AND CONSTANT FLUX HEAT SOURCE ON THE BOTTOM WALL

Faculty of process Engineering, University Salah Boubnider Constantine 3, Algeria
Sacia KACHI[1*], Saadoun BOUDEBOUS[2]
Corresponding Author: kachi.sacia@gmail.com

1. INTRODUTION

The study of the movement of a fluid in mixed convection in the cavities is frequently encountered in nature and in different industrial systems. This movement results from complex interactions within this medium (fluid) or between different media as soon as there is a temperature gradient. This interaction is responsible for the resulting diversity of fluid flows (the bifurcation), such as several studies both numerically and experimentally concerning this phenomenon of changing the very nature of the flow in different geometric configurations. natural convection regime, forced or mixed have been reported in the literature.

The present study differs from Aydin and Yang [1] and Guo and Sharif [2] in that here the direction of the displacement of the sidewalls are reversed thus creating a competition between the forced convection and the natural Rayleigh-Bénard convection. Therefore, it is interesting to establish the flow pattern and to predict the various critical values of the Richardson number for the occurrence of loss of symmetry and bifurcations, if these are indeed present in the fluid flow.

2. OBJECTIF

The main purpose of this study is to determine the different characteristics of the flow in this cavity.

3. MODEL DESCRIPTION

The physical model considered here is shown in Figure 1; which is a square cavity filled with air. The vertical walls are maintained at a constant cold temperature T_C and move simultaneously with an upward constant velocity V_0. A heat source is located at the medium of the lower wall of the cavity having a length L equal to the fourth fifth of that of the cavity and subjected to a constant heat flux q''. The other walls are supposed to be adiabatic.

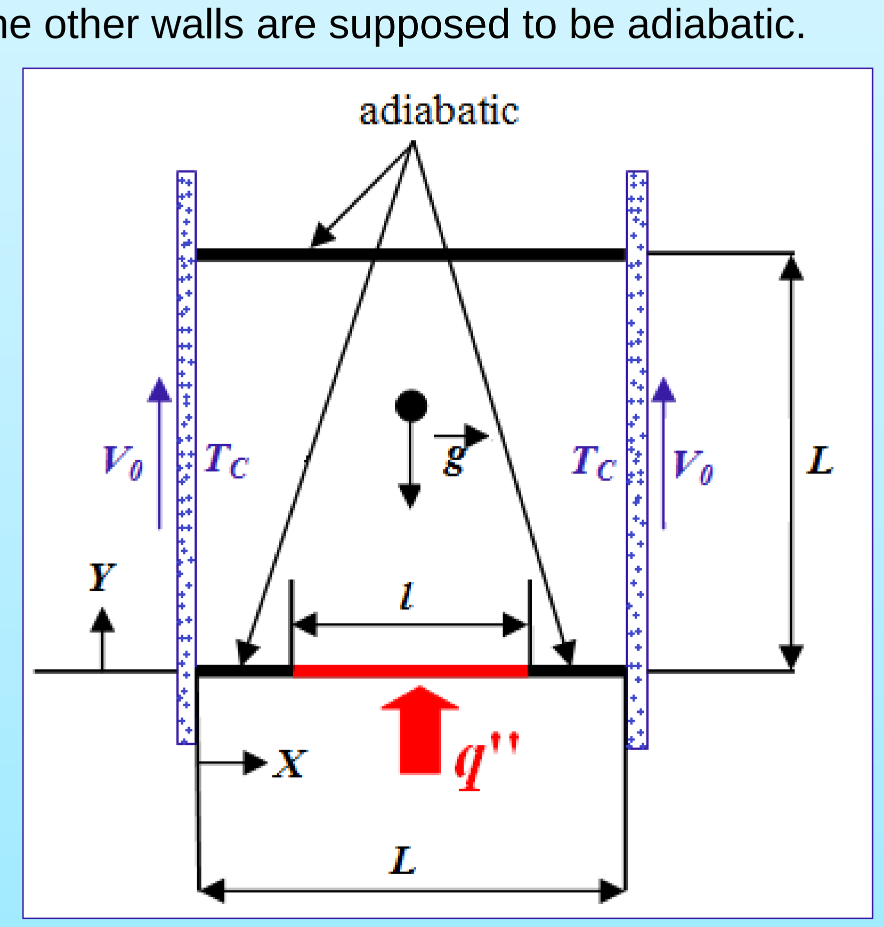

Figure 1: Schematic diagram of the physical model.

