Dynamical Pauli-Villars Regularization

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Introduction. Pauli-Villars regularization

In particle physics we have deal with expressions which contain singular functions. The simplest example is the Green function

$$<0|T\varphi(x)\varphi(x')|0>=G(x-x')=\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-x')}}{k^2+m^2}$$

which may be used to define the vacuum value of the stress-energy tensor $T_{\mu\nu}(x)=\partial_\mu\varphi(x)\partial_\nu\varphi(x)-\frac{1}{2}g_{\mu\nu}\left(\partial_\sigma\varphi(x)\partial^\sigma\varphi(x)-m^2\varphi(x)^2\right)$ as the limit

$$<0|T_{\mu\nu}(x)|0>==\lim_{x\to x'}\left[\partial_\mu\partial_{\nu'}-\frac{1}{2}g_{\mu\nu}\left(\partial_\sigma\partial^{\sigma'}-m^2\right)\right]G(x-x')\to\infty.$$

Pauli-Villars regularization [1,2] suggests the formal way of removing divergences from such expressions. It is achieved by means of introducing auxiliary masses into the Green function as follows

$$\frac{1}{k^2+m^2}\to\frac{1}{k^2+m^2}\frac{{M_1}^2}{k^2+{M_1}^2}\frac{{M_2}^2}{k^2+{M_2}^2}\cdots$$

If we introduce sufficient number of auxiliary masses, then this defines formally finite values for all analogous expressions, e.g.,

$$<0|T_{\mu\nu}(x)|0>_{reg}=\frac{1}{4}g_{\mu\nu}\int\frac{k^2+2m^2}{k^2+m^2}\frac{M_1^2}{k^2+{M_1}^2}\frac{M_2^2}{k^2+{M_2}^2}\cdots\frac{d^4k}{(2\pi)^4}=g_{\mu\nu}\Lambda_{reg}(M_1,M_2,..).$$

In final calculations one considers the limit when all auxiliary masses tend to infinity. Then, if in a given field theory all infinities can be absorbed into a finite number of constants, the theory is said to admit renormalization. Quantum gravity is not the renormalizable theory. This means that every new order in the perturbation theory introduces a new set of constants whose total number is infinite.

Virtual Wormholes as generators of auxiliary masses

We suggest the mechanism which generates additional massive excitations (extremely heavy particles) for all kinds of fields [3]. The auxiliary masses acquire the status of real physical particles.

Figure 1 illustrates the basic idea. Wormhole throats have the Planck size and the energy of particles starts from Planck values. The number of wormholes and of auxiliary masses can be an arbitrary big.

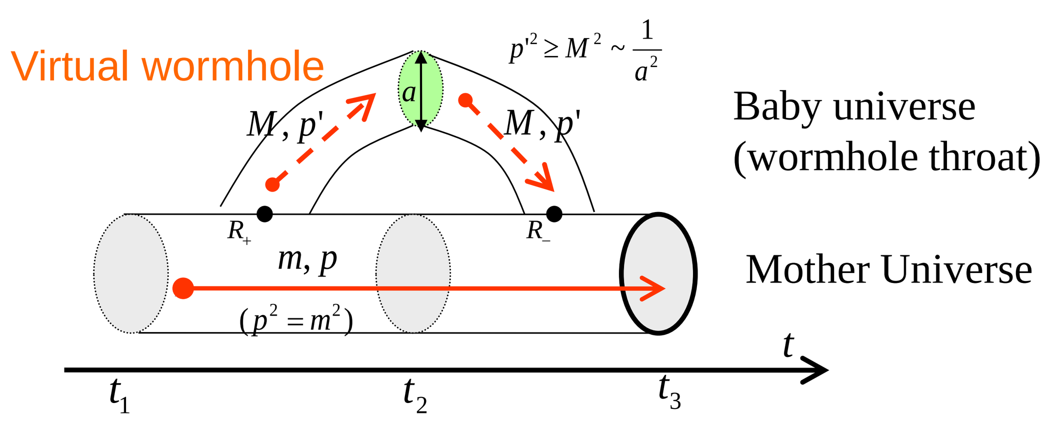

Figure 1 When particles pass through the wormhole throat, they acquire masses $M\sim 1/a$. Here a is the throat radius and $R_\pm$ are points where the baby joints to the Mother Universe.

Generating functional

Consider the linear theory (free particles) and add virtual wormholes in space. We add the dynamical value $\boldsymbol{F}$ which defines the number density of wormholes and their parameters. The action is

$$S_E=\frac{1}{2}(\varphi(-\Delta+m^2)\varphi)-(J\varphi).$$

The generating functional is given by the sum over wormholes $\boldsymbol{F}$ (topologies) and over fields as

$$Z_{tot}(J)=\sum_{F,\varphi}e^{-S_E}=\int[DF]Z_0(G)e^{\frac{1}{2}(JGJ)}=Z_{tot}(0)e^{W(J)},$$

$G=G_0+\delta G$ is the Green function for a given distribution $\boldsymbol{F}$ [4,5],

$G_0(x)=\frac{m^2}{4\pi^2}\frac{K_1(mx)}{mx}$ is the Green function in the absence of wormholes.

