of reference



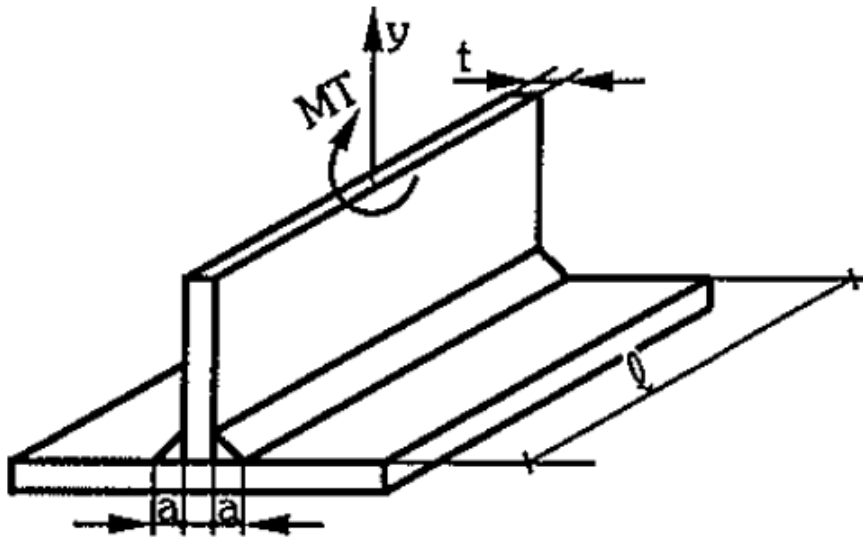
## Technical Mechanics and Resistance of Materials

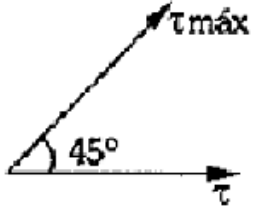
Melconian, Sarkis; Ed. 18, São  
Paulo, Brasil, 2008.



# Solution of reference

It is based on normal stress from bending.



$$\tau = \frac{M}{J} Y_{\max} = \frac{12Mt(\ell / 2)}{(2a)\ell^3} = \frac{3Mt}{a\ell^2}$$
$$\tau_{\max} = \frac{\tau}{\cos 45^\circ}$$


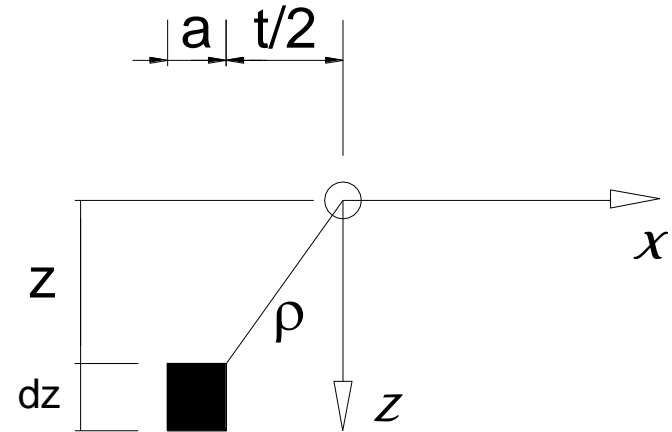
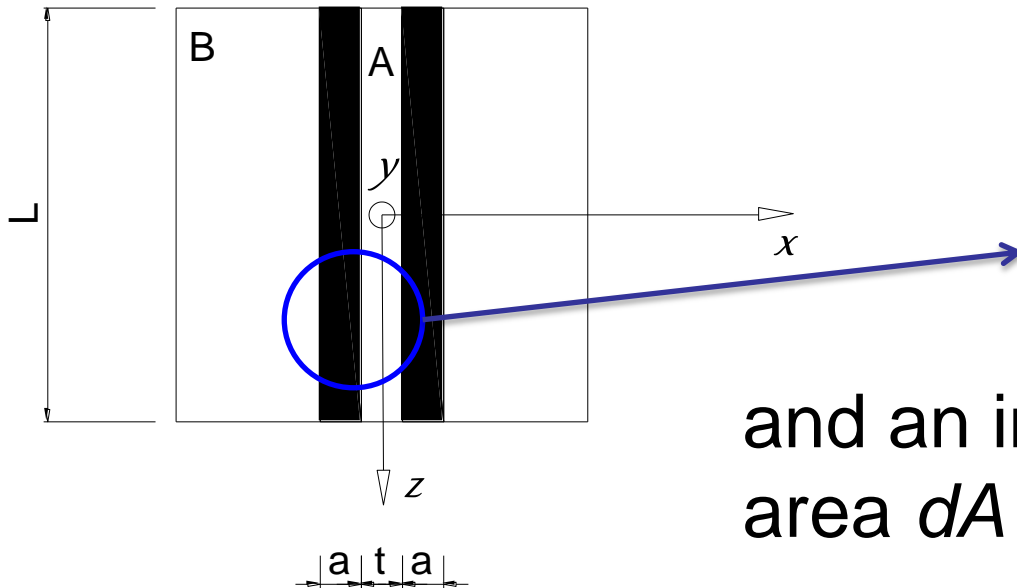
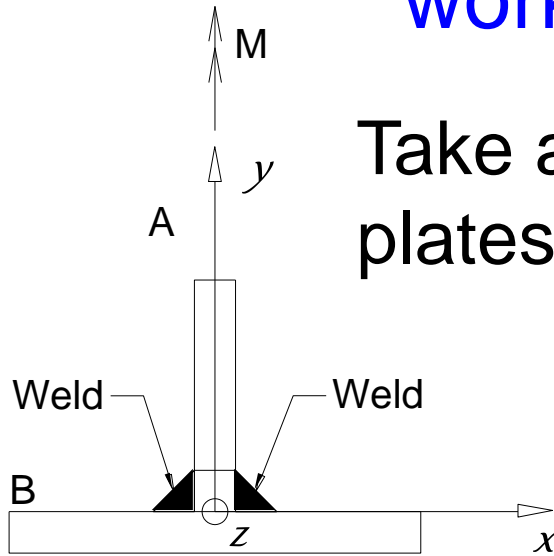
$$\tau_{\max} = \frac{3Mt}{a \cdot \ell^2 \cos 45^\circ}$$

