THE WEIGHTED MEAN AND ITS IMPROVED DISPERSION

E.L. GRIGORESCU

National Institute for Nuclear Physics and Engineering, 30 Reactorului Str, P.O.Box MG-6, RO-077125 Bucharest-Magurele, Romania, E-mail: elgrigorescu@yahoo.com

INTRODUCTION

The weighted mean $\bar{x}_w$ appears when a physical quantity is measured by different methods in different laboratories, producing different x_i results. It is the case of determination of physical constants (as Nuclear Data analysis), International Comparisons of radioactive sources and still others. (The formula 1) with w_i - absolute weights and p_i- relative weights.

A relation exists (2) with σ_i- the individual standard deviations including possible systematic uncertainty as which do not affect the logical (1). Formula (2) is obtained with some complicated calculations in [1], [2], [3].

THE DISPERSIONS

To $\bar{x}_w$, two different dispersions are associated, D_1 (internal) and D_2 (external).

The formula (3) is obtained by error propagation applied on formula (1) and is valid for small σ_i. It supposes a satisfactory distribution of x_i results.

The formula (4) is the classical formula, valid for any σ_i and then, to be recommended. In literature suggestions are made, to take as valid the grater from D_1, D_2.

The ratio D_2/D_1 is in formula (5).

For $(D_2/D_1)>1$ is sufficient that a single x_i produces $r_i > 1$ or add many r_i not greater than one. So D_2 is the confident one.

A SIMPLE CALCULUS FOR $\bar{x}_w$

In principle $\bar{x}_w$ should produce as great deviations $(x_i - \bar{x}_w)$, as the associated σ_i are great.

For equal treatment of these deviations, relative deviations may be considered the formula (6) which express the deviations in units σ_i. They may be appreciated as "classical" deviations for a single σ.

The n values from (6) must have same near (equivalent) values.

Their arithmetical mean tends to zero (both positive and negative deviations).

Thus, as in the case of a unique σ, we may reach the minimum of the expression in formula (7).

The annulation of the x derivative for $\bar{x}_w$, gives the formula (8) and finally, the formula (9) where formula (10) (the well known formulae). This calculus accepts great σ_i even with systematic uncertainties.

AN IMPROVED DISPERSION

The value σ_i offered by the experimenters, represent the experimental standard deviations which fluctuates together x_i.

In practice the theoretical unknown values σ_i, are replaced in formulae (1), (2) by the experimental value s_i^2, which fluctuate together with x_i. A dispersion D_3 is obtained considering this, by error propagation, and added to D_1, D_2.

First are calculates the formula (11).

Then the formula (12) with $\varepsilon(w_i)$- the relative standard deviation for w_i and n_i is the values of measurements which provided x_i. For (12) it was used the equality (13). See [4] [5].

The relation (13) is valid for s^2 related to both a simple result as a complex one. It is obtained from χ_i.

$\varepsilon(s_i^2)$ express the fluctuation of the only statistical part of s_i^2 (systematic components do not fluctuate) and so formulae (13) may be applied.

Adding D_1 with D_3, a correct D_{1c} results the formula (14).

If σ_i are great, w_i are small but the spread $(x_i - \bar{x}_w)$ may be great, and vice-versa. So, to neglect D_3, n_i must have same values.

The corrected D_{2c} is obtained as the formula (15).

Again, to neglect D_3, n_i must have same values.

Many p_i values (many x_i) produce small D_3. The importance of D_3 depends of a given concrete case.

CONCLUSIONS

- σ_i may contain systematic uncertainties.
- D_2 is more to trust than D_1.
- A simple calculus may provide formulae for $\bar{x}_w$ and w_i.
- One may add to D_1, D_2, a D_3 representing the fluctuation of w_i.
- With n_i great, D_3 may be practically neglected.

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$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i} = \sum p_i x_i \quad (1)$$

$$w_i = \frac{1}{\sigma_i^2} \quad (2)$$

$$D_1 = \frac{1}{\sum w_i} \quad (3)$$

$$D_2 = \frac{\sum w_i (x_i - \bar{x}_w)^2}{\sum w_i} \quad (4)$$

$$\frac{D_2}{D_1} = \sum w_i (x_i - \bar{x}_w)^2 = \sum \frac{(x_i - \bar{x}_w)^2}{\sigma_i^2} = \sum r_i^2 \quad (5)$$

$$d_i = \frac{x_i - \bar{x}_w}{\sigma_i} \quad (6)$$

$$\sum_1^n \frac{(x_i - \bar{x}_w)^2}{\sigma_i^2} \quad (7)$$

$$\sum_1^n \frac{(x_i - \bar{x}_w)}{\sigma_i^2} = \sum_1^n \frac{x_i}{\sigma_i^2} - \bar{x}_w \sum_1^n \frac{1}{\sigma_i^2} = 0 \quad (8)$$

$$\bar{x}_w = \frac{\sum_1^n \frac{x_i}{\sigma_i^2}}{\sum_1^n \frac{1}{\sigma_i^2}} = \sum p_i x_i \quad (9)$$

$$p_i = \frac{\frac{1}{\sigma_i^2}}{\sum_1^n \frac{1}{\sigma_i^2}} \quad (10)$$

$$\frac{\partial \bar{x}_w}{\partial w_i} = \frac{x_i \sum w_i - \sum w_i x_i}{(\sum w_i)^2} = \frac{x_i - \bar{x}_w}{\sum w_i} \quad (11)$$

$$D_3 = \frac{\sum (x_i - \bar{x}_w)^2 \sigma^2(w_i)}{(\sum w_i)^2} = \frac{\sum w_i^2 (x_i - \bar{x}_w)^2 \varepsilon^2(w_i)}{(\sum w_i)^2} = \frac{\sum w_i p_i (x_i - \bar{x}_w)^2 \frac{2}{n_i}}{\sum w_i} \quad (12)$$

$$\varepsilon(w_i) = \varepsilon(s_i^2) = \sqrt{\frac{2}{n_i}} \quad (13)$$

$$D_{1c} = \left(\frac{1}{\sum w_i}\right) D_1 \left[1 + \sum w_i p_i (x_i - \bar{x}_w)^2 \frac{2}{n_i}\right] \quad (14)$$

$$D_{2c} = \sum p_i (x_i - \bar{x}_w)^2 + \sum p_i^2 (x_i - \bar{x}_w)^2 \frac{2}{n_i} = \sum p_i (x_i - \bar{x}_w)^2 \left(1 + p_i \frac{2}{n_i}\right) \quad (15)$$