Function ***W(J)*** defines irreducible moments. Assuming ***J*** to be small and expanding ***Z(J)*** and ***W(J)*** in the series we find

$$W(J)=\frac{1}{2}<0|(JGJ)|0>+\frac{1}{8}<0|(J\Delta GJ)^2|0>+\ldots$$

All this terms can be expressed via the moments of the wormhole density $\boldsymbol{F}$ in the configuration space as

$$<0|G|0>=G_0+\int\delta G(\xi)\rho(\xi)d\xi,$$

$$<0|\Delta G_1\Delta G_2|0>=\int\delta G_1(\xi_1)\delta G_2(\xi_2)\rho(\xi_1,\xi_2)d\xi_1d\xi_2,$$

where $\rho(\xi)=<0|F(\xi)|0>,$

$$\rho(\xi_1,\xi_2)=<0|\Delta F(\xi_1)\Delta F(\xi_2)|0>=\rho(\xi_1)\delta(\xi_1-\xi_2)+\lambda(\xi_1,\xi_2),$$

and we denote $\Delta F(\xi)=F(\xi)-\rho(\xi).$

ξ labels the set of parameters of a single wormhole.

In the approximation of a rarefied gas of wormholes $\lambda(\xi_1,\xi_2)\approx 0$ and we find

$$W=W_0(J)+\int\left(\frac{1}{2}(J\delta G(\xi)J)+\frac{1}{8}(J\delta G(\xi)J)^2\right)\rho(\xi)d\xi+\ldots$$

Standard free field

scattering of particles on nontrivial topology

Nonlinearity = self-interactions

Different terms here reflect modification of the free particles propagation and correspond to different processes.

Effective action

To analyze the modification we consider the effective action. Let us define the vacuum classical field

$$\varphi(J)=<0|\varphi|0>_J=\frac{\partial W}{\partial J}$$

$$\varphi(J)=G_0J+\int\delta G(\xi)J\rho(\xi)d\xi+\ldots$$

resolve with respect to $J(\varphi)$

and define the effective action by the Legendre transformation

$$\Gamma(\varphi)=(J\varphi)-W(J(\varphi)).$$

Using the first monopole term of the exact Green function [4]

$$\delta G^1=-2\pi^2a^2\gamma(L)(G_0(x-R_+)-G_0(x-R_-))(G_0(x'-R_+)-G_0(x'-R_-))$$

where ***a*** is the throat radius, ***L*** is the length of neck, and $R_\pm$ are positions of wormhole entrances in space, we find

$$\Gamma(\varphi)=\frac{1}{2}(\varphi(-\Delta+m^2)\varphi)+V_1(\varphi)+V_2(\varphi)+\ldots$$

Effective potential

$$V_1(\varphi)=\pi^2\int a^2(\varphi(R_+)-\varphi(R_-))^2\rho(\xi)d\xi$$

Defines renormalization of the Green function

scattering of particles on virtual wormholes

$$V_2(\varphi)=-\frac{\pi^4}{2}\int a^4(\varphi(R_+)-\varphi(R_-))^4\rho(\xi)d\xi$$

self-interactions

Effective Action

Expanding the effective potential in series we find

$$\Gamma(\varphi)=\int\left[\frac{1}{2}\varphi\left(-\Delta\left(\alpha+\frac{\Delta}{M_1^2}\right)+m^2\right)\varphi-\frac{(\Delta\varphi)^4}{4!M_2^4}\right]d^4x+\ldots$$

where parameters are $\alpha=1+\frac{\pi^2}{2}n<a^2(R_+-R_-)^2>,$

$$\frac{1}{M_1^2}=\frac{\pi^2}{16}n<a^2(R_+-R_-)^4>,\quad\frac{1}{M_2^4}=\frac{\pi^4}{6}n<a^4(R_+-R_-)^4>\qquad etc$$

n is the density of wormholes, and <x> is the mean value.

Linear part of the effective action

The characteristic size of a wormhole throat has the Planck value and so do the values of masses. Let us set m=0, then we get

quadratic part

$$m=0\quad\Gamma_0(\varphi)\approx\frac{1}{2}\int\left(\alpha k^2-\frac{1}{M_1^2}k^4+\frac{1}{M_3^4}k^6+\ldots\right)|\varphi_k|^2d\tilde{k}\qquad k\geq M_{P1}>>m$$

Neglect terms higher then k^6

$$\Gamma_0(\varphi)\approx\frac{1}{2}\int k^2\frac{(k^2+M_3^2\omega)(k^2+M_3^2\omega*)}{M_3^4}|\varphi_k|^2d\tilde{k}$$

Has the structure of Pauli-Villars action

$$\Gamma_0(\varphi)=\frac{1}{2}\int\varphi(x)(-\Delta+m^2)\prod_s\left(\frac{-\Delta+{M_s}^2}{{M_s}^2}\right)\varphi(x)d^4x$$

General structure of the PV action

In general the number of masses is infinite.

Conclusion

Virtual wormholes
generate
Dynamical Pauli-Villars Regularization.

- Scattering of particles in vacuum on virtual wormholes generates additional very heavy particles and fields. Those play the role of auxiliary masses and fields in the Pauli-Villars scheme.
- Auxiliary masses are very big but have finite values and this makes Quantum Field Theory to be finite (free from divergences).
- Virtual wormholes generate additional self-interaction between particles.

Literature Cited

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