$M$  = Bending Moment;  $t$  = Thickness of the perpendicular plate;  
 $\ell$  = Length of weld line;  $a$  = Base of the weld line.



# The analytical solution of this work is based on shearing stress

Take a welded joint of perpendicular plates requested by torque  $M$



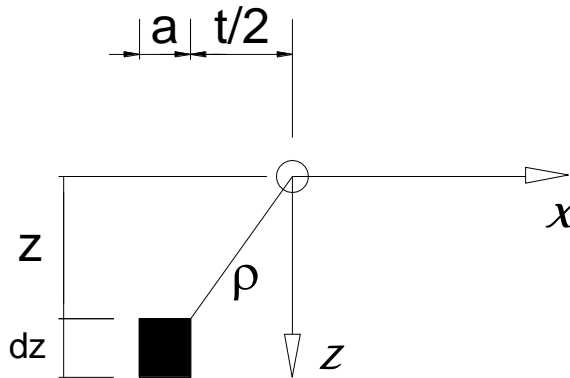
and an infinitesimal element of area  $dA$  in weld.  $dA = a dz$

# The analytical solution based on shearing stress

The distribution of the **shearing stress** obeys the law from Resistance of Materials, where “ $\rho$ ” is the generic distance in relation to the center of joint and “ $J$ ” is the polar moment of inertia.

$$\tau = \frac{M \rho}{J}$$

The **polar moment of inertia** to be determined by



$$\rho^2 = \left( a + \frac{t}{2} \right)^2 + z^2$$

## The analytical solution based on shearing stress

The **polar moment of inertia** is obtained by the equation.

$$J = \int_A \rho^2 dA$$

Substituting the **polar moment of inertia** on that equation, with the integration limits appropriate to the problem, **one has**

$$J = \int_{-L/2}^{+L/2} \left[ \left( a + \frac{t}{2} \right)^2 + z^2 \right] adz \longrightarrow J = La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3$$

## The analytical solution based on shearing stress

Replacing the polar moment of inertia **obtained** and **knowing** that the linear distribution of stresses requires that the **maximum stress** occurs at the **end** of the **weld**, we can approximate to  $\rho_{\max}$  equals to  $L/2$ , and write the **equation** of **maximum shear stress**

$$\tau_{\max} = \frac{M \frac{L}{2}}{La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3}$$

## The analytical solution based on shearing stress

The prior expression can be used for the design of the base "a" of the bead weld. To this must be put it in the polynomial form

$$a^3 + a^2t + a\xi - \eta = 0$$

Where  $\xi = \frac{t^2}{4} + \frac{L^2}{12}$  and  $\eta = \frac{M}{4\tau_{\max}}$

