Rossby modes in neutron stars as sources of gravitational waves

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INTRODUCTION

We explore the Rossby mode instabilities in neutron stars as sources of gravitational waves. The intensity and time evolution of the emitted gravitational waves in terms of the amplitude of the strain tensor are estimated in the slow rotation approximation using β-equilibrated neutron star matter obtained from density dependent M3Y effective interaction [1,2]. For a wide range of neutron star masses, the fiducial gravitational and various viscous time scales, the critical frequencies and the time evolutions of the frequencies are calculated. The dissipative mechanism of the Rossby modes is considered to be driven by shear viscosity along the boundary layer of the solid crust-liquid core interface as well as in the core and bulk viscosity. It is found that neutron stars with slower frequency of rotation, for the same mass, radius and surface temperature, are expected to emit gravitational waves of higher intensity.

Core-Crust Transition in β-equilibrated Neutron Star matter

In a Fermi gas model of interacting neutrons and protons, with isospin asymmetry $X = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$; $\rho = \rho_n + \rho_p$; where ρ_n, ρ_p and ρ are the neutron, proton and baryonic number densities respectively, the energy per baryon for isospin asymmetric nuclear matter can be derived as $E/A = \epsilon = \left[\frac{3\hbar^2 k_F^2}{10m}\right] F(X) + C(1-\beta\rho^n)\rho J_v/2$ …..(1)

where m is the baryonic rest mass, $K_F = (1.5\pi^2\rho)^{1/3}$ which equals Fermi momentum in case of SNM, the kinetic energy per baryon $\epsilon_{kin} = \left[3\hbar^2 k_F^2/10m\right] F(X)$ with $F(X) = \left[(1+X)^{5/3} + (1-X)^{5/3}\right]/2$ and $J_v = J_{v00} + X^2 J_{v01}$, J_{v00} and J_{v01} represent the volume integrals of the isoscalar and the isovector parts of the M3Y interaction.

The β-equilibrated nuclear matter EoS is obtained by evaluating the asymmetric nuclear matter EoS at the isospin asymmetry $X = 1 - 2x_p$ determined from the β-equilibrium proton fraction $x_p [= \rho_p/\rho]$, obtained by solving $\hbar c(3\pi^2\rho x_p)^{1/3} = -\frac{\partial\epsilon(\rho, x_p)}{\partial x_p} = +2\frac{\partial\epsilon}{\partial X}$ ……(2)

The thermodynamical method requires the system to obey the intrinsic stability condition $V_{thermal} > 0$ which is given by $V_{thermal} = \rho^2\left(2\rho\frac{\partial\epsilon^b}{\partial\rho} + \rho^2\frac{\partial^2\epsilon^b}{\partial\rho^2} - \rho^2\frac{\left(\epsilon^b_{\rho x_p}\right)^2}{\epsilon_{x_p}\epsilon_{x_p}}\right)$ ……(3),

it goes to zero at the inner edge separating the liquid core from the solid crust since it corresponds to a phase transition from the homogeneous matter at high densities to the inhomogeneous matter at low densities. The core-crust transition density ρ_t, pressure P_t and proton fraction x_p (t) of the neutron stars are obtained [10] by setting $V_{thermal}$ = 0 which goes to negative with decreasing density.

Crustal Fraction of Moment of Inertia in Neutron Stars

The crustal fraction of the moment of inertia $\frac{\Delta I}{I}$ can be expressed as a function of M(gravitational mass of the star) and R (radius of the star) with the only dependence on the equation of state arising from the values of transition density ρ_t and pressure P_t. Actually, the major dependence is on the value of P_t, since ρ_t enters only as a correction in the following approximate formula [3]

$$\frac{\Delta I}{I} = \frac{28\pi P_t R^3}{3Mc^2}\left(\frac{1-1.67\zeta-0.6\zeta^2}{\zeta}\right)\left(1 + \frac{2P_t(1+7\zeta)(1-2\zeta)}{\rho_t m_b c^2\zeta^2}\right)^{-1} \quad …….(4)$$

where $\zeta = \frac{GM}{Rc^2}$. The angular momentum requirements of glitches in the Vela pulsar indicate that more than 0.014 of the moment of inertia drives these events. So, if glitches originate in the liquid of the inner crust, this means that $\frac{\Delta I}{I} > 1.4\%$.

