Impact of Partial Slip on MHD Non-aligned Stagnation Point Flow of Casson Fluid over a Stretching Sheet

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BACKGROUND

Almost all fluids used in industry disobey Newtonian law of viscosity. Therefore, study of non-Newtonian fluid flow is of supreme importance due to their inevitable presence in petrochemical industries, food and beverage processing and biotechnology. Casson fluid is one of the non-Newtonian fluids whose characteristics include human blood, jelly, honey, fruit juice with fibres etc. Flows past a stretching sheet are highly regarded considering various physical situations for both Newtonian and non-Newtonian fluids (Cortell, 2005). Stagnation point flows along with heat transfer aspect are quite evident in paper production, rotating filaments, crystal puffing, continuous moulding and melt spinning process. Keeping in view the importance of such study, many researchers have studied the flow near stagnation point considering various aspects of the problem (Yian et al., 2007; Pop et al., 2010; Khan et al., 2016; Jalilpour et al., 2017; Tabassum et al., 2018).

OBJECTIVES

The aim of the present investigation is to discuss the mixed convection, thermal radiation and partial slip effect on Casson fluid flow over a stretching convective surface in the presence of magnetic field when the fluid strikes the wall in an oblique manner.

PROBLEM FORMULATION

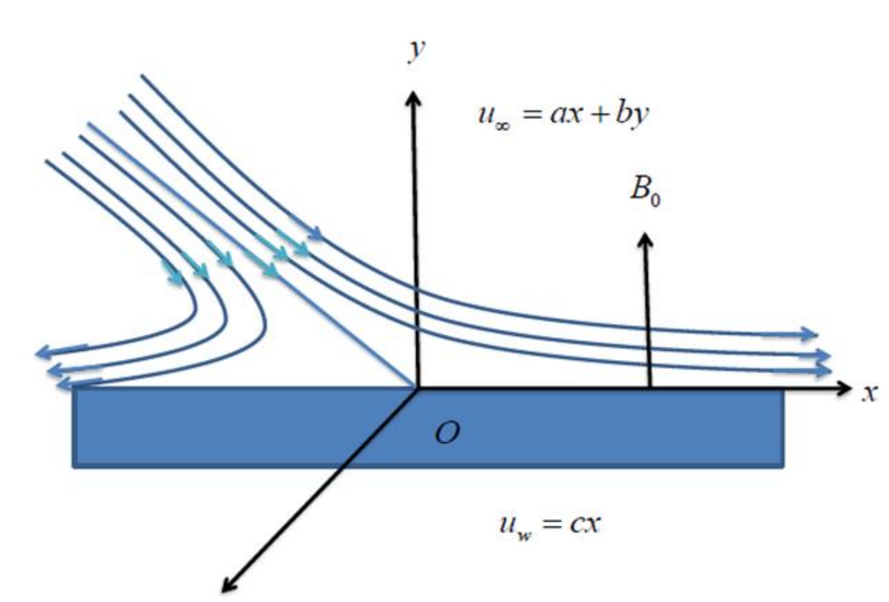

Fig. 1: Schematic diagram of the physical problem

The equations of conservation of mass, momentum and energy in the presence of magnetic field and thermal radiation and using Rosseland approximations, can be expressed as

$$\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}=0, \tag{1}$$

$$\rho\left(u^*\frac{\partial u^*}{\partial x}+v^*\frac{\partial u^*}{\partial y}\right)=-\frac{\partial p^*}{\partial x}+\mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u^*}{\partial x^2}+\frac{\partial^2 u^*}{\partial y^2}\right)-\sigma B_0^{\,2}u^*+g\left(\rho\beta\right)\left(T^*-T_\infty\right), \tag{2}$$

$$\rho\left(u^*\frac{\partial v^*}{\partial x}+v^*\frac{\partial v^*}{\partial y}\right)=\frac{\partial p^*}{\partial y}+\mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 v^*}{\partial x^2}+\frac{\partial^2 v^*}{\partial y^2}\right), \tag{3}$$

$$\left(\rho C_p\right)\left(u^*\frac{\partial T^*}{\partial x}+v^*\frac{\partial T^*}{\partial y}\right)=k\left(\frac{\partial^2 T^*}{\partial x^2}+\frac{\partial^2 T^*}{\partial y^2}\right)-\frac{16\sigma^* T_\infty^{*3}}{3k^*}\frac{\partial^2 T^*}{\partial y^2}, \tag{4}$$

