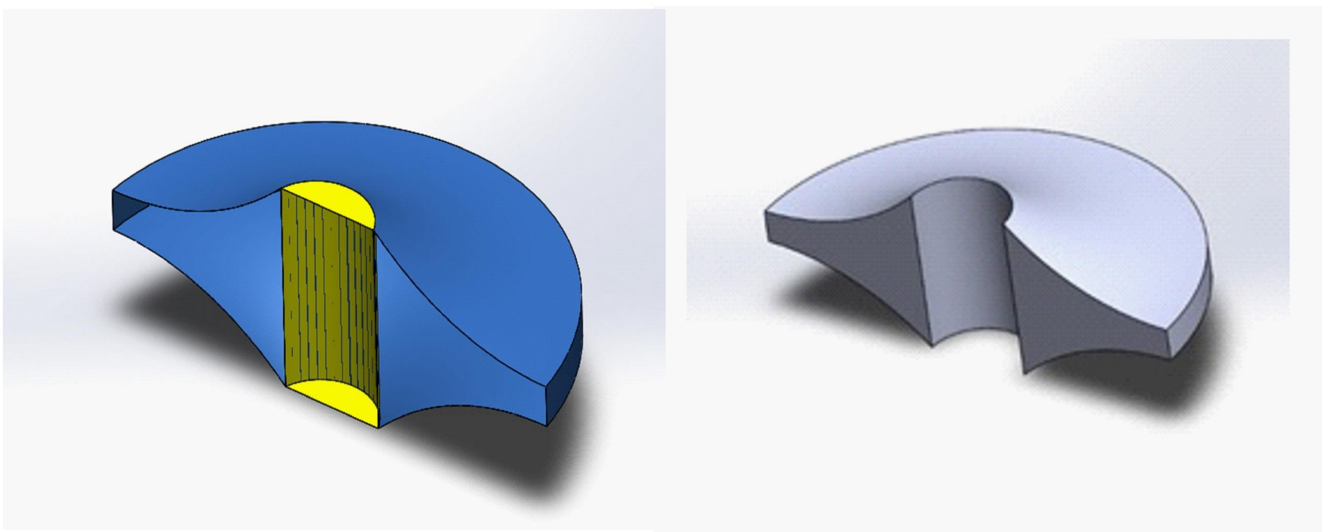
Solutions for the polar orthotropic functionally graded annular disks having variable profile