Whose real root is

$$a = \frac{\Pi}{6} - \frac{2\xi - \frac{2t^2}{3}}{\Pi} - \frac{t}{3}$$

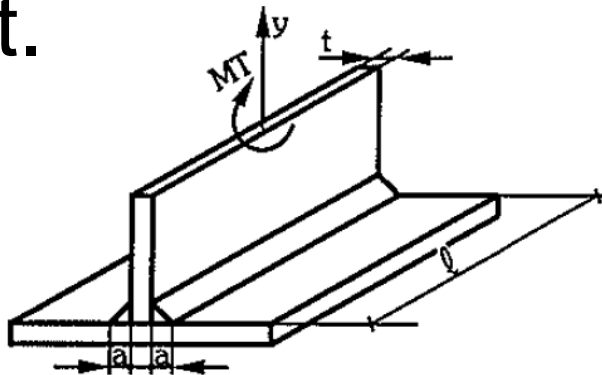
in which

$$\Pi = \sqrt[3]{36\xi t + 108\eta - 8t^3 + 12\sqrt{(12\xi^3 - 3\xi^2t^2 + 54\xi t\eta + 81\eta^2 - 12\eta t^3)}}$$



# NUMERICAL SIMULATIONS

Consider two steel plates welded perpendicularly through a weld bead length of  $L = 500 \text{ mm}$  and weld base  $a = 12 \text{ mm}$ . By the specifications of the American Welding Society, the allowable stress indicated is  $\tau_{adm} = 70 \text{ MPa}$ . One wants to know the maximum torque that can act at the joint.



“M” is the torsional moment,  
“a” is the base of bead weld and  
“L” is the length of bead weld.





# NUMERICAL SIMULATIONS

Solving the problem by the proposal for [Sarkis](#), one has.

$$M = \frac{3\tau_{adm}}{aL^2} \quad \text{and}$$

$$M = \frac{3\tau_{adm}}{aL^2 \cos 45^\circ}$$

and by [Wahrhaftig](#)

$$M = \frac{\tau_{adm} \frac{L}{2}}{La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3}$$

and

$$M = \frac{\tau_{adm} \frac{L}{2}}{\left( La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3 \right) \cos 45^\circ}$$



# NUMERICAL SIMULATIONS



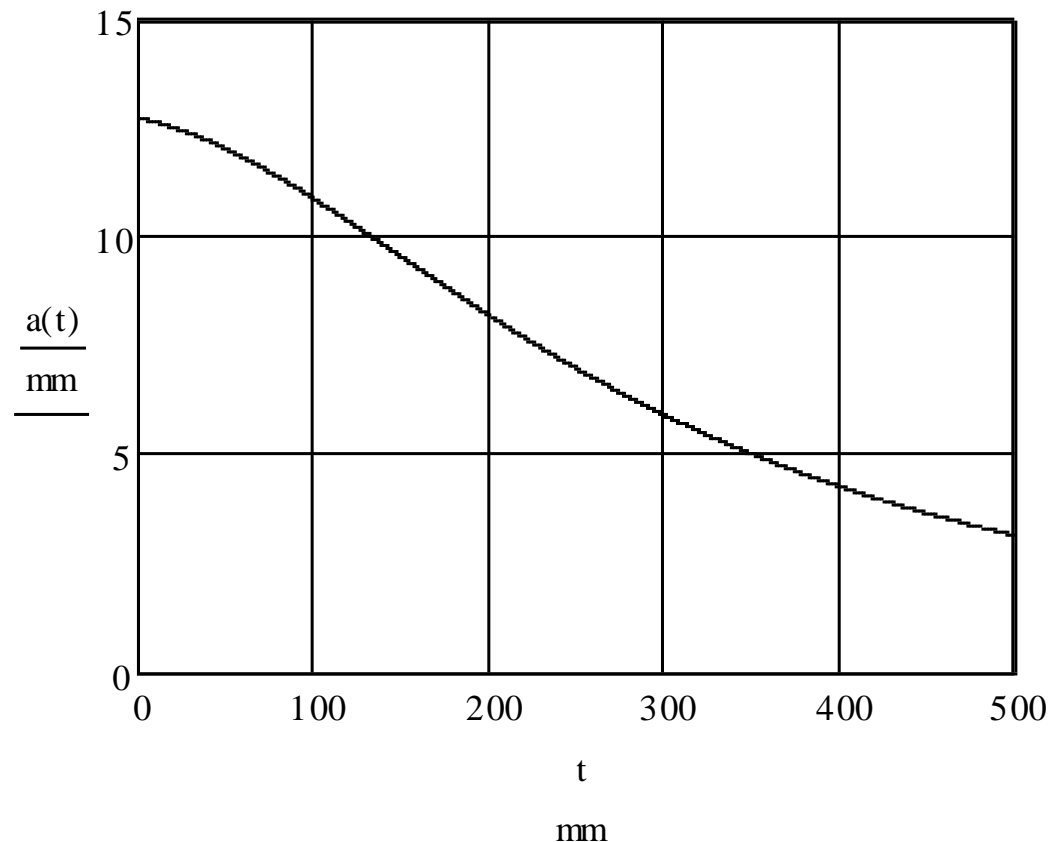
Results	$M$ (Nm)	$M\cos 45^\circ$ (Nm)	Difference
Wahrhaftig	70483.840	49839.601	342.127 (Nm)
Sarkis	70000.000	49497.475	0.686 (%)