Theoretical Method

The quantity which is of crucial importance in the evaluation of various timescales is the integral $\int_0^{R_c}\rho(r)r^6 dr$. This integral can be rewritten in terms of energy density $\epsilon(r) = \rho(r)c^2$ and can be expressed in the dimensionless form as: $I(R_c) = \int_0^{R_c}\left[\frac{\epsilon(r)}{\text{MeV fm}^{-3}}\right]\left(\frac{r}{\text{km}}\right)^6 d\left(\frac{r}{\text{km}}\right)$. The fiducial gravitational radiation timescale $\tilde{\tau}_{GR}$ is given by $\frac{1}{\tau_{GR}} = -\frac{32\pi G\Omega^{2l+2}}{c^{2l+3}}\frac{(l-1)^{2l}}{[(2l+1)!!]^2}\left(\frac{l+2}{l+1}\right)^{(2l+2)} \times \int_0^{R_c}\rho(r)r^{2l+2}dr,$ where R_c and r are in km and M is in $M_\odot$. Bulk and shear viscous time scales in the fluid core and viscous dissipation in the crust-core boundary layer are:

$$\frac{1}{\tau_{BV}} = \frac{4\pi R^{2l-2}}{690}\left(\frac{\Omega}{\Omega_0}\right)^4\left(\int_0^{R_c}\rho(r)r^{2l+2}dr\right)^{-1} \times \int_0^{R_c}\xi_{BV}\left(\frac{r}{R}\right)^6\left[1+0.86\left(\frac{r}{R}\right)^2\right]r^2 dr \quad (10)$$

where ξ_{BV} is the bulk viscosity, the shear viscous dissipation time scale $1/\tau_{SV}$ in the fluid core is given by [3]

$$\frac{1}{\tau_{SV}} = (l-1)(2l+1)\left(\int_0^{R_c}\rho(r)r^{2l+2}dr\right)^{-1}\int_0^{R_c}\eta_{SV}r^{2l}dr \quad (11)$$

where η_{SV} is the shear viscosity, and for the viscous dissipation in the crust-core boundary layer

$$\frac{1}{\tau_{VE}} = \left[\frac{1}{2\Omega}\frac{2^{l+3/2}(l+1)!}{l(2l+1)!!I_l}\sqrt{\frac{2\Omega R_c^2\rho_c}{\eta_c}}\right]^{-1} \times \left[\int_0^{R_c}\frac{\rho(r)}{\rho_c}\left(\frac{r}{R_c}\right)^{2l+2}\frac{dr}{R_c}\right]^{-1}, \quad (12)$$

Result and Discussion

- In Fig.1, plots of the fiducial timescales with neutron star gravitational mass are shown for the DDM3Y EoS. It is found that the gravitational radiation time scale falls rapidly with increasing mass. At the junction the shear viscous damping time scales increase approximately linearly while the shear viscous damping time scales in the core show opposite trend. We have also found that the contributions from the neutron-neutron scattering are more than that for the electron-electron scattering. However, the bulk viscous damping time scale remains almost constant with respect to NS masses.
- In Ref.[4], the spin frequencies and core temperatures (measurements & upper limits) of observed low-mass x-ray binaries and millisecond radio pulsars are listed, and in Fig.2, their positions in the critical frequency versus temperature plot are shown to compare with observational data. From Fig.2, it is interesting to note that according to our model of the EoS with a rigid crust and a relatively small r-mode amplitude, all of the observed neutron stars lie in the stable r-mode region, which is consistent with the absence of observation of gravitational radiation due to r-mode instability.
- In Fig.-3, angular frequency has been plotted as functions of time for rotating NSs of 3 different masses.
- In Fig.-4, the amplitude of the strain tensor h_0 has been plotted as a function of distance for two different NS masses rotating with two fixed frequencies of 200 Hz and 600 Hz. In Fig.-5, the amplitude of the strain tensor h_0 has been plotted as a function of evolving angular velocity of the star in units of rad s^{-1} resulting from the Gravitational Wave emission for three different NS masses.