Saad Essa

Erbil Technical Engineering College – Erbil Polytechnic University - Iraq

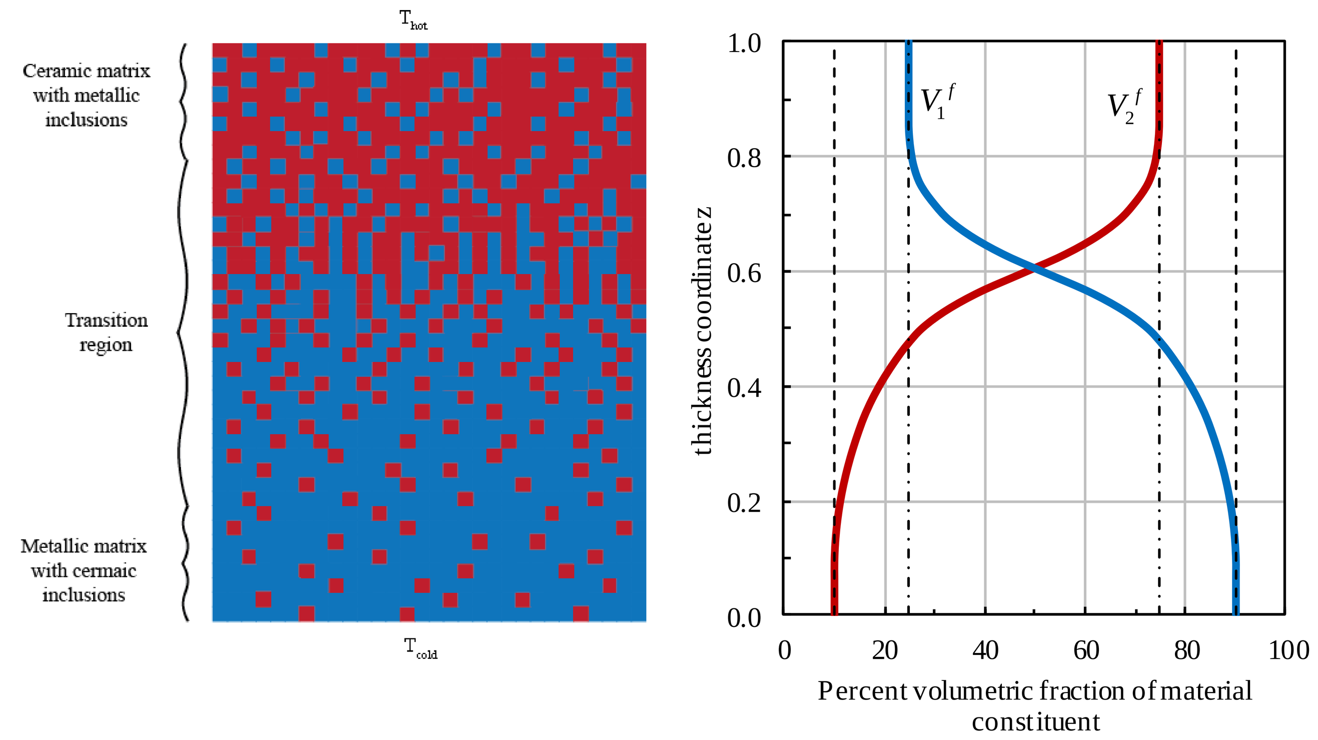
OVERVIEW

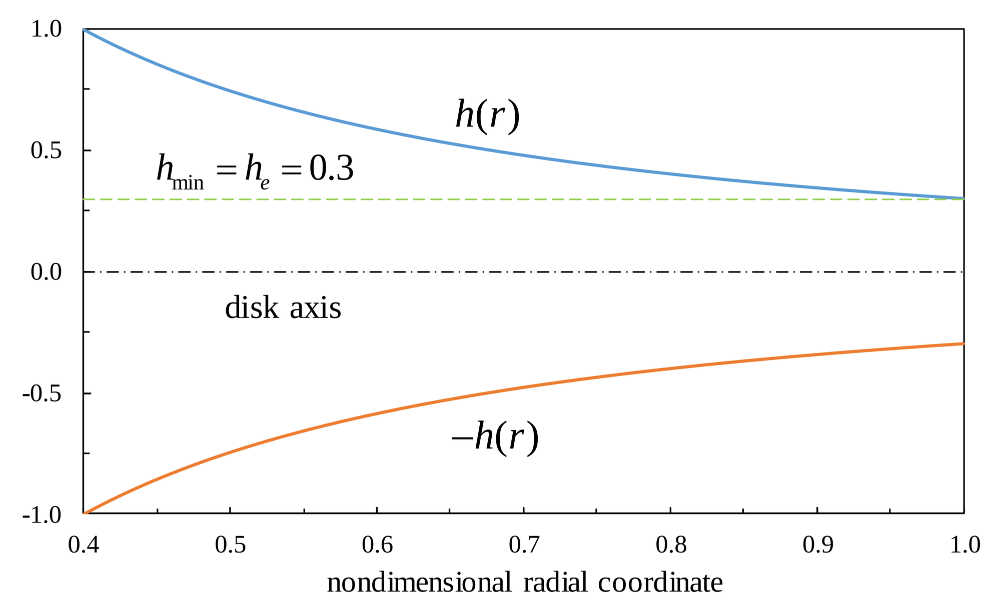
In this study, analytical solutions are presented for the polar orthotropic functionally graded annular disks having variable profile and rotating with constant angular velocity. The formulations are performed by referring to polar coordinate system. Small deformations, state of plane stress and rotational symmetry are presumed in the formulations. Elasticity moduli, disk thickness and density vary radially according to power law while Poisson's ratios are constant valued. Rotating disks having two different types of boundary conditions are studied. These are (i) annular disks having traction free inner and outer surfaces, and, (ii) annular disks mounted on a circular rigid shaft having traction free outer surface. Two sample problems are presented that accounts the solutions developed for the two different boundary conditions considered in the study. The stress and displacement profiles are determined at the elastic limit angular velocities which are evaluated according to Hill's quadratic yield criteria. The effects of the degree of orthotropy are also examined.



OBJECTIVE

This research investigates an elastic problem of polar orthotropic rotating annular disks having functionally graded materials based on constant angular velocity. The influence of orthotropy degree on the elastic field of annular rotating disks in particular the radial and circumferential stress distribution is considered.