The boundary conditions for this problem are given by

$$u^*=cx+N\left(1+\frac{1}{\beta}\right)\mu\frac{\partial u^*}{\partial y};\ v^*=0;\ \ -k\frac{\partial T^*}{\partial y}=h_f\left(T_f-T^*\right)\ \text{at}\ \ y=0, \tag{6a}$$

$$u^*=ax+by;\ T^*\to T_\infty \quad \text{as} \quad y\to\infty. \tag{6b}$$

We use the following scaling transformation

$$\left.\begin{aligned}&\xi=x\sqrt{\frac{c}{\upsilon}},\ \eta=y\sqrt{\frac{c}{\upsilon}},\ u=\frac{u^*}{\sqrt{c\upsilon}},\ v=\frac{v^*}{\sqrt{c\upsilon}},\\ &p=\frac{p^*}{(\rho\upsilon)c},\ T(\eta)=\frac{T^*-T_\infty}{T_f-T_\infty}.\end{aligned}\right\} \tag{7}$$

Now using the procedure given in Jalipour et al. (2017), we get the final equations in non-dimensional form as

$$\left(1+\frac{1}{\beta}\right)f'''+\left(ff''-f'^2+\lambda_1^2\right)-M\left(f'-\lambda_1\right)=0, \tag{8}$$

$$\left(1+\frac{1}{\beta}\right)h''+\left(fh'-f'h-A\right)-M\left(h-\lambda_2\right)+\frac{\gamma}{\lambda_2}\theta=0, \tag{9}$$

$$\theta''+\mathrm{Pr}_{eff}\ f\theta'=0 \tag{10}$$

The corresponding boundary conditions take the form

$$f(0)=0,\ \ f'(0)=1+L_1\left(1+\frac{1}{\beta}\right)f''(0),\ h(0)=L_1\left(1+\frac{1}{\beta}\right)h'(0),$$
$$\theta'(0)=-Bi\left(1-\theta(0)\right), \tag{11a}$$

$$f'(\infty)=\lambda_1,\ h'(\infty)=\lambda_2,\ \theta(\infty)=0. \tag{11b}$$

SUMMARY

Non-aligned stagnation point flow of Casson fluid in the presence of magnetic field has been investigated. Effect of Mixed convection, partial slip and thermal radiation are also taken into account. The profiles of fluid velocity and temperature are plotted and analysed corresponding to various pertinent flow parameters such as magnetic parameter, stagnation parameter, mixed convection parameter, velocity slip parameter, radiation parameter, Biot number etc. Also, rate of heat transfer at the surface are computed and explained in detail. For positive and negative obliqueness, stream line patterns are also plotted.

It is found that, streamlines are tilted towards the left side of the origin for positive shear rate and it falls on the right side when the value of shear rate is taken to be negative.

NUMERICAL SOLUTION

Eqs. (8) to (10) are coupled non-linear differential equations which are numerically solved by making use of shooting technique via Runge-Kutta-Fehlberg method. Suitable finite value of $\eta\to\infty$ say η_∞ is taken as $\eta_\infty\approx 10$. The step size $\Delta\eta=0.001$ is issued to obtain the numerical solution. Suitable guesses values of $f''(0)$, $h'(0)$ and $\theta'(0)$ are choosen and the integration is carried out. Our results for $f''(0)$ and $h'(0)$ agree excellently with the numerical results presented in Pop et al. [2010] (see Table 1).

Default values for parameters are taken as

$$\mathrm{Pr}=6.2,\ R=2,\ \gamma=0.5,\ \lambda_1=0.1,\ \lambda_2=1,\ \beta=0.5,\ L_1=0.5,\ Bi=0.5.$$