4. MATHEMATICAL EXPRESSIONS

The mixed convection phenomena to be investigated here are described by the complete Navier-Stokes and energy equations for two-dimensional laminar incompressible flow. The viscous dissipation term in the energy equation is neglected and the classical Boussinesq approximation is invoked for the buoyancy inducted body force term in the momentum equation. The 2-D governing equations are transformed into stream function-vorticity (Ψ-Ω) formulation and can be written in non-dimensional forms:

4.1. Model Equations:

4.1.1. Energy Transport Equation:

$$\frac{\partial\theta}{\partial\tau}+U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{1}{\mathrm{Re.Pr}}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \quad (1)$$

4.1.2. Vorticity Transport Equation:

$$\frac{\partial\Omega}{\partial\tau}+U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right)+\frac{Gr}{\mathrm{Re}^2}\cdot\frac{\partial\theta}{\partial X} \quad (2)$$

4.1.3. Stream Function Equation:

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega \quad (3)$$

4.1.4. Components of the Velocity:

$$U=\frac{\partial\Psi}{\partial Y}\ et\ V=-\frac{\partial\Psi}{\partial X} \quad (4)$$

The dimensionless vorticity is defined by:

$$\Omega=\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}$$

4.2. Initial and boundary conditions:

- The initial conditions $(\tau=0)$ are :

$$0\langle Y\langle 1\ \&\ 0\langle X\langle 1:\theta=0;\Psi=0;\Omega=0$$

- The boundary conditions $(\tau\rangle 0)$ are :
- ✓ Top wall:

$$Y=1;0\langle X\langle 1\Rightarrow U=V=\Psi=0\ \&\ \frac{\partial\theta}{\partial Y}=0 \quad (4a)$$

- ✓ Bottom wall:

$$Y=0\Rightarrow U=V=\Psi=0\ \&\ \frac{\partial\theta}{\partial Y}=\begin{cases}0;\ for:0\langle X\langle\frac{1-\varepsilon}{2}\\ -1;\ for:\frac{1-\varepsilon}{2}\langle X\langle\frac{1+\varepsilon}{2}\\ 0;\ for:\frac{1+\varepsilon}{2}\langle X\langle 1\end{cases} \quad (4b)$$

- ✓ Right and left wall:

$$\left.\begin{matrix}X=0\\X=1\end{matrix}\right\}\ \&\ 0\langle Y\langle 1\Rightarrow U=\theta=\Psi=0;V=1 \quad (4c)$$

The condition $\frac{\partial\theta}{\partial Y}=-1;\ for:\frac{1-\varepsilon}{2}\langle X\langle\frac{1+\varepsilon}{2}$ at the bottom wall arises as a consequence of constant heat flux q''.

Where *Re*, *Pr* and *Ri* denote, respectively, Reynolds, Prandtl and Richardson numbers and the dimensionless variables are defined as:

$$X=\frac{x}{L};Y=\frac{y}{L};U=\frac{u}{V_0};V=\frac{v}{V_0};\tau=t\frac{V_0}{L};\theta=\frac{k(T-T_C)}{q''L};\Omega=\frac{L}{V_0}\omega;$$

$$\Psi=\frac{\psi}{LV_0};\varepsilon=\frac{l}{L};\mathrm{Re}=\frac{V_0L}{\nu};\mathrm{Pr}=\frac{\nu}{\alpha};Ri=\frac{Gr}{\mathrm{Re}^2};Gr=\frac{g\beta q''L^4}{k\nu^2}$$

Where $\boldsymbol{\varepsilon}$ is the dimensionless length of the heat source.