Annular disk profile, $h_0 = 1.0$, $h_e = 0.3$ and $k_h = -1.31396$

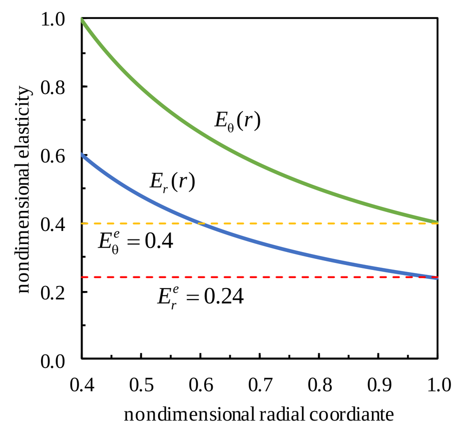
MATERIALS AND METHODS

The complex microstructure of FGM is the key to interpret their mechanical properties, so it has been widely studied by many authors for years. For example, cermet material is composed of two phases: one is the ceramic phase (titanium carbonitride hard phase) and the other is metal binder phase (nickel or cobalt or a mixture of them) which bonds the ceramic phase. In this work, it is assumed that, the material properties are graded according to power law as

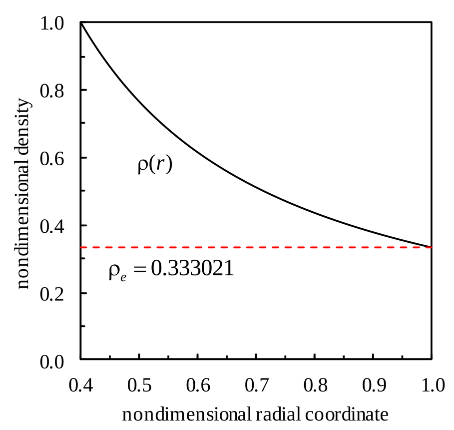
$$E_\theta(r) = \Phi_\theta^0 r^k \quad ; \quad E_r(r) = \Phi_r^0 r^k \quad ; \quad \rho(r) = \Phi_\rho^0 r^{k_\rho}$$

$$\Phi_\theta^0 = E_\theta^0 a^{-k} \quad ; \quad \Phi_r^0 = E_r^0 a^{-k} \quad ; \quad \Phi_\rho^0 = \rho^0 a^{-k_\rho}$$

and the material yields according to Hill's yield criteria.



Variations of and $E_\theta(r)$, $E_r(r)$ $k = -1.0$ and $S = 5/3$

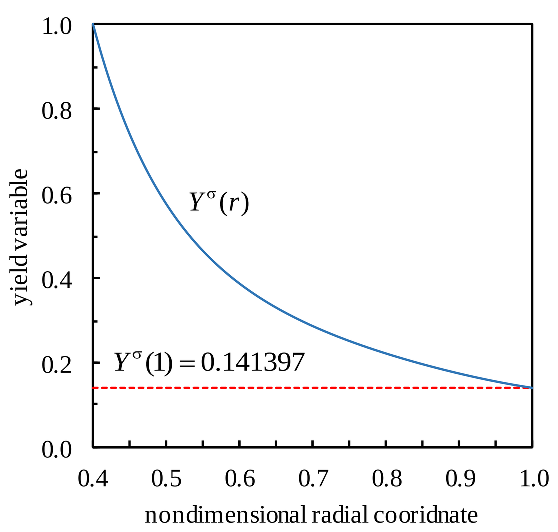


Variation of density along radial coordinate. $k = -1.2$

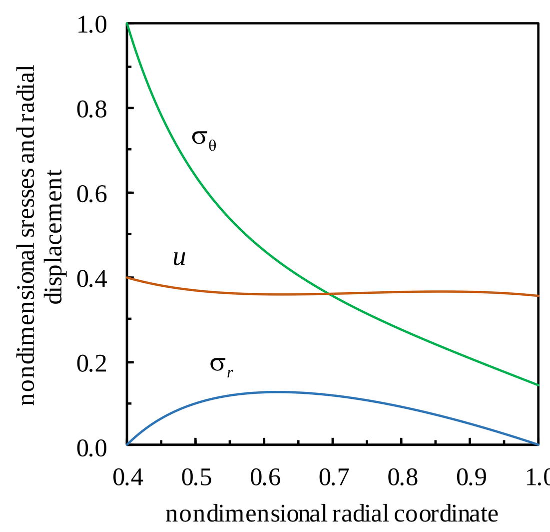
$$\frac{d^2Y}{dr^2} - \frac{1}{r}\left[-1 + k + k_h\right]\frac{dY}{dr} + \frac{1}{r^2} S\left[-1 + \left(k + k_h\right)\nu\right] Y = -r^{1+k_h+k_\rho}\left[3 - k + k_\rho + S\nu\right]\Phi_h^0 \Phi_\rho^0 \omega^2$$