RESULTS

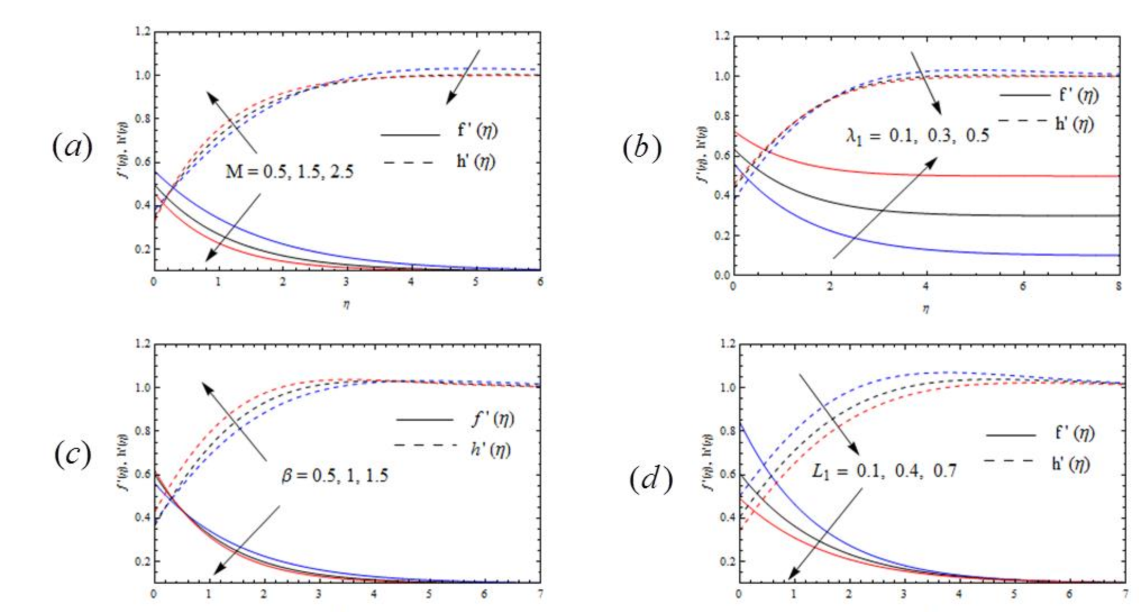

Fig.2. Normal and Tangential velocity profiles $f'(\eta)$ and $h'(\eta)$ for different values of (a) magnetic parameter M, (b) stagnation parameter λ_1, (c) Casson parameter β and (d) slip parameter L_1.

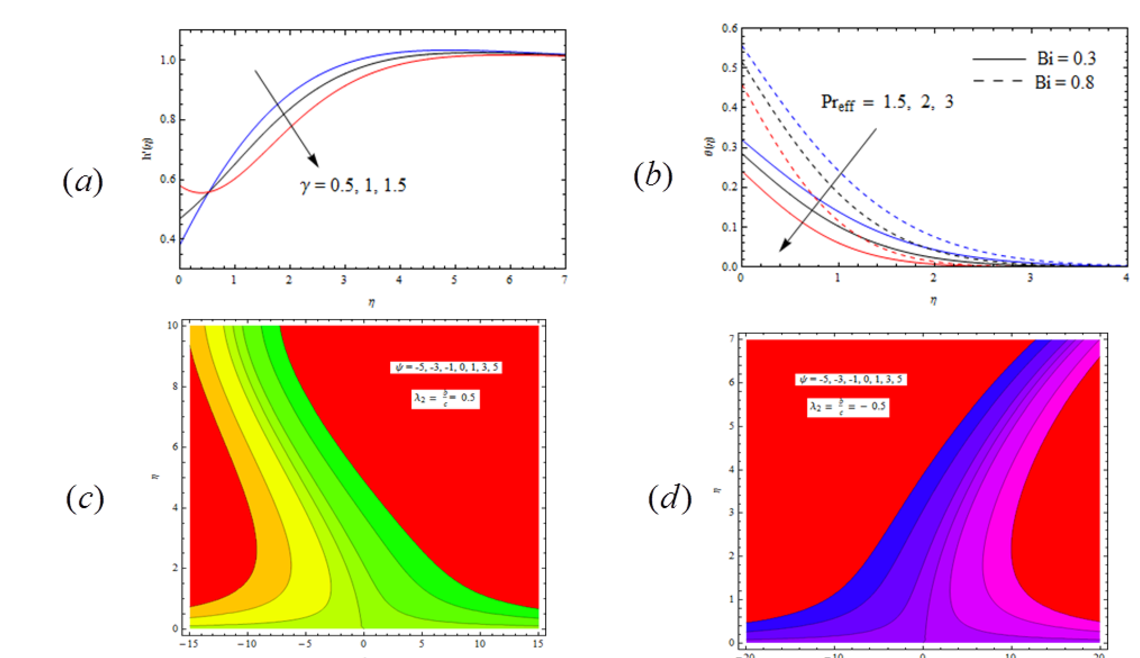

Fig.3. (a) Tangential velocity profiles for mixed convection parameter γ, (b) Temperature profiles for effective Prandtl number $\mathrm{Pr}_{\mathrm{eff}}$ and Biot number Bi, (c,d) streamline plot for obliqueness parameter $\lambda_2=0.5$ *and* $\lambda_2=-0.5$.

Table 1. Comparison of present results with previous published results when $M=\gamma=L_1=0$ and $\beta\to\infty$

λ_1	Pop et al. [2010]		Present Results	
	$f''(0)$	$h'(0)$	$f''(0)$	$h'(0)$
0.1	- 0.96938	0.26278	- 0.9693846	0.2630140
0.3	- 0.84942	0.60573	- 0.8494187	0.6062142
1	0.00	1.00	0.00	1.000000
2	2.01750	1.16489	2.0175016	1.1650280

Table 2. Nusselt Number variations:

Pr_{eff}	Bi	$-(1+R)\theta'(0)$
2	0.5	0.900807
1.5	0.5	1.121080
3		0.653626
2	0.3	0.643269
	0.8	1.162635

CONCLUSIONS

The important findings of this investigation are summarised below :

- Magnetic field, stagnation parameter and mixed convection parameter accelerates the tangential component of velocity near the sheet but opposite behaviour is observed far away from the sheet.
- The impact of Casson parameter and slip parameter on both the normal and tangential components of velocity are same.
- Fluid temperature intensifies when thermal radiation parameter and Biot number augments.
- There exist a flow separation on increasing values of stagnation parameter.
- Thermal radiation and Biot number enhance the rate of heat transfer.

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