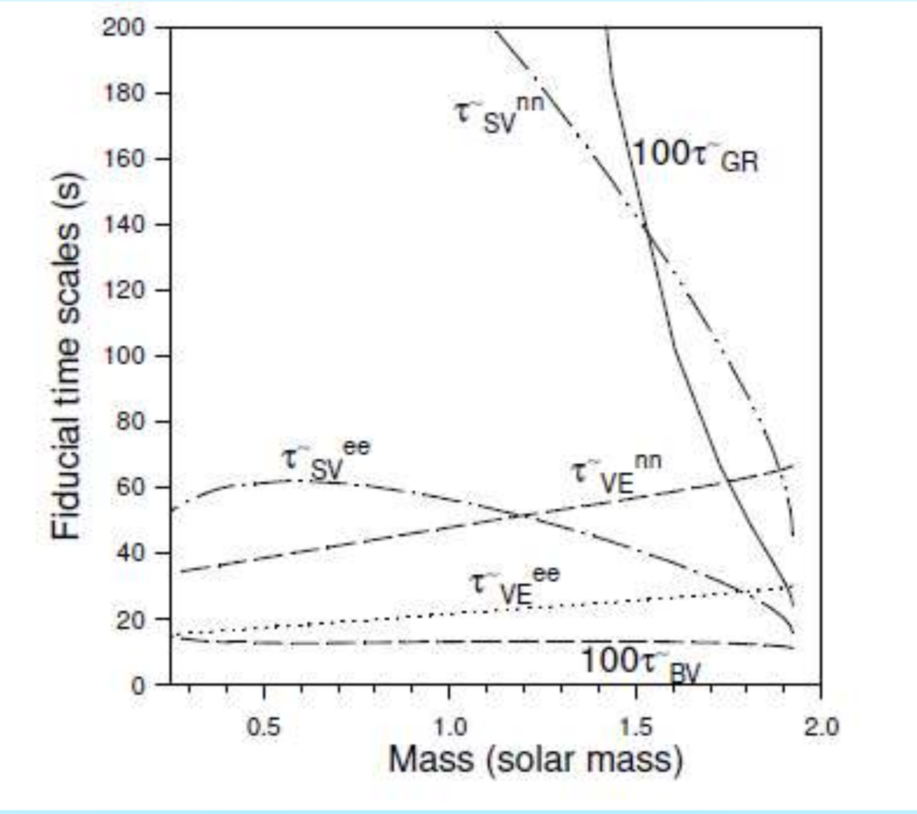

FIG.1: Plots of fiducial timescales with gravitational mass of NSs.