5. NUMERICAL METHOD

5.1. Discretization:

The system of Eqs. (1-4), together with the boundary conditions Eq. (4a-c) have been discretized and solved using the finite difference method. For solving nonlinear systems of differential partial equations, the fourth-order Runge-Kutta method is known to be quite effective compared to other methods. The convective terms in Eqs. (1-2) are discretized using the accurate third order upwind scheme of Kawamura [2] taking into account the sign of the velocity. A fourth-order centered scheme was adopted for the discretization of the diffusive terms, the source term in Eq. (2), and the explicit evaluation of the *U* and *V* components of the velocity vector in Eq. (4).

The mirror-point technique due to Leonar [3] was used to maintain the fourth-order centered scheme of the first and second spatial partial derivatives in the grid points adjacent to the walls. An iterative procedure based on successive Non Linear Over Relaxation method (NLOR) [4] was used to solve the discretized stream function Eq. (3) in each time step of Runge-Kutta procedure. The FORTRAN language was elaborated like a calculation program. The iterative procedure is stopped when the maximum relative change in stream function between two consecutive iterations is less than 10^{-6}.

The dimensionless local and average Nusselt numbers of the hot part of the bottom wall are defined, respectively by Guo and Sharif [2] :

$$Nu(X)=\frac{h.L}{\lambda}=\frac{1}{\theta w(X)};\overline{Nu}=\frac{1}{\varepsilon}\int_0^{\varepsilon}Nu(X)dX$$

The average Nusselt number (Nu_{av}) is integrated using Simpson's rule.

5.2. Choice of the grid:

In order to make the numerical solution independent of the step values, we preceded different simulations, by comparing the values of the average Nusselt number with *Ri = 10, Re = 100 and Pr = 0.71*. We noted a weaker difference between the values of the average Nusselt number, determined with a grid of 81x81, 101x101, 161x161 and of 201x201 according to the following table 1:

Table 1: Comparison of the average Nusselt number for various grid dimensions.

Grid	Average Nusselt number	Relative error in %
201x201	5.68913	-----
161x161	5.68334	0.10
101x101	**5.66761**	**0.40**
81x81	5.65734	0.56

These comparisons allow us the choice of the grid **(101x101)** because it provides a good compromise between the duration of the computing time and the precision of these calculations.

6. RESULTATS AND DISCUSSION

The working fluid is chosen as air with Prandtl number, *Pr* = 0.71 and Reynolds number, fixed at 100. The normalized length of the constant flux heat source at the bottom wall, ε, is equal to 0.8, with a step of the non-dimensional time fixed at 10^{-4}.

The bifurcation of the regime of the flow in mixed convection was highlighted for Richardson numbers equal to 15.6 and 15.7. The results are presented in the forms of the flow and temperature fields (streamlines contour and isotherms contour) and the evolution of the average Nusselt number.

6.1. Temperature contours:

These contours are represented in figure 2, for Ri = 15.6, we note that a thermal stratification exists near the heated part. The upward movement of the side walls drives the fluid layers adjacent to the walls upwards by the viscous forces; we also notice that the cold temperature prevails throughout the upper part of the cavity. In fact, the low heat flows recovered by the fluid from the heat source are directly evacuated through the lower part of the vertical walls, whereas for Ri = 15.7. We find that the distribution of the temperature in the cavity is mainly characterized by the loss of the symmetry. The heat recovered from the heated part of the cavity is transported mainly by the right wall of the cavity.