# NUMERICAL SIMULATIONS

$$\tau_{\max} = \frac{M \frac{L}{2}}{La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3}$$

It allows performing to study the influence of the thickness of vertical plate over dimensions of the weld.

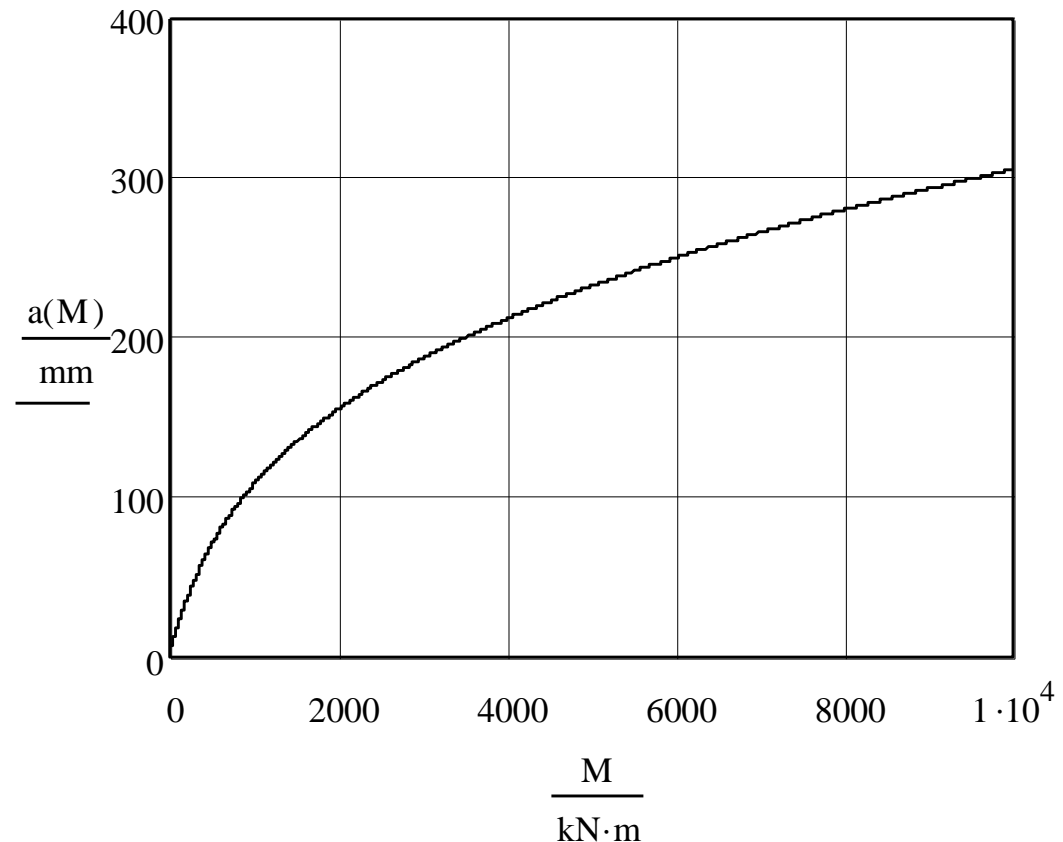




# NUMERICAL SIMULATIONS

$$\tau_{\max} = \frac{M \frac{L}{2}}{La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3}$$

It allows to obtain the **weld dimensions** in function of the **torsional moment** acting.