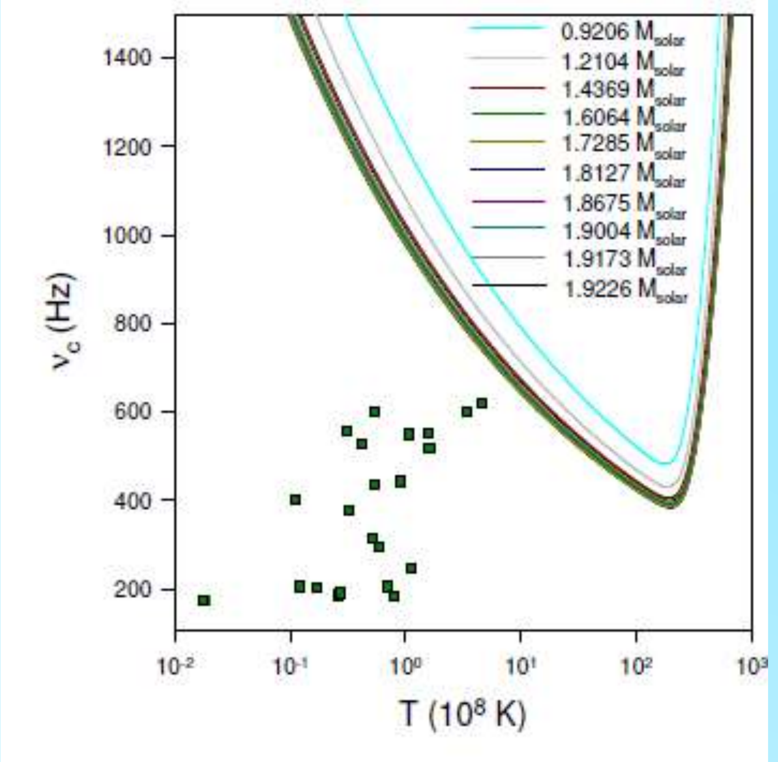

FIG.2: Plots of critical frequency with temperature for different masses of neutron stars. The square dots represent observational data [4].

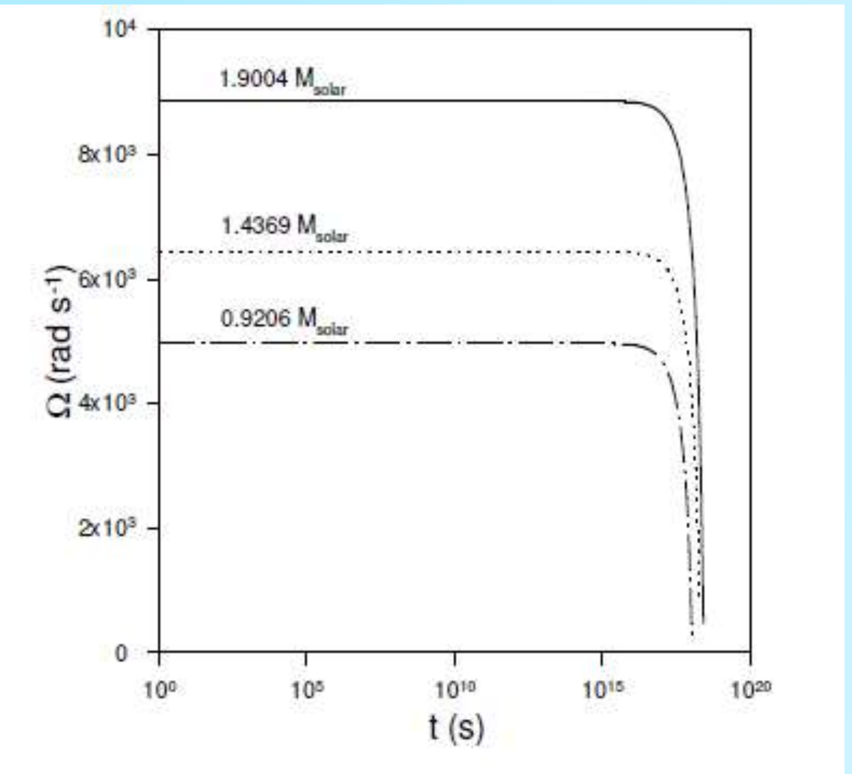

FIG.3: Plots of time evolution of angular frequencies.