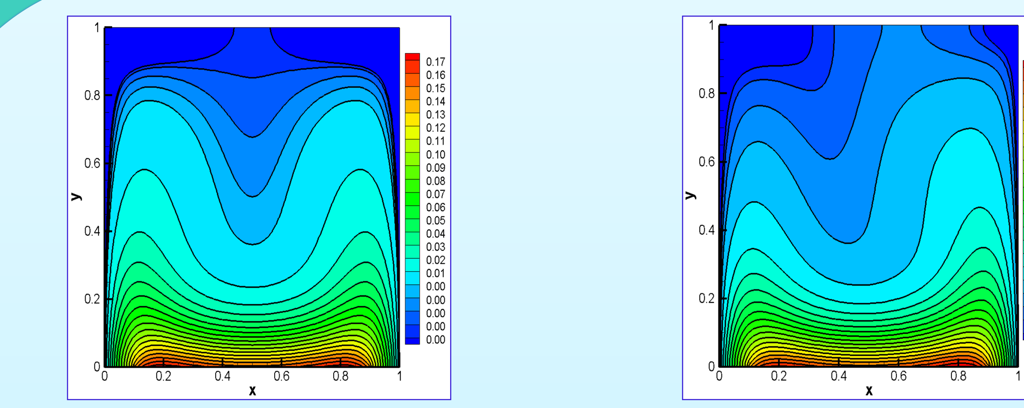

Figure 2: Isotherms for Ri = 15.6 on the left and Ri = 15.7 on the right.

6.2. Streamlines contour:

These contours are shown as iso-currents in Figure 6 for the two Richardson number values considered. For Ri = 15.6, the flow consists of two symmetric main cells because the boundary conditions are symmetric. A bifurcation towards an asymmetric flow regime characterized by the sudden appearance of two main but dissymmetrical cells is demonstrated when the value of this same parameter increases by just 0.1 (Ri = 15.7). The two contra-rotating cells of different shapes and intensities are observed, the first, counterclockwise, occupying two thirds of the cavity and the second clockwise on the left.

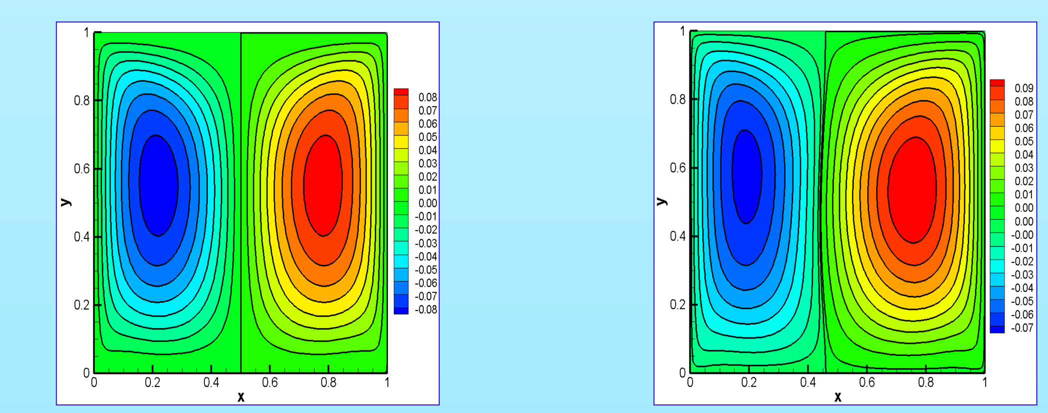

Figure 3: Streamlines for Ri = 15.6 on the left and Ri = 15.7 on the right.

7. CONCLUSION

A mathematical model to simulate mixed convective heat transfer in a two-dimensional square cavity and the associated computer coding has been developed. The model is applied to analyze mixed convection in a square cavity where the cold isothermal vertical sidewalls are moving with constant upward velocity and are subjected to a cold temperature. A constant flux heat source is placed at the bottom wall. The moving sidewalls are an idealization of cold air jet blown across the cavity. The cooling airflow caused by the shearing action of the moving sidewalls interacts with the buoyancy-driven flow due to the heat source at the bottom. The other parts of this cavity are considered adiabatic. The preliminary results show the transition from a structure to two strictly symmetric cells for a value of the Richardson number equal to 15.6 to two asymmetric cells structure for a value of the same number equal to 15.7.

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