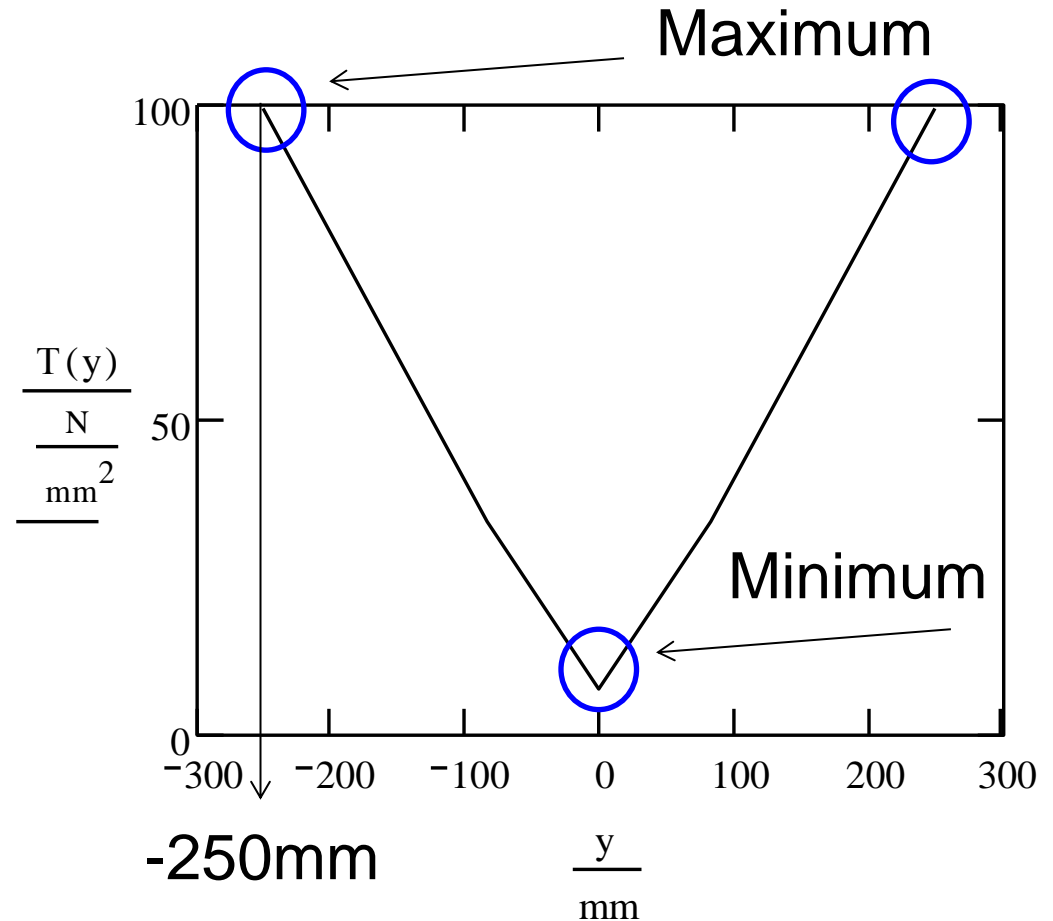




# NUMERICAL SIMULATIONS

$$\tau = \frac{M \rho}{La^3 + La^2t + \frac{1}{4}aLt^2 + \frac{1}{12}aL^3}$$

Shearing  
stress  
distribution  
on weld line  
to  $t = 12$  mm.



# CONCLUSIONS



- Analytical solution presented in this work (**Wahrhaftig**) is **appropriate** for the **design** and **verification** of **bead weld** to joints of perpendicular plates subjected to torsional moment;
- It allows evaluating of the **horizontal shearing stresses** induced in the way that it really occurs;





# CONCLUSIONS



- Difference of the polar moment of inertia between **Sarkis and Wahrhaftig** is 1.51%;
- Results by **Wahrhaftig** and **Sarkis** are consistent in order of magnitude; **but**
- **Sarkis**, considers the bending theory, while **Wahrhaftig** the torsion theory.

The author express its  
**gratitude to:**

**UFBa** – Federal University of  
Bahia, Brazil.

**Mech Aero 2015** – San  
Francisco, USA.

**Thank you  
very much!**