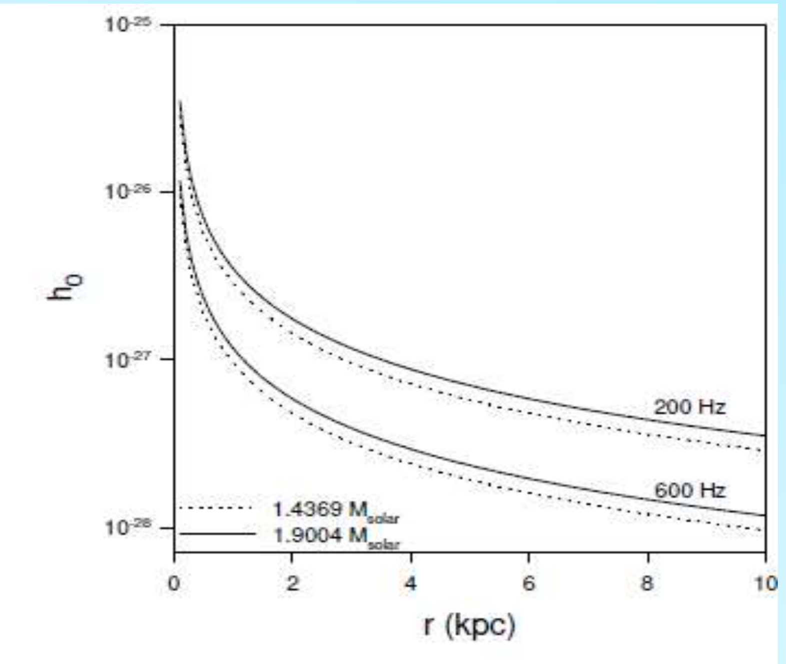

FIG.4: Plots of strain tensor amplitude as a function of distance.

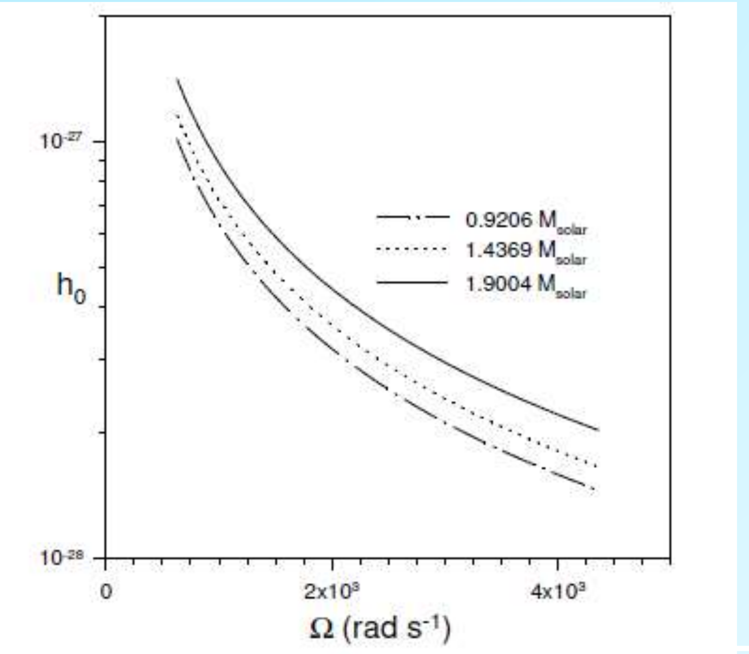

FIG.5: Plots of strain tensor amplitude as a function of reduced angular frequency from 'thermal equilibrium'.

Summary and Conclusion

- The fiducial time scales are calculated for gravitational radiation, bulk viscosity, shear viscosity in the core and at the junction for r-mode damping mechanism. It is observed that the gravitational radiation and shear viscosity in the core time scales decrease with increasing neutron star mass, the shear viscous damping time scales at the junction exhibit an approximate linear increase while bulk viscous time scale remains almost constant.
- We have studied the temperature dependence of the critical angular frequency for different neutron star masses. It is observed that the majority of the neutron stars do not lie in the r-mode instability region [5].
- In Fig.-2, the straight line at $\nu_c/\nu_0 = 2/3$ intersects each curve at two points providing two values of the critical temperature T_c. For temperatures below lower T_c. any perturbation is suppressed due to the dominance of shear viscosity while for temperatures above higher T_c. any perturbation is suppressed due to the dominance of bulk viscosity.
- The conclusion is that massive hot neutron stars are more susceptible to r-mode instability through gravitational radiation. The spin down rates and angular frequency evolution of the neutron stars through gravitational emission due to r-mode perturbation are calculated. The saturation value of r-mode amplitude α_r has been estimated from considerations of 'spin' and 'thermal' equilibria. Subsequently, the amplitude of the strain tensor h_0 has been calculated as a function of and observer distance.

References

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