RESULTS

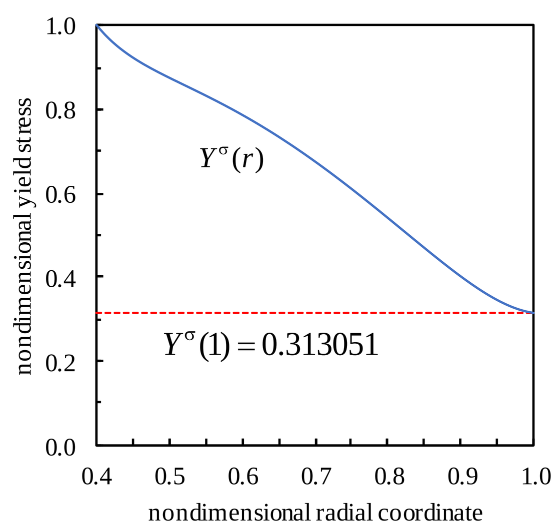
The degree of orthotropy is described by the orthotropy parameter (orthotropy index) $S = E_\theta(r)/E_r(r)$ which represents the ratio of circumferential to radial elasticity modulus. The disk model is variable thickness and material properties rotating disk subjected to angular velocity in free-free and fixed-free boundary conditions. The orthotropy degree is selected as $S = 5/3$ and Poisson's ratios $\nu = 0.3$. Figures below depict the stress, displacement and yield variable profiles. Note that $Y(r)$ is the nondimensional yield variable and yielding commences when $Y(r) = 1.0$. The maximum angular velocities for both FF and RF are respectively $\omega = 1.448109$ and $\omega = 3.228618$



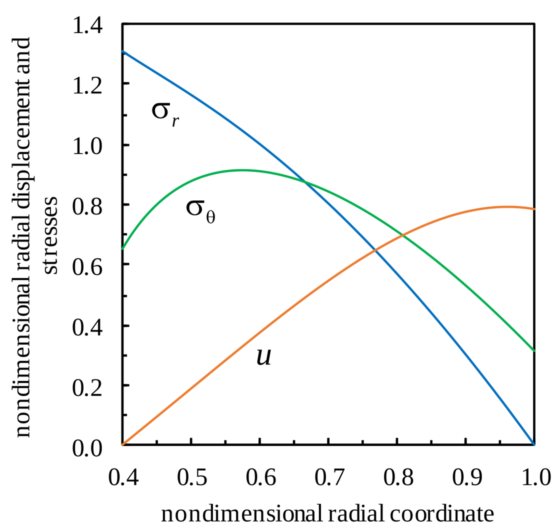
The variation yield variable $Y^\sigma(r)$ along nondimensional radial coordinate for the FF VD



The variation of nondimensional stresses and radial displacement with nondimensional radial coordinate for the FF VD



The variation yield variable $Y^\sigma(r)$ along nondimensional radial coordinate for the RF VD



The variation of nondimensional stresses and radial displacement with nondimensional radial coordinate for the RF VD

CONCLUSIONS

Within this study, analytical solution is considered for polar orthotropic annular functionally graded rotating disks by taking also thickness variation into account. Disks are rotating with constant angular velocity and the two types of boundary conditions are considered. The first one is an annular disk having traction free inner and outer surfaces, whereas, the second has a rigid inclusion within and traction free outer surface. The problem is treated under the assumptions of small deformations and plane stress. Elastic limit angular velocities are also determined assuming that the plastic deformation starts according to Hill's yield criteria. It is seen that, the effect of orthotropy degree on the stress distributions has more significant effects on the disks that have a rigid inclusion within and traction free outer surface.

The aim of this study may help engineering designers to select an appropriate gradient, orthotropic degree, and materials to achieve an optimal state of a rotating functionally graded orthotropic annular